Date 10/4

まず $\frac{\partial f}{\partial x}(x,y)=\frac{\partial f}{\partial y}(x,y)=0$ → xを固定 → yを固定

report問題

次の関数の極値を求めよ

(1) $f(x,y)=(x^2+y^2)^2-2(x^2-y^2)$

(2) $f(x,y)=xy(x^2+y^2-1)$

(3) $f(x,y)=(y-x^2)(y-4x^2)$

$$\frac{\partial f}{\partial x}(x,y)=\frac{\partial f}{\partial y}(x,y)=0 \quad \leftarrow \text{必要条件}$$

xで極大値 $\Leftarrow f'(x)=0$ かつ $f''(x)<0$

xで極小値 $\Leftarrow f'(x)=0$ かつ $f''(x)>0$

命題

(x,y)で極大値 $\Leftrightarrow$ 任意の $\forall(a,b)\in\mathbb{R}^2$ $t\in\mathbb{R}\longmapsto f(x+at,y+bt)\in\mathbb{R}$

$$\varphi_{a,b}(t)=f(x+at,y+bt)$$

$$\varphi'_{a,b}(t)=a\frac{\partial f}{\partial x}(x+at,y+bt)+b\frac{\partial f}{\partial y}(x+at,y+bt)$$

$\varphi'_{a,b}(0)=0$ となるためには $\frac{\partial f}{\partial x}(x,y)=\frac{\partial f}{\partial y}(x,y)=0$

$$\varphi''_{a,b}(t)=a\left\{a\frac{\partial}{\partial x}\frac{\partial f}{\partial x}(x+at,y+bt)+b\frac{\partial}{\partial x}\frac{\partial f}{\partial y}(x+at,y+bt)\right\}$$
$$+b\left\{a\frac{\partial}{\partial x}\frac{\partial f}{\partial y}(x+at,y+bt)+b\frac{\partial}{\partial y}\frac{\partial f}{\partial y}(x+at,y+bt)\right\}$$
$$=a^2\frac{\partial^2 f}{\partial x^2}(x+at,y+bt)+ab\frac{\partial^2 f}{\partial x\partial y}(x+at,y+bt)$$
$$+ab\frac{\partial^2 f}{\partial x\partial y}(x+at,y+bt)+b^2\frac{\partial^2 f}{\partial y^2}(x+at,y+bt)$$

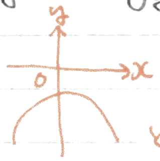
$$\varphi''_{a,b}(0)=a^2\frac{\partial^2 f}{\partial x^2}(x,y)+2ab\frac{\partial^2 f}{\partial x\partial y}(x,y)+b^2\frac{\partial^2 f}{\partial y^2}(x,y)<0$$

2次方程式っぽい。 $b\neq0$として b^2で割る

$$\frac{\varphi''_{a,b}(0)}{b^2}=\boxed{\left(\frac{a}{b}\right)^2\frac{\partial^2 f}{\partial x^2}(x,y)}+2\cdot\boxed{\frac{a}{b}}\frac{\partial^2 f}{\partial x\partial y}(x,y)+\frac{\partial^2 f}{\partial y^2}(x,y)<0$$

$\frac{a}{b}$の2次方程式

$\rightarrow \frac{\partial^2 f}{\partial x^2}(x,y)<0$



となればよい

$b^2-ac<0$ っぽくすればよいので

$$\left(\frac{\partial^2 f}{\partial x\partial y}(x,y)\right)^2-\frac{\partial^2 f}{\partial y^2}(x,y)\cdot\frac{\partial^2 f}{\partial x^2}(x,y)<0$$

$$f(x,y)=(x^2+y^2)^2-2(x^2-y^2)$$

$$\frac{\partial f}{\partial x}=4x^3+4xy^2-4x=0$$

$$\frac{\partial f}{\partial y}=4y^3+4x^2y-4y=0$$

$$\frac{\partial}{\partial x}\frac{\partial f}{\partial x}=\frac{\partial^2 f}{\partial x^2}$$

$$\frac{\partial}{\partial y}\frac{\partial f}{\partial y}=\frac{\partial^2 f}{\partial y^2}$$