Characteristic Analysis of Android Smartphone Applications Based on Usage Patterns

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Abstract

Along with the wide spread of mobile devices to access the Internet from anywhere at any time, the smartphone application market has been growing with amazing speed. The specific areas concerned with smartphone application market such as application popularity, usage patterns and the like are attracting the interest of researchers around the world. However, due to practical limitations of acquiring sufficient amount of data, the studies on smartphone application usage by individual users have been rather limited in the literature. Existing limited literature mostly rely upon actual data collected from either a limited number of volunteers or the network level information at the sacrifice of details of individual users. In this work, we employ a massive amount of actual smartphone application usage data acquired via a joint research project with a Japanese software development company named Fuller, Inc. Based on this massive data acquired from Fuller, Inc., this research investigates smartphone application characteristics in different aspects such as application downloads, application usage patterns and product lifecycle concerned with smartphone applications. We introduce five key competitive performance measures for assessing smartphone applications and establish a novel statistical approach for estimating the values of those key competitive performance measures. Further, we develop segmentation criteria for classifying smartphone applications based on the concept of the product lifecycle. The proposed ideas and the findings of this work will provide useful business implications of interest to application developers.

Initial chapters of this thesis discuss about the necessary background information for the work. Next, we introduce application performance measures based on the monthly application download information. However, considering the distinct behaviors of the individual users, in the next section we introduce ten application usage patterns considering a

six month period of time. More specifically, we define a usage pattern $up_j(a, I(t))$ for application a considering a six month period specified by $I(t) = \{\, t-5, t-4, \cdots, t \,\}$, where t is the month under consideration. We re-define the application performance measures based on the usage patterns, in order to verify the suitability of using the usage patterns to describe smartphone application characteristics. In the following chapters we describe a novel statistical approach for estimating the application performance measures by optimal convex combination of ARIMA models and the linear regression approach. In the final section we present the smartphone applications segmentation criteria developed based on the concept of the product lifecycle. More specifically, we put together the application usage patterns to form three groups: Group a, Group b and Group c. We then employ the three groups to classify smartphone applications into three categories based on the concept of the product lifecycle: Class A for smartphone applications that are active in lifecycle; Class B for smartphone applications that are in the tail of the lifecycle; and Class C for smartphone applications that are already diminishing. We then develop a discrete time Markov chain model where the Markov chain has the state space $\{a, b, c, d\}$ with d being diminished from the market. We develop a numerical procedure for computing the survival functions of the first passage times $T_{a\rightarrow d}$ and $T_{b\rightarrow d}$ and a quantifiable criteria for classifying smartphone applications into Class A, Class B and Class C. All the proposed approaches are supported by the numerical examples exhibiting the appropriateness of the idea.

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Notation

$\boldsymbol{A}$	Whole set of applications under consideration
$\boldsymbol{D}$	Whole set of devices under consideration
$\boldsymbol{T}$	Period of time
$\boldsymbol{S}$	Application sub-category under consideration
$\boldsymbol{A}(\boldsymbol{S})$	Set of applications in the considered application sub-category
$\boldsymbol{DL}(\boldsymbol{A}(\boldsymbol{S}))$	Set of devices which have downloaded any application in $A(S)$
$\boldsymbol{DL}(\boldsymbol{a},\boldsymbol{k})$	Set of devices which have downloaded application $a \in A(S)$ in month $k \in T$
$\boldsymbol{D}(\boldsymbol{A}(\boldsymbol{S}))$	Set of devices having any application in $A(S)$
$\boldsymbol{b}_{\boldsymbol{DL}}(\boldsymbol{a},\boldsymbol{d},\boldsymbol{k})$	Download of application $a \in A(S)$ on device $d \in D(S)$ in month $k \in T$
$\boldsymbol{b}(\boldsymbol{a},\boldsymbol{d},\boldsymbol{k})$	Presence of an application a in device d in month k
$\boldsymbol{sign}(\boldsymbol{a},\boldsymbol{d},\boldsymbol{k})$	The switch of the status of application a in device d from month $k-1$ to month k where, $k \neq 1$
$\boldsymbol{UP}(\boldsymbol{a},\boldsymbol{t})$	The set of usage patterns
$\boldsymbol{UP}_{\boldsymbol{j}}(\boldsymbol{a},\boldsymbol{t})$	The set of devices having the usage pattern j for application a in month t
$\boldsymbol{I}(\boldsymbol{t})$	Six month period ending in month $t \in T$. i.e. $I(t) = \{t-5, t-4, \dots, t\}$
$\boldsymbol{D}(\boldsymbol{a},\boldsymbol{k})$	Set of devices having application $a \in A$ in month $k \in T$
$\boldsymbol{D}(\boldsymbol{a},\boldsymbol{I}(\boldsymbol{t}))$	The set of devices having application $a \in A(S)$ in at least one month during the period $I(t) \subseteq T$
$\boldsymbol{D}(\boldsymbol{A}(\boldsymbol{S}),\boldsymbol{k})$	The set of devices having some applications $a \in A(S)$ in month k
$\boldsymbol{D}(\boldsymbol{A}(\boldsymbol{S}),\boldsymbol{I}(\boldsymbol{t}))$	The set of devices having some application $a \in A(S)$ in at least one month during the period $I(t) \subseteq T$

$\mathcal{N}$	A state space
$\boldsymbol{N(k)}$	A stochastic process at time k
$\boldsymbol{a_{ij}(k)}$	Transition from state i to state j at time k
$\boldsymbol{n_{ij}}$	Number of devices moving from i to j
$\boldsymbol{T_{G \to d}}$	Minimum time taken to move from state G to state d for the first time
$\boldsymbol{\bar{S}_{G \to d}(k)}$	Survival function of $T_{G \to d}$

Chapter 1

Introduction

1.1 Background

With the rapid development of mobile technologies, smartphones have become an essential equipment in most people's life, serving not only the communication needs, but also requirements such as entertainment, business, knowledge gathering and so on. According to (Gartner Inc., 2016), global smartphone sales totaled 349 million in the first quarter of 2016. Further, (eMarketer, 2014) forecasts that the worldwide smartphone users would surpass 2.5 billion by the year 2018. Similar situation can be observed in Japan with 55.8 million smartphone users, which is around 44% of its population (eMarketer, 2016). The smartphone market is highly influenced by two leading Operating systems Android and Apple OS where they possess 65% and 34% of market share in Japan respectively (KantarWorldpanel, 2016).

The practical value of a smartphone lies not only on the value of the equipment, but also on the applications installed in it, for making the life of the user much easier and productive. In the recent years the smartphone application market has been growing with an amazing speed. As of (Statistica, 2016), the number of smartphone applications available in online app stores exceeded 2 million for both Android and Apple OS. This rapid growth of the smartphone population has been sustained by a variety of enriched applications, covering such areas as Finance, Businesses, Communication, Music, Movies, Sports, Dictionaries, Education and Games. As of ("App Brain," 2016), by the end of the year 2016 'Google Play' the official online app-store for Android smartphone applications hosted 2.6 million Android applications under 33 application categories and 17 sub-categories of Games.

Typical interactions between smartphone applications and a smartphone device can be listed as application download or installation in the device, application launch, application version update, uninstallation of the application and reinstallation of the application. These interactions can take place dynamically and in a repeated manner and are generally independent of other applications and devices.

Smartphones and the related ecosystems has become a new field of interest where smartphone application usage holds a greater importance. However, the amount of work carried-out in this area has been largely limited mainly due to the difficulty of obtaining actual information on smartphone application usage. Accordingly, the study of the relevant markets is quite limited in the literature. In our work, we had a rare opportunity of having access to a large volume of information related to Android smartphone applications in Japan. This information was obtained via a joint research project between the University of Tsukuba and a Japanese software development company named Fuller, Inc. This special dataset is consisted of information on how smartphone applications were downloaded, used and removed in individual smartphone devices along the time axis. The data collection has been performed by the battery-saver smartphone application named “Mr. Mobile, the battery saver” developed by Fuller, Inc. in 2012. This application enables the company to acquire the information about which smartphone applications are installed in and removed from the devices that are registered with agreement of customers. In exchange, the customers receive the free assessment of the level of energy consumption for each smartphone application in their device, along with recommendations concerning which idle applications should be removed to increase the battery life.

In this work, we focus on studying the characteristics of smartphone applications employing the actual smartphone application data. We introduce several key competitive performance measures for smartphone applications, which can explain smartphone application characteristics in different aspects. These competitive performance measures are discussed in terms of application downloads as well as by using newly introduced ten usage patterns. We develop statistical approaches to better estimate these key performance measures and introduce a classification algorithm for smartphone applications developed based on the concept of product lifecycle.

1.2 Purpose of this Thesis

With the rapid growth smartphone application market, it is important for the stakeholders of smartphone applications to have the capability of assessing the applications in the market. This may give one an insight on the current situation and trends in the smartphone application market and the ability of identify the characteristics of model applications that might enter and conquer the market in future. In this work, we perform analysis of smartphone application characteristics. In order to achieve this purpose, we introduce application usage patterns and key competitive performance measures for smartphone applications. Further, we propose a novel approach of estimating the key performance measures of smartphone applications, by considering the optimal convex combination of the estimations based on linear regression approach and the ARIMA models approach. We develop quantifiable criteria for classifying smartphone applications based on the concept of the product lifecycle, by developing a discrete time Markov chain model and a numerical procedure for computing the survival functions of the first passage times for moving from good states to diminished state in the lifecycle. In this thesis, we present the above with comprehension on development of the concept and validation of the concept by numerical examples.

The structure of the thesis is as follows. In this chapter, we provide a succinct summary of the entire thesis for the ease of understanding. In Chapter 2, literature relevant for this work is presented. Chapter 3 introduces the dataset employed to construct the proposed concepts in Chapters 4 though Chapter 8. Chapter 4 introduces two key performance measures based on the smartphone application downloads on individual devices. In Chapter 5, we introduce a set of ten smartphone application usage patterns defined over a period of six months. By employing the usage patterns introduced in Chapter 5, we introduce five key competitive performance measures in Chapter 6. In Chapter 7, we construct statistical models for the five performance measures and propose a statistical approach based on linear regression method and ARIMA (Auto Regressive Integrated Moving Average) models for estimating them accurately. Next, in Chapter 8 we present an algorithm to classify smartphone applications based on the concept of product lifecycle. Finally we provide concluding remarks in Chapter 9.

The contents of Chapter 4 is published as a journal paper in (U Perera, Dewi, Sari, & Sumita, 2014). Chapters 5, 6 and 7 are based on (Umesha Perera, Shigeno, Sumita, & Yamamoto, 2016).

Chapter 2

Literature Survey

2.1 Smartphone Applications

In this section we discuss about the literature relevant to smartphone applications and the related areas such as application usage, usage patterns, performance measures and the like.

With the rapid growing smartphone and smartphone application markets, interest has grown on different aspects related to smartphone and smartphone application usage. By investigating into how smartphone applications are being used by different users over a stretch of time, it is possible to understand various characteristics of the applications, which will convey useful business implications. However, it is quite difficult for researchers to obtain real data concerning how such applications are downloaded and removed by users along the time axis in a consistent manner. Hence, the study of the relevant markets has been quite limited and relying upon a limited set of usage data collected by a small number of volunteers. Some of the researchers have overcome the difficulty of acquiring actual data on application usage and management by the users by collecting detailed usage information of mobile devices from a limited number of volunteers. One example of this is the study carried-out by (Verkasalo, López-Nicolás, Molina-Castillo, & Bouwman, 2010) on behavioral factors driving users for using or not using smartphone applications by considering usage and non-usage of three specific smartphone applications of 579 users. In the same year, (Falaki et al., 2010) carried out another analysis on smartphone application usage based on a set of real data collected from 255 smartphone users from two different smartphone platforms for a period of 7 to 28 weeks focusing on the network traffic and energy drain. Using application usage events, location data and blue tooth data of 77 smartphone users for a period 9 months on a set of applications, (Minh, Do, Blom, & Gatica-perez, 2011) carried out an analysis to understand the application usage in the contexts of location and proximity. However, some drawbacks one can note in above literature are the limitations of sample size and the data period. To improve smartphone performance and user experience, (Chang, Qi, Enhong, & Hui, 2012) proposed a mechanism of Prediction Algorithm with Fixed Cycle Length (PAFCL) to predict mobile user's application usage on the device, which was tested against a data set collected from 38 volunteers.

An alternative approach for conquering the difficulty of collecting real data is to rely upon the network level information at the sacrifice of details of individual users. For example, (Xu et al., 2011) analyzed the diurnal patterns of different application types, network access patterns and their relations with the location based on IP level network data from a cellular network in US for one week time period. (Böhmer, Hecht, Schöning, Kr¨uger, & Bauer, 2011), carried-out a study on application usage by using tool to collect application usage information from 4100 users for 127 days. Using the information of mobile in-app advertisements in network traces for a large number of applications for two days, (Tongaonkar, Dai, Nucci, & Song, 2013) introduced a method of understanding mobile application usage patterns of Android applications. Here the application usage patterns refer to the diurnal patterns in usage of different application categories. Another example of this approach is given in (Li et al., 2015) which they carried out a research on application usage behaviors, using information obtained from a Chinese Android application marketplace from up to 2 million users for a period of one month. They study about information such as number of application downloads, number of unique devices downloading the application, aggregated data traffic generated by the application and the aggregated access time that users interact with the application to discuss about popularity distribution of applications, diurnal usage patterns, co-installed applications, app installation/uninstallation patterns and the network activity patterns. However, above literature are limited by the length of time of the data period.

Perhaps due to practical limitations in obtaining actual usage information on individual smartphone applications, some studies have focused on the more general level of usage; the usage of smartphone device by its users without focusing into details of application usage. One example is the study done by (Kang, Seo, & Hong, 2011) on smartphone usage pattern analysis with the interest of mobile and network traffic. In this study, the data has been collected from 20 users over a two month period by using a log data collection application. By defining 5 states of smartphone usage (waiting, voice calls, data communication via WiFi, data communication via 3G and Other) analysis has been carried-out in user level to understand their smartphone usage and they discuss the existence of own usage patterns for each user. In another study by (Karikoski & Soikkeli, 2013), by employing data collected from 140 Symbian smartphone users for average period of 134 days, investigates about the relationships between user context (Home, Office, Other meaningful, Elsewhere and Abroad) and the usage patterns in smartphone communication services such as voice calls, SMS and MMS, and mobile internet communication services such as email, instant messaging, VoIP and social media. Their findings are such that smartphone communication services usage differs depending on the use context of the people. In (Chmielarz, 2015), by collecting data via a survey on 314 university students, performed an analysis of characteristics of the users of smartphones and their opinion on the quantity and conditions of their usage in terms of the quality of applications and the convenience of using websites with mobile applications. However, these literatures are limited by the facts of sample size and by confining usage to the level of smartphone itself. In above literature, authors have tried to give an insight on smartphone/smartphone application usage patterns in a scope however limited by the number of users, individual user details or the duration of data period. Therefore the findings are more focused on quite small time periods. In our research we try to bridge the gaps mentioned above by employing actual smartphone application usage information from large number of individual smartphone users over period up to 40 months to study about the application usage patterns.

The performance measures of a certain entity may highly characterized by the type of the entity and there exist some literature on the development of performance measures in different fields of interest. One such example is (Wills, Mikhailov, & Shang, 2003) that propose a methodology to assess the relative popularity for any Internet application based on the data servers it uses. Another example is the study carried-out by (Olugu, Wong, & Shaharoun, 2011) on development of key performance measures for the automobile green supply chain. Another similar work is described in (Ahi, Searcy, & Jaber, 2016). However, the little literature existing related to performance measures of smartphone applications are mainly concerned with the smartphone application popularity or popularity of the online content. One example of such literature is the analysis presented in (Ratkiewicz, Fortunato, Flammini, Menczer, & Vespignani, 2010) on the dynamics of online content popularity in the

Wikipedia. Using user rating and comments of daily top 200 iTunes applications for the period of three months (Chen & Liu, 2011) proposes a mechanism to determine and predict application popularity based on the CART (Classification And Regression Tree) algorithm by "poking" the network requests local Domain Name servers (LDNSs). In (Borghol et al., 2011) presents a framework for studying the popularity dynamics of user-generated videos, using a dataset that tracks the views to a sample of recently uploaded YouTube videos and the viewing rate and total views distributions over time. Using the regression and classification algorithms, (Bandari, Asur, & Huberman, 2012) study the efficiency of properties of news articles to predict online popularity of News on Twitter. In (Yin, Luo, Wang, & Lee, 2012) uses Conformer-Maverick (CM) model to simulate the voting process for online content in iPhone application named 'JukeBox' and to rank the potentially popular items based on the early votes. In (Kawai, Murata, & Sumita, 2014) proposes an approach for understanding complementary relationships and countervailing relationships among smartphone applications. In a more recent research based on the information obtained from a Chinese Android application marketplace named 'Wandoujia,' (Li et al., 2015) define and measure the popularity of a smartphone applications simply by the number of application downloads, number of unique devices downloading the application, aggregated data traffic generated by the application and the aggregated access time that users interact with the application.

2.2 ARIMA Models Approach of Future Value Estimation

Time series is an area of exceptional growth in interest in numerous fields such as statistics, econometrics, finance, forecasting and so on for the value of practical relevance. It is concerned with analyzing sequences of data points having sorts of periodical order. One can employ time series analysis to excerpt meaningful attributes of the time series data, while time series forecasting enables to predict future values based on the historical values of those data. In this work, we employ a hybrid approach linear regression methods and of well-known concept in time series, ARIMA models. ARIMA refers to an integration of Auto Regressive (AR) models and Moving Average (MA) models. The Auto Regressive (AR) models were first introduced by (Yule, 1926). Then the Moving Average (MA) schemes were introduce by Slutsky in 1937 and these two models were combined in to Auto Regressive Moving Average (ARMA) models by Wold in 1938 (Makridakis & Hibon, 1995). In year

1976 (Box, Jenkins, Reinsel, & Ljung, 2015) introduced the ARIMA models by incorporating the integrated part. Since then ARIMA models have been in discussion and used in many areas of literature for predictions based on the historical values of time series data. It One such example is (Mcdonald, 1994) which investigated the forecasting performance of ARIMA models. In a study by (Kurawarwala & Matsuo, 1998) ARIMA models are employed for empirical validation, comparison and performance forecasting of the PC models. In (Babu & Reddy, 2014) proposes a combined approach of ARIMA and ANN (Artificial Neural Network) by experimenting on several datasets such as sunspot data, electricity price data, and stock market data. In (Hassan, 2014) regression methods as well as ARIMA models are employed separately to model hourly and daily measurements of total and diffuse solar radiation and sunshine duration. (Ramos, Santos, & Rebelo, 2015) compared the performance of the forecasting by state space models and ARIMA models using retain sales of women footwear. In the existing literature it is possible to find many instances of employing ARIMA models for future value estimation. Further, many researchers have tried to combine ARIMA models with other statistical approaches such as Artificial Neural Network (Nury, Hasan, & Alam, 2015), Adaptive Neuro Fuzzy Inference System (Barak & Sadegh, 2016), GARCH (Generalized AutoRegressive Conditional Heteroscedastic) model and so on. However, we found it difficult to find literature on hybrid approach of ARIMA models and linear regression methods which we use in our work.

2.3 Markov Chains Approach

Markov chains were first introduced by A. A. Markov in 1913 (Hayes, 2013) by investigating the patterns of text in Alexander Pushkin's novel called 'Eugene Onegin.' This was an opening to a new area of probability theory by applying mathematics to poetry and later applied in various fields such as information theory hidden Markov chains (Baum & Petrie, 1966), (Shannon, 1948), computer performance (Frenkel & Scherr, 1987), web search (Brin, Page, Motwami, & Winograd, 1999) and so on. In (Cao, Jiang, Pei, Chen, & Li, 2009) proposes an approach for context-aware search by using variable length Hidden Markov Model on log data of search sessions. (C.-H. Liu, Chiu, & Chiu, 2011) carried-out a value analysis of strategic partners in business using both Markov chain and Hierarchical Bayesian model. In (Bocchini, Saydam, & Frangopol, 2013) discusses the use of Markov chain model for the life-cycle analysis and risk analysis of bridges. In (Mizusawa, Sumita, & Takano,

2015) uses a combined approach of ARIMA models and Markov chains to predict the number of downloads smartphone applications. In much recent work by (Zhang et al., 2016) propose a framework to model users' mobile online behavior based on a multi-state model and a hidden Markov model. Although there is literature on various applications of Markov chains, to our best knowledge, little or no literature exists on the application of Markov chains for smartphone application lifecycle segmentation. From our work, we try to bridge this gap.

2.4 Product Lifecycle

The concept of Product Life Cycle (PLC) has been discussed heavily by various scientists in the past (Osland, 1991). The emergence of this concept in Marketing field traces many decades back in to the history to the days as early as in 1930's, where (Kleppner, 1933) discussed the three-staged phases the products go through; namely, 'Pioneering,' 'Competitive' and 'Retentive.' This concept was termed as 'Product Life Cycle' for the first time by (Dean, 1950). In (Forrester, 1959) provided the first graphical presentation of Product Life Cycle, with the four stages, 'Introduction,' 'Growth,' 'Maturity' and 'Decline.' In 1965 (Levitt, 1965) proposed the concept of managerial theory relating the PLC. In 1966, (Buzzell, 1966) carried-out one the first empirical experiments on the PLC concept employing sales data of food products over a period of 16 years, verifying the PLC with different stages and also identifying different types of 'Maturity Stages' in PLC. In the study by (Cox Jr. 1967), they identify six shapes of PLC, discussing that the conceptual distribution is not apparent in every product. Several other studies and discussions related to PLC can be seen later literature; (Bass, 1969), (Rink & Swan, 1979), (Polli & Cook, 1969) and (Day, 1986) to name some of the historical studies. However, it is also noticeable that all these research are relying upon ordinary consumer products and industrial products, perhaps owing to the difficulty of acquisition of data. Some of the recent studies such as (Peres, Muller, Mahajan, Mahajan, & Muller, 2010), (Norton & Bass, 1987) and (Kurawarwala & Matsuo, 1998) have been taken place on brand-levels and technological products such as semi-conductors, computer models. However, it is somewhat difficult to find literature on PLC related to Internet or online content except for (X. Liu, Jia, & Guo, 2011), where classical Product Life Cycle theory has been applied to mobile applications for Apple devices. In this work, researchers have employed publicly available daily download rank data as an indicator for applications' market popularity. Using regression methods, the correlations between Application Life Cycle and latent application features are examined for top 200 of most downloaded applications in various categories. However, the App Store information is not consisted of any information on the applications' actual usage or removal by the users, which may be a crucial factor in application life cycle characterization. In our work, bridge this gap by performing application life cycle analysis based on the generally unavailable actual application usage data.

Chapter 3

Data Description

This chapter describes the data employed for the research. In order to carry-out this work, we utilize large volumes of data related with Android smartphone applications in Japanese market. This data, obtained from three data sources; namely 1) Fuller, Inc., 2) Google Play and 3) AppBrain can be mainly classified into three groups.

(1) Application data

Application data which is available on Google Play contains basic information of individual Android smartphone applications. This includes basic information such as, Application ID, name, category, price, size and developer. Additionally, each application record is supplemented with current information such as Application rank, number of downloads, system requirements and the like holding values relevant to the time the data was extracted.

(2) Application usage data

This is a portion of mass volumes of information received from Fuller, Inc. Application usage data is comprised of information such as, application installations and removals on smartphone devices, presence of applications on the devices and counts of application launches on the devices.

(3) Application change log data

Application change log data refers to records on changes made to a specific application during its life cycle. It is consisted of such information as Initial release on the App-store, version updates, reach of benchmark number of installs (e.g. 100000, 500000, 1000000, …), category changes and un-publishing from App-store. This information is extracted from AppBrain per each considered application.

3.1 Data Structures

In this section, we explain the data structures of the three groups of data used in this research in detail. Application data is consisted of information on individual applications, and an example of typical records is presented in Table 3.1. While many of the table headings are explanatory, ‘Application ID’ is a uniquely identifiable string corresponding to each application and ‘Category’ represents the application category, or application sub-category when it is referring to a Game application. ‘Average User Rating’ is the rating given by the Google Play based on user votes. ‘Content Rating’ specifies the suitability of application’s contents for different age groups.

Table 3. 1 - Application Information

Application ID	Name	Category	Price	Description	Developer	Current Version	Downloads	Number of Votes	Average User Rating	Content Rating	File Size	System Requireme-nts
jp.naver.SJLINEPANG	LINE POP	BRAIN	0	Everyone's fa	NAVER	1.1.0	50000000	42596	3.5	1	16000	20200
com.mojang.minecraftp	Minecraft - Pocket Edition	ARCADE	555	Minecraft is ..	Mojang	Varies with device	5000000	69107	4.5	3	-1	-1
com.skype.raider	Skype	COMMUNICATION	0	Skype keeps t	Skype	3.1.0.6458	500000000	920473	4	3	15000	20100

Application usage data is received on monthly basis and maintained in separate tables as given in Table 3.2. Here, ‘Device ID’ is a uniquely identifiable string for the device. However, ‘Device ID’ provides no information which enables one to identify the device in reality and is different from the device’s serial number which can be used to physically distinguish the smartphone device. ‘Removed’ represents whether the application was removed from the device or not by the end of the month. More specifically, ‘1’ in the ‘Removed’ field corresponds to a device that the specific application being removed during the considered month. ‘Number of Application Launches’ provides the number of times the specific application has been launched by the specific device during the considered months. Number of days that the specific application has been launched at least once is given by ‘Number of Active Days.’

Table 3. 2 - Application Usage Information for a Specific Month

Device ID	Application ID	Removed	Number of Application Launches	Number of Active Days
50854252d02aa605a394f091	com.halfbrick.fruitninjafree	1	5	3
50e433eead4c767ee0e55043	jp.naver.SJLINEPANG	0	2	2
…				

For the convenience of the work we constructed the table structure given in Table 3.3 by combining all the monthly data tables of application usage. Here, 1 represents the specific application was present on the specific device in the given month, while 0 represents that the application was not present on the device throughout the given month.

Table 3. 3 - Monthly Application Usage Information over a *k* Month Period

Device ID	Application ID	Month_01	Month_02	…	Month_k
50854252d02aa605a394f091	com.halfbrick.fruitninjafree	1	1		0
50e433eead4c767ee0e55043	jp.naver.SJLINEPANG	0	0		1
…					

Application change log data lists changes made to single applications along the timeline. Given in Table 3.4 is a typical set of records relevant to a single application which has completed its life cycle. Records are listed in the table in the descending order of time.

Table 3. 4 - Application Change Log Data for an Individual Application

Application ID	Date	Change	Description
com.bushiroad.bushidora	Nov 29, 2015	Unpublished	
com.bushiroad.bushidora	…	…	…
com.bushiroad.bushidora	Mar 18, 2014	Category	Moved from Casual to Casual
com.bushiroad.bushidora	…	…	…
com.bushiroad.bushidora	Jun 6, 2013	Update	Version 1.06
com.bushiroad.bushidora	…	…	…
com.bushiroad.bushidora	Apr 9, 2013	Installs	100,000+ installs
com.bushiroad.bushidora	Apr 9, 2013	Update	Version 1.03
com.bushiroad.bushidora	Apr 5, 2013	Installs	50,000+ installs
com.bushiroad.bushidora	Apr 5, 2013	Update	Version 1.01
com.bushiroad.bushidora	Apr 1, 2013	Installs	10,000+ installs
com.bushiroad.bushidora	Mar 31, 2013	Installs	1,000+ installs
com.bushiroad.bushidora	Mar 29, 2013	New App	Version 1 in Casual for Free

In the study all these three groups of data are used together and they are linked through the key, ‘Application ID’ as depicted in the Figure 3.1.

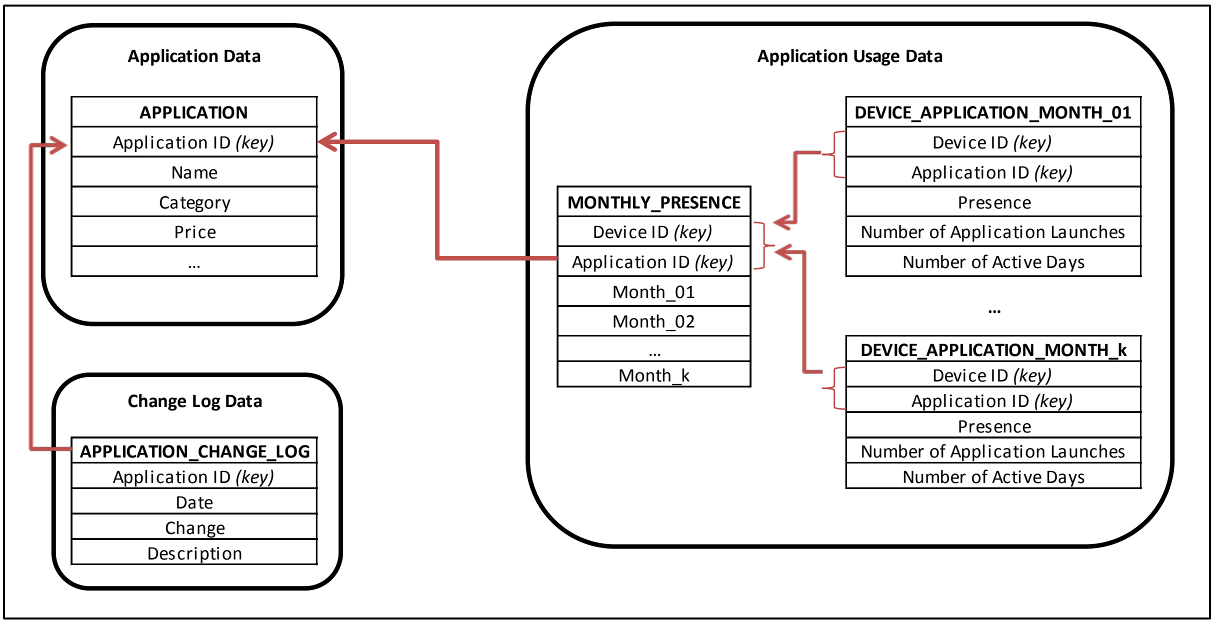


Figure 3. 1 - Data Structure

3.2 Size of the Whole Dataset

For the purpose of this research, we employed data of Android smartphone applications, application usage information and application change log information spanning over a period of 40 months from January 2013 to April 2016. The whole dataset contains information on 464174 Android smartphone applications under 25 application categories. The distribution of applications is as given in Figure 3.2. Here, the application categories are listed in the alphabetical order.

Application Category	Number of Applications
Books and Reference	17811
Business	8558
Comics	4169
Communication	8001
Education	24706
Entertainment	41445
Finance	6753
Games	135551
Health and Fitness	11453
Libraries and Demo	2305
Lifestyle	25596
Media and Video	9105
Medical	5129
Music and Audio	16733
News and Magazines	7615
Personalization	43522
Photography	11959
Productivity	11540
Shopping	5428
Social	9134
Sports	7457
Tools	32041
Transportation	4683
Travel and Local	10569
Weather	2911
Total	**464174**

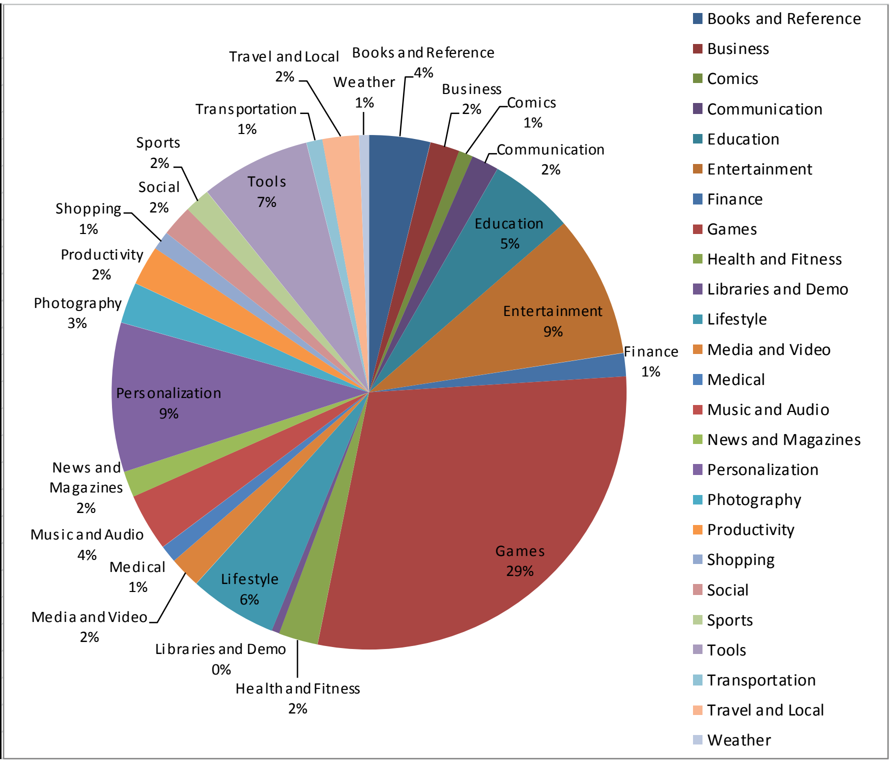


Figure 3. 2 - Distribution of Applications in the Dataset among the Application Categories

Because of the massive amount of data, we restrict our focus in to one category. As clearly seen in Figure 3.2, ‘Games’ constitute the largest of the total number of applications, having 29% of the total number of applications in the dataset. Among the game applications, 90% were found to be free applications, as given in Figure 3.3. Therefore, naturally focus on the ‘Free Games’ to proceed with this study.

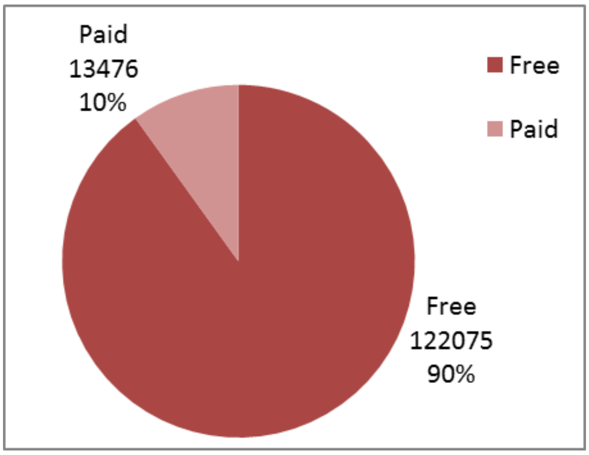


Figure 3. 3 - Distribution of Free and Paid Games

'Games' category is further divided into several sub-categories such as 'Action,' 'Arcade,' 'Card,' 'Puzzle' and so on. Figure 3.4 provides the distribution of 'Free Game' applications in the dataset among these sub-categories. Here, applications in the sub-category specified by 'Other' refer to the applications which had category change during the data period or the ones which had its category specified as 'Games' in raw data.

Game Sub-category	Number of Applications
Action	8297
Adventure	5453
Arcade	15160
Board	2108
Card	4008
Casino	2667
Casual	26320
Educational	4084
Music	1599
Puzzle	22655
Racing	5316
Roleplaying	3300
Simulation	7808
Sports	3913
Strategy	2553
Trivia	3145
Word	2409
Other	1280
Total	**122075**

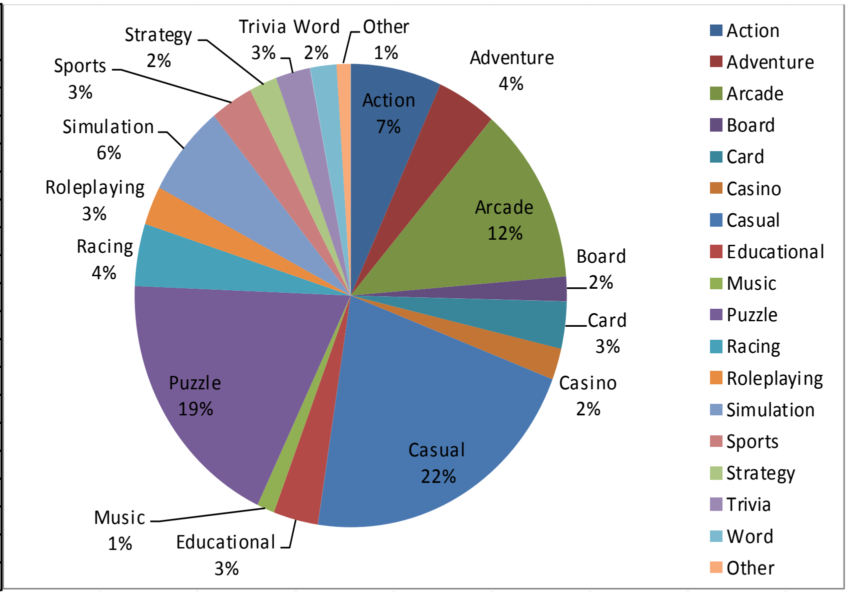


Figure 3. 4 - Distribution of Free Game Applications in the Dataset among the Game Sub-Categories

3.3 The Three Datasets

For the different areas of this research, we extract three sets of data from the whole dataset explained above, varied by the length of the data period. This is based on the progression of the work and the availability of data at time that the respective work take place. At the inception of this research, the availability of data was limited for only six months and rest of the data became available gradually with time. For each research problem, we employed the maximum available data of that time. For the consistency of the study and the reliability of the results, we employ suitable data selection criteria for each dataset. Only those applications satisfying the conditions are chosen for further analysis. By this we expect to exclude the applications with too small user-base, which might erroneously affect the general models. The three datasets are as described below:

3.3.1 Dataset 1

Dataset 1 is dedicated for the work discussed in Chapter 4, and it is extracted from the data available at the inception of the research. It is consisted of application data and application usage data over a data period of six months starting from January 2013 through June 2013. Data selection criteria is as given by (C1.1) and (C1.2).

(C1.1) Applications which were present in 50 or more smartphone devices in each month during the data period

(C1.2) Applications which were within top 70% in terms of the number of devices in at least one month during the data period

The set of applications satisfying above data selection criteria is comprised of 63 applications chosen across 6 game sub-categories present at that time. The distribution of these 63 applications across the sub-categories is as depicted in Figure 3.5. The difference in sub-category names between the whole dataset and in Figure 3.5 is due to category restructuring and renaming took place in android smartphone applications.

Game Sub-category	Number of Applications	Percentage	Cumulative Percentage
Casual	25	40%	40%
Arcade	14	22%	62%
Brain	14	22%	84%
Cards	4	6%	90%
Racing	3	5%	95%
Sports	3	5%	100%
Total	**63**	**100%**	**100%**

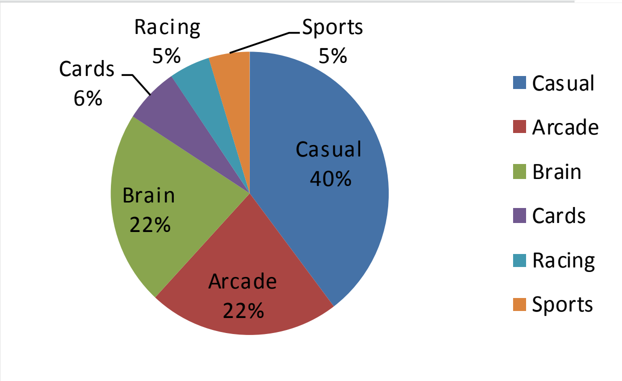


Figure 3. 5 - Free Game Applications Satisfying (C1.1) and (C1.2)

For the computational purposes, we consider the top 3 sub-categories ‘Casual,’ ‘Arcade’ and ‘Brain’ which contributes to a relatively large portion of applications as ‘Dataset 1.’ This dataset, is consisted of 53 applications and related usage information. The number of unique smartphone devices in ‘Dataset 1’ is 87394 and many of those have more than one application installed in them. The total number of application usage records of the selected applications and devices varies between 36351 and 67910 during the considered 6 month period.

3.3.2 Dataset 2

This data is employed for the research work discussed from Chapter 5 through Chapter 7. In this specific dataset, we consider the initial 21 months period from January 2013 through September 2014 as the data period. We focus on the application data and application usage data in this data period and apply the following data selection criteria given by (C2.1) through (C2.3).

(C2.1) Applications which were present in 50 or more smartphone devices in at least one month during the data period

(C2.2) Applications which were within top 70% in terms of the number of devices having the corresponding application in at least one month during the data period

(C2.3) Applications which are not an outlier in terms of the number of devices having the corresponding application in each month

For condition (C2.3) we employ an outlier detection method based on mean and standard deviation. The outline detection criterion is explained in Section 3.4.

The set of applications satisfying the conditions (C2.1) through (C2.3) consists of 16 game sub-categories with 255 applications, as given in the Figure 3.6.

Game Sub-category	Number of Applications	Percentage	Cumulative Percentage
Casual	83	33%	33%
Puzzle	47	18%	51%
Arcade	28	11%	62%
Action	18	7%	69%
Role playing	11	4%	73%
Simulation	11	4%	78%
Card	10	4%	82%
Family	9	4%	85%
Racing	6	2%	87%
Trivia	6	2%	90%
Casino	5	2%	92%
Music	5	2%	94%
Sports	5	2%	96%
Strategy	5	2%	98%
Board	4	2%	99%
Educational	2	1%	100%
Total	**255**	**100%**	**100%**

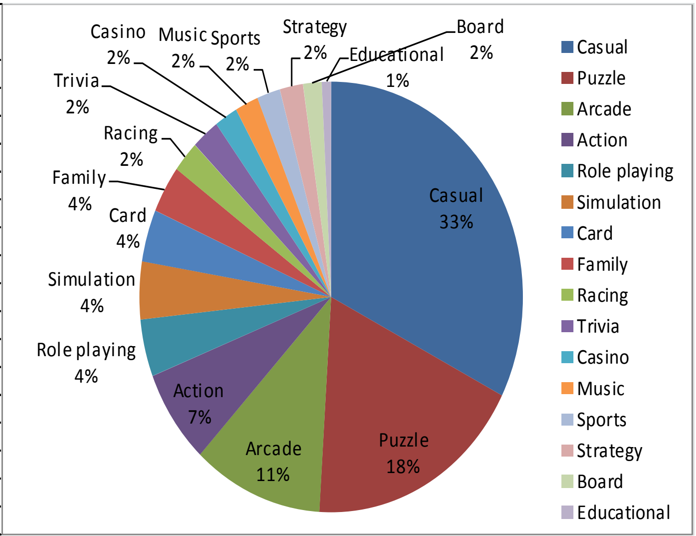


Figure 3. 6 – Free Game Applications Satisfying (C2.1) through (C2.3)

As clearly seen in the figure, the sub-categories 'Casual' and 'Puzzle' account for more than half of the satisfactory applications in 'Free Games.' In order to achieve the computational efficiency in the analysis, for Dataset 2 we exclusively focus on the 130 applications in 'Casual' and 'Puzzle' sub-categories and the application usage information relevant those applications. The number of unique smartphone devices in 'Dataset 2' is 192,529 where majority of those have more than one application installed in them.

3.3.3 Dataset 3

Dataset 3 is utilized for the research work discussed in Chapter 8. This dataset is derived by extracting fewer applications from Dataset 2 by tightening the data selection criteria and extending the data period to 40 months from January 2013 to April 2016. In here, we consider application data, application usage data and application change log data throughout the 40 months period from January 2013 to April 2016. We introduce the following data selection criteria given by (C3.1) through (C3.3) for Dataset 3.

(C3.1) Applications within top 30% in terms of the monthly number of devices having the underlying application in each month over the data period

(C3.2) Applications with the maximum of such monthly number of devices in (C1) over the data period being 5000 or more

(C3.3) Applications which are not an outlier in terms of the number of devices having the corresponding application in each month

The resulting set of applications satisfying the conditions (C3.1) through (C3.3) is consisted of 20 smartphone applications distributed among 7 game sub-categories, as given in the Figure 3.7.

Game Sub-category	Number of Applications	Percentage	Cumulative Percentage
Puzzle	7	35%	35%
Casual	5	25%	60%
Arcade	3	15%	75%
Action	2	10%	85%
Card	1	5%	90%
Casino	1	5%	95%
Roleplaying	1	5%	100%
Total	**20**	**100%**	**100%**

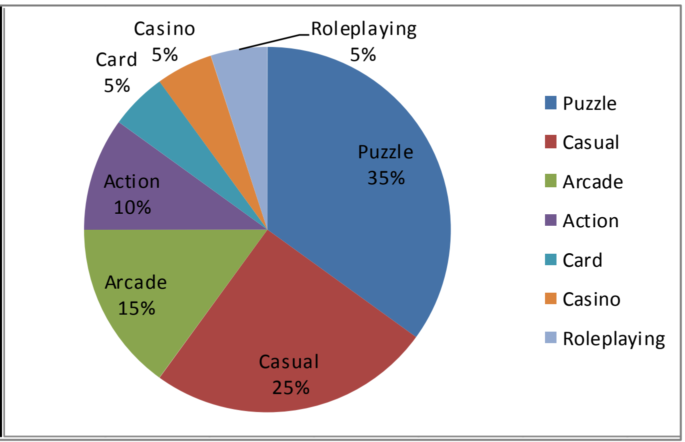


Figure 3. 7 – Free Game Applications Satisfying (C3.1) through (C3.3)

We consider the above set of satisfactory applications along with the related application usage data and change log information, as the 'Dataset 3.' The top 3 free game sub-categories 'Puzzle,' 'Casual' and 'Arcade' consisting 75% of the satisfactory applications with 415699 unique smartphone devices are employed as the development data. The rests of the applications in the remaining sub-categories with 151174 unique devices are used as the test data for validating the concept.

3.4 Outlier Detection

In data selection, we employ an outlier detection method based on mean and standard deviation. We specify an outlier in the following manner.

Let,

X: variable under consideration

$T = \{1, 2, \dots, M\}$ be the set of months constituting the data period

A: set of applications under consideration

$x(a, k)$: value of the variable under consideration with respect to application $a \in A$ in month $k \in T$

$\mu(k)$: the mean of X over $a \in A$

$\sigma(k)$: the standard deviation of X over $a \in A$

β_-, β_+: positive numbers

Then assuming the normal distribution of X, we say $x(a, k)$ is an outlier if,

$$x(a, k) < \mu(k) - \beta_- \sigma(k) \text{ or } \mu(k) + \beta_+ \sigma(k) < x(a, k).$$

In this work X is set to be the logarithm of number of devices having corresponding application in each month. More specifically,

$D(a, k)$: set of devices having application $a \in A$ in month $k \in T$

$x(a, k) = log|D(a, k)|$, where $|X|$ denotes the cardinality of a set X

We determined β_- and β_+ to be 2, so that the probability of a regular value to be at least 95%.

As an example, Figure 3.8 below illustrates the distribution of data values (with outliers marked in red) for the two sub-categories 'Casual' and 'Puzzle' during the initial 3 months of data.

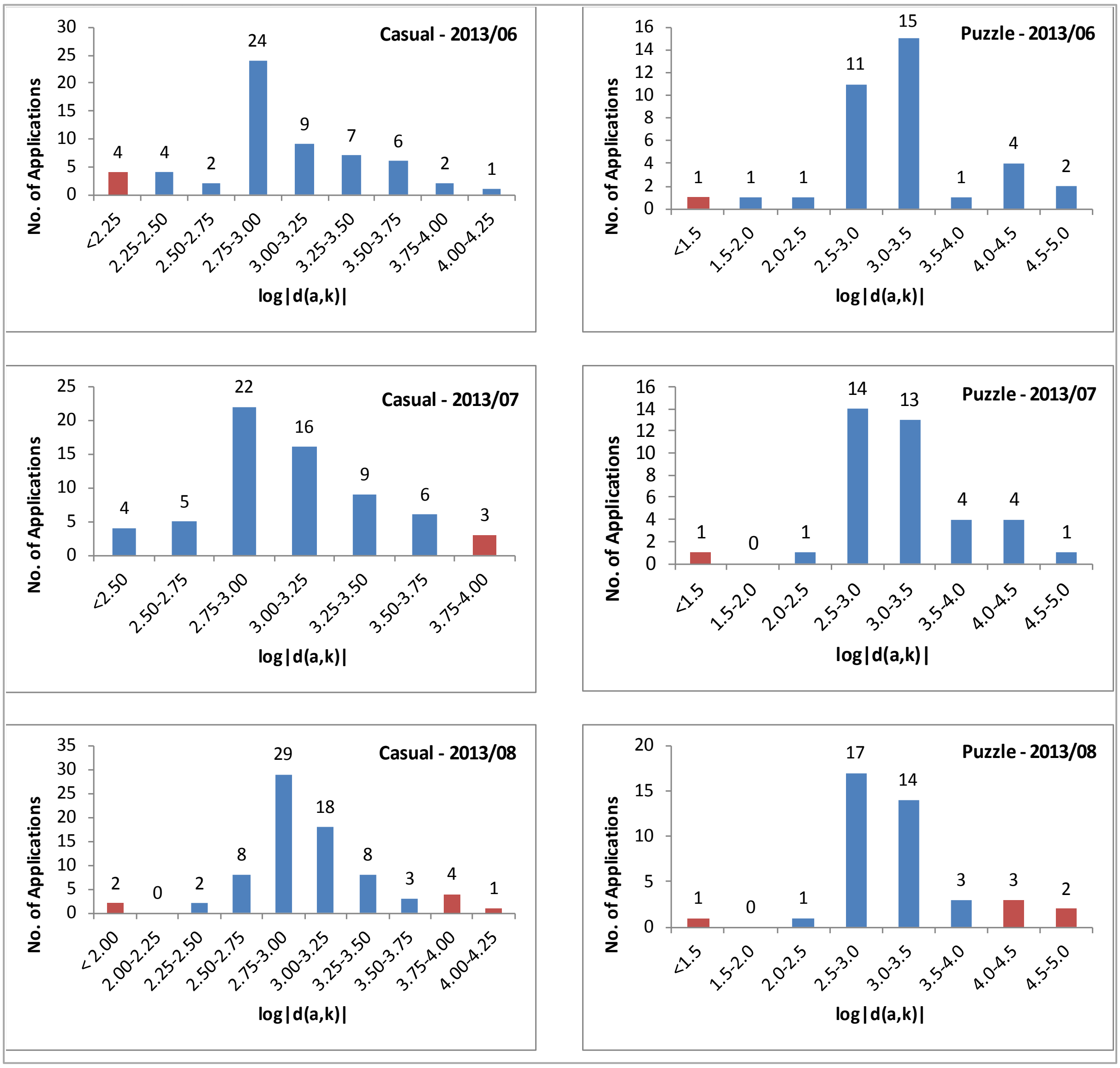


Figure 3. 8 - Distribution of Data and Outliers

Chapter 4

Key Performance Measures Based on Number of Downloads

As we know, one who goes searching for smartphone applications can come across millions of different applications. These applications perform in the market in various levels, with different user-bases and different levels of consistency. However, to get a better idea of the characteristics and market performance of these applications it is important to have the capability of measuring and comparing similar applications. To make such a comparison effective, it is pivotal to determine a set of application characteristics which can perform as competitive performance measures. Thereby, one can measure the competitiveness of an application or carry out comparisons between applications quantitatively. This may be especially beneficial in drawing useful business implications for smartphone application development companies to determine their future applications. In this chapter, we discuss about application characteristics by introducing two key competitive performance measures based on monthly application downloads.

4.1 Application Performance Measures

In this chapter we introduce two key performance measures based on application download counts. They are namely, Application Device Ratio and Application Stability. For the convenience in defining these performance measures, we first introduce following notation.

T_1: Period of time where, $T_1 = \{ 1, 2, \ldots, 6 \}$

A: Whole set of applications

S: Application category or sub-category under consideration

$A(S)$: Set of applications in the considered application category or sub-category

$DL(A(S))$: Set of devices which have downloaded any application in $A(S)$

$DL(A(S), k)$: Set of devices which have downloaded any application in $A(S)$ in month $k \in T_1$

Download of application $a \in A(S)$ on device $d \in D(S)$ in month $k \in T_1$ can be written as

$$(4.1) \quad b_{DL}(a, d, k) = \begin{cases} 1 & application\ a\ is\ downloaded\ in\ device\ d\ in\ month\ k \\ 0 & otherwise \end{cases} \quad .$$

$DL(a, k)$: Set of devices which have downloaded application $a \in A(S)$ in month $k \in T_1$
We can formally write $DL(a, k)$ as

$$(4.2) \quad DL(a, k) = \left\{ d \in DL(A(S)) |\, b_{DL}(a, d, k) = 1 \right\} \quad .$$

The cardinality of a set X is denoted by $|X|$. Therefore, the number of devices in $DL(A(S), k)$ is written as $|DL(A(S), k)|$. In the same manner we write the number of devices in $DL(a, k)$ as $|DL(a, k)|$. Now we are in a position to define the two key performance measures introduced in this Chapter.

The first key performance measure introduced in this chapter is named as Application Device Ratio. It is concerned with relative size of application users (devices) of an application at a given time. This measure may come useful in assessing smartphone applications in terms of the size of the user-base and the market share and also act as an indicator of application's popularity. The Application Device Ratio of application $a \in A(S)$ in month $k \in T_1$ is written as,

$$(4.3) \quad App\text{-}Dev(a,k) = \frac{|DL(a,k)|}{|DL(A(S),k)|} \quad .$$

We next introduce the second key performance measure, Application Stability. The concept of Application Stability is concerned with the size of the user-base that continues to use the application. This performance measure may be useful in measuring and comparing the market continuity of applications. More specifically, we define application $a \in A(S)$ to be stable in device $d \in D(S)$ if the device has continuously used the application during the last 3 months of the period. Let,

$ST(a,k)$: Set of stable devices associated with application $a \in A(S)$ in month k

This is formally defined as,

$$(4.4) \quad ST(a,k) = \{d \in DL(\mathrm{A(S)}) | b_{DL}(a,d,k-2) = b_{DL}(a,d,k-1) = b_{DL}(a,d,k) = 1\}$$

$$for\ k \in T_1 = \{3,4,5,6\} \quad .$$

We now define Application Stability as,

$$(4.5) \quad App\text{-}Stab(a,k) = \frac{|ST(a,k)|}{|DL(A(S),k)|} \quad .$$

4.2 Numerical Example

In this section we present the numerical results of the proposed key performance measures applied on real data. For this purpose we use the 'Dataset 1' introduced in Section 3.3.1, which is consisted of application and application usage data of 63 applications across the 3 game-sub categories of 'Casual,' 'Arcade' and 'Brain.'

The Table 4.1 lists the $App\text{-}Dev(a, k)$ values defined by (4.3) for the selected applications $a \in A(S)$ in the months $k = 1, 2, \ldots, 6$, for $S \in \{Casual, Arcade, Puzzle\}$.

Table 4. 1 - App-Dev (a, k) Values

Game Sub-category	Application Name	App-Dev(a,k) k = 1 (2013/01)	k = 2 (2013/02)	k = 3 (2013/03)	k = 4 (2013/04)	k = 5 (2013/05)	k = 6 (2013/06)
CASUAL	Mushroom Garden Deluxe	0.502	0.484	0.497	0.505	0.497	0.505
	Kanji ability diagnostic FREE	0.072	0.076	0.077	0.072	0.071	0.077
	Gunma's Ambition	0.061	0.060	0.061	0.057	0.052	0.053
	Fire the match	0.054	0.048	0.050	0.053	0.048	0.040
	Walk Band - Multitracks Music	0.038	0.054	0.051	0.049	0.050	0.052
	Piano Lesson PianoMan	0.038	0.033	0.033	0.036	0.034	0.035
	Hamster Life	0.027	0.027	0.028	0.030	0.028	0.026
	Ayakashi: Ghost Guild	0.026	0.025	0.024	0.022	0.022	0.024
	Paper Toss	0.024	0.036	0.028	0.025	0.024	0.024
	Oddbot Workshop	0.023	0.021	0.019	0.017	0.015	0.015
	Raise! My puppy	0.015	0.015	0.016	0.015	0.015	0.015
	Can Knockdown	0.014	0.014	0.016	0.013	0.013	0.011
	Haraguro Checker	0.011	0.008	0.007	0.006	0.007	0.009
	What's her Boobs Size?	0.010	0.009	0.008	0.007	0.006	0.006
	Diagnosis of Mental Age	0.010	0.008	0.007	0.006	0.006	0.006
	Bubble Shooter	0.010	0.018	0.011	0.013	0.011	0.011
	Droid Paint Free	0.009	0.009	0.010	0.011	0.013	0.014
	Mizukiri	0.008	0.007	0.007	0.009	0.008	0.007
	Jewels Deluxe	0.008	0.010	0.009	0.007	0.007	0.007
	1 Million eggs	0.008	0.008	0.012	0.019	0.044	0.037
	Delightful! Owata shiritori	0.008	0.006	0.005	0.005	0.005	0.004
	Cotton Candy Master	0.008	0.007	0.007	0.007	0.007	0.007
	Landing Cat [Sweet Cat Puzzle]	0.007	0.007	0.008	0.007	0.009	0.008
	Find Differences Touch	0.007	0.005	0.005	0.004	0.004	0.004
	Wasserstein Alien!	0.006	0.005	0.005	0.005	0.004	0.004
ARCADE	Touch the Numbers for Android	0.240	0.216	0.238	0.256	0.266	0.270
	Game Cloud	0.198	0.175	0.206	0.219	0.220	0.218
	NoruhitoA	0.153	0.123	0.123	0.119	0.111	0.107
	free Goldfish Fishing game RPG	0.089	0.070	0.073	0.068	0.071	0.066
	Angry Birds Space	0.075	0.151	0.100	0.073	0.070	0.066
	Doriland Expedition	0.059	0.052	0.062	0.065	0.063	0.066
	Mole!Mole!!Mole!!!	0.038	0.033	0.036	0.033	0.029	0.029
	Princess Punt	0.029	0.022	0.024	0.023	0.022	0.022
	PAC-CHOMP!	0.028	0.026	0.024	0.027	0.026	0.024
	Minecraft - Pocket Ed. Demo	0.027	0.067	0.050	0.055	0.061	0.069
	Miniature Garden GREE	0.023	0.023	0.024	0.028	0.027	0.031
	Hamabeno Futari	0.016	0.013	0.013	0.012	0.012	0.012
	Yoo Ninja! Free	0.015	0.022	0.019	0.015	0.014	0.012
	Zombie Street	0.009	0.007	0.008	0.008	0.008	0.007
BRAIN	Jewels Star	0.190	0.236	0.167	0.153	0.159	0.170
	Block Puzzle	0.130	0.127	0.125	0.136	0.130	0.121
	TRARIS Deluxe	0.126	0.110	0.117	0.119	0.127	0.136
	Jigsaroid - Jigsaw Generater	0.085	0.068	0.066	0.065	0.064	0.065
	Ultima Reversi	0.080	0.084	0.129	0.146	0.158	0.174
	100 Floors™ - Can You Escape?	0.070	0.076	0.080	0.070	0.063	0.056
	Find Differences	0.053	0.048	0.043	0.043	0.041	0.040
	NemoNemo Picross	0.052	0.043	0.040	0.039	0.039	0.043
	Puzzle & Dragons -ID exchange bulletin board	0.048	0.059	0.067	0.072	0.053	0.030
	Birzzle	0.043	0.036	0.033	0.028	0.032	0.032
	Blade Master	0.042	0.033	0.032	0.027	0.027	0.028
	Andoku Sudoku 2 Free	0.033	0.036	0.039	0.042	0.047	0.052
	Sheep Spongy♪	0.025	0.024	0.041	0.041	0.036	0.030
	Puzzle Family	0.022	0.022	0.020	0.019	0.022	0.023

In the table, applications are listed in the descending order of the $App\text{-}Dev(a, 1)$ inside each sub-category. One can notice that the applications in general possess relatively small values for $App\text{-}Dev(a, k)$. It's worth noting that the 'Casual' application 'Mushroom Garden Deluxe' holds significantly high $App\text{-}Dev(a, k)$ values which are more than six times of the next largest $App\text{-}Dev(a, k)$ value in the same sub-category, exhibiting it's higher relative market share. This could be thought as an application that enjoys a huge user attraction. This idea is supported by the application's historical information which shows a gain of an amazing amount of downloads over 50,000 by the second day and over 250,000 downloads by the fourth day and similar continuous growth afterwards since its introduction in July 2012.

For the ease of comparing the distributions of $App\text{-}Dev(a, k)$ values in Table 4.1, they are graphically presented for the sub-categories 'Casual,' 'Arcade' and 'Brain' by the Figures 4.1 through 4.3 respectively. In the figures, the smartphone applications are listed along the horizontal axis in the descending order of the $App\text{-}Dev(a, 1)$. For the convenience of observing the distributions of applications, we have excluded the application 'Mushroom Garden Deluxe' which has distinctively large $App\text{-}Dev(a, 1)$values from the Figure 4.1

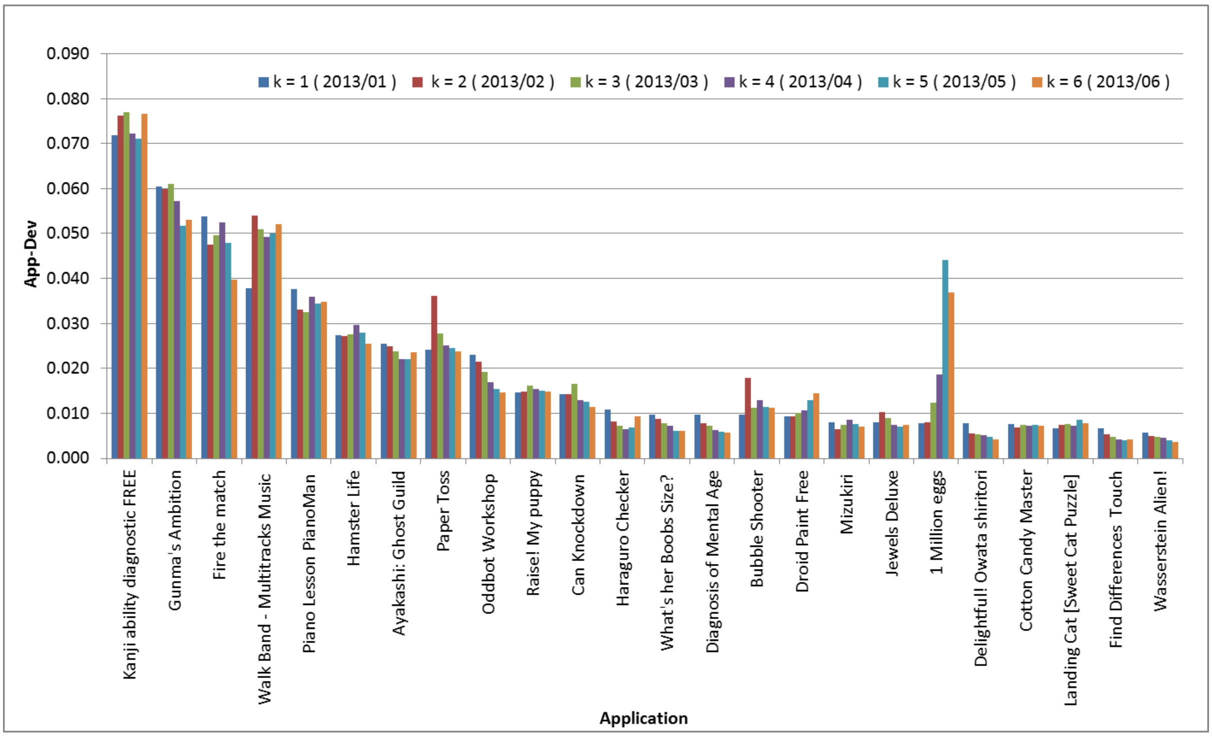


Figure 4. 1 - App-Dev(a, k) Values (Sub-category: Casual, Excluding the Application 'Mushroom Garden Deluxe')

In the Figure 4.1, one can observe the application '1 Million eggs' is experiencing exponential growth in the values for $App\text{-}Dev(a,k)$, this could be explained by its initial introduction to the market during the same period.

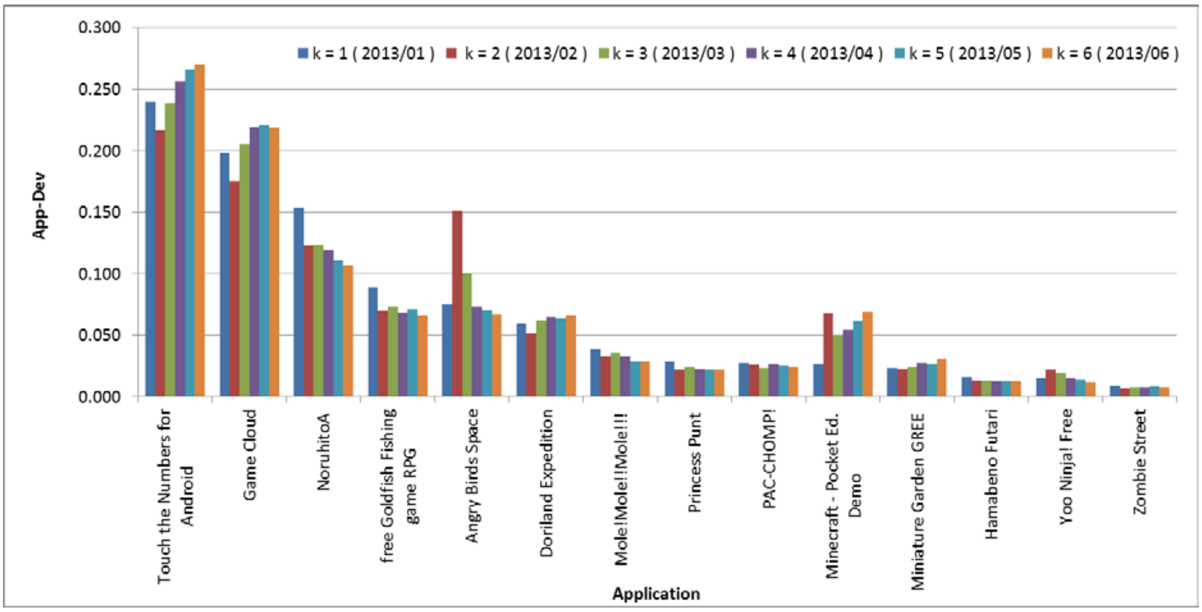


Figure 4. 2 - App-Dev(a, k) Values (Sub-category: Arcade)

In Figure 4.2 one can observe that the majority of applications' $App\text{-}Dev(a,k)$ value is ceasing over the time showing the loss of interest from the users.

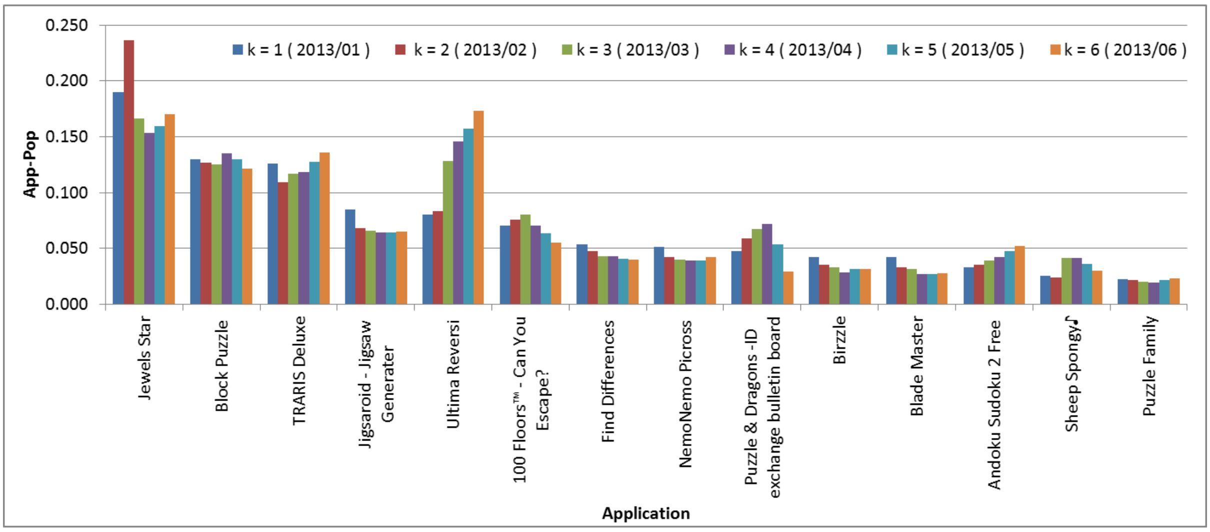


Figure 4. 3 - App-Dev(a, k) Values (Sub-category: Brain)

Next, we list in Table 4.2, the $App\text{-}Stab(a, k)$ values defined by (4.5) of the selected applications $a \in A(S)$ for the months $k = 3, \dots, 6$, where $S \in \{Casual, Arcade, Puzzle\}$.

Table 4. 2 - App-Stab (a, k) Values

Game Sub-category	Application Name	App-Stab (a,k) k = 3 (2013/03)	k = 4 (2013/04)	k = 5 (2013/05)	k = 6 (2013/06)
CASUAL	Mushroom Garden Deluxe	0.548	0.549	0.548	0.546
	Kanji ability diagnostic FREE	0.068	0.073	0.076	0.078
	Gunma's Ambition	0.061	0.061	0.059	0.055
	Fire the match	0.044	0.039	0.039	0.041
	Piano Lesson PianoMan	0.034	0.031	0.031	0.034
	Walk Band - Multitracks Music	0.032	0.037	0.039	0.042
	Ayakashi: Ghost Guild	0.029	0.028	0.027	0.025
	Oddbot Workshop	0.023	0.022	0.020	0.017
	Paper Toss	0.022	0.026	0.026	0.024
	Hamster Life	0.021	0.020	0.019	0.022
	Raise! My puppy	0.014	0.014	0.015	0.013
	Can Knockdown	0.013	0.011	0.012	0.010
	Droid Paint Free	0.009	0.009	0.010	0.011
	What's her Boobs Size?	0.009	0.008	0.007	0.007
	Jewels Deluxe	0.008	0.008	0.008	0.008
	Bubble Shooter	0.008	0.010	0.010	0.012
	Haraguro Checker	0.008	0.007	0.007	0.006
	1 Million eggs	0.008	0.008	0.010	0.013
	Cotton Candy Master	0.008	0.007	0.007	0.007
	Diagnosis of Mental Age	0.007	0.007	0.006	0.005
	Delightful! Owata shiritori	0.006	0.005	0.004	0.004
	Mizukiri	0.006	0.005	0.005	0.006
	Wasserstein Alien!	0.006	0.005	0.005	0.004
	Find Differences Touch	0.006	0.004	0.004	0.004
	Landing Cat [Sweet Cat Puzzle]	0.005	0.006	0.006	0.006
ARCADE	Touch the Numbers for Android	0.245	0.246	0.252	0.255
	Game Cloud	0.230	0.226	0.232	0.239
	NoruhitoA	0.156	0.138	0.132	0.123
	free Goldfish Fishing game RPG	0.081	0.072	0.066	0.066
	Doriland Expedition	0.064	0.063	0.067	0.069
	Angry Birds Space	0.053	0.073	0.071	0.066
	Mole!Mole!!Mole!!!	0.037	0.038	0.034	0.032
	PAC-CHOMP!	0.027	0.028	0.026	0.025
	Miniature Garden GREE	0.026	0.027	0.028	0.031
	Princess Punt	0.025	0.022	0.019	0.019
	Hamabeno Futari	0.018	0.015	0.014	0.014
	Minecraft - Pocket Ed. Demo	0.015	0.030	0.036	0.042
	Yoo Ninja! Free	0.014	0.014	0.014	0.012
	Zombie Street	0.009	0.008	0.009	0.008
BRAIN	Jewels Star	0.179	0.185	0.175	0.175
	Block Puzzle	0.135	0.133	0.132	0.139
	TRARIS Deluxe	0.122	0.119	0.118	0.125
	Jigsaroid - Jigsaw Generater	0.097	0.089	0.081	0.075
	Ultima Reversi	0.092	0.101	0.135	0.147
	100 Floors™ - Can You Escape?	0.073	0.072	0.067	0.061
	NemoNemo Picross	0.058	0.051	0.047	0.046
	Andoku Sudoku 2 Free	0.041	0.045	0.050	0.050
	Find Differences	0.041	0.046	0.042	0.040
	Blade Master	0.041	0.034	0.031	0.028
	Puzzle & Dragons -ID exchange bulletin board	0.037	0.045	0.040	0.034
	Birzzle	0.034	0.033	0.033	0.032
	Sheep Spongy♪	0.027	0.023	0.027	0.026
	Puzzle Family	0.023	0.023	0.023	0.022

In this table the applications are listed along the horizontal axis in the descending order of $App\text{-}Stab(a, 3)$ inside each sub-category. Here as well, one can observe outstanding values in application 'Mushroom Garden Deluxe' for $App\text{-}Stab(a, k)$.

For the convenience of comparing the values given in Table 4.2, we graphically present those values by the Figures 4.4 through 4.6 separated into sub-categories 'Casual,' 'Arcade' and 'Brain' respectively. Again, for the convenience of observing the distributions of applications, from the Figure 4.1 we have excluded the application 'Mushroom Garden Deluxe' which has $App\text{-}Stab(a, 3)$ values about 8 times larger than the other applications.

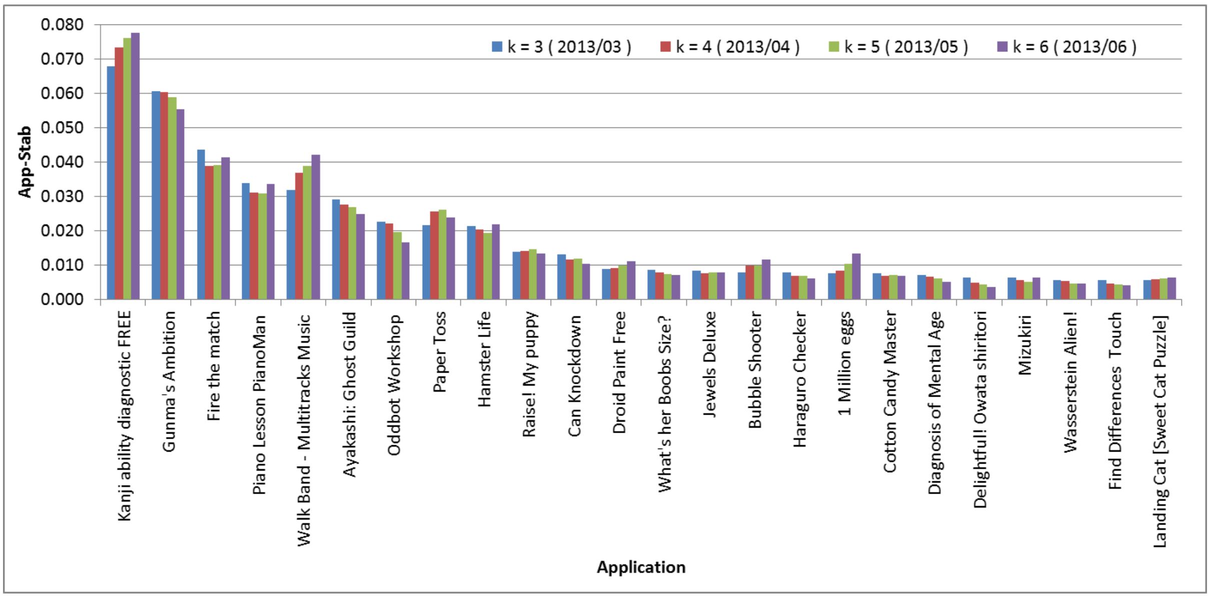


Figure 4. 4 - App-Stab(a, k) Values (Sub-category: Casual, Excluding the Application 'Mushroom Garden Deluxe')

By comparing Figure 4.4 with Figure 4.1, one can notice that the order of the top 4 applications in the 'Casual' sub-category remains the same for both $App\text{-}Dev(a, k)$ and $App\text{-}Stab(a, k)$. It is worth nothing that here we cannot observe any exponential growth in the application '1 Million eggs' as in $App\text{-}Dev(a, k)$. We can assume that although the early users were keen on downloading this application, they must have removed the application in the next months, without contributing to the stability of the application in the coming months.

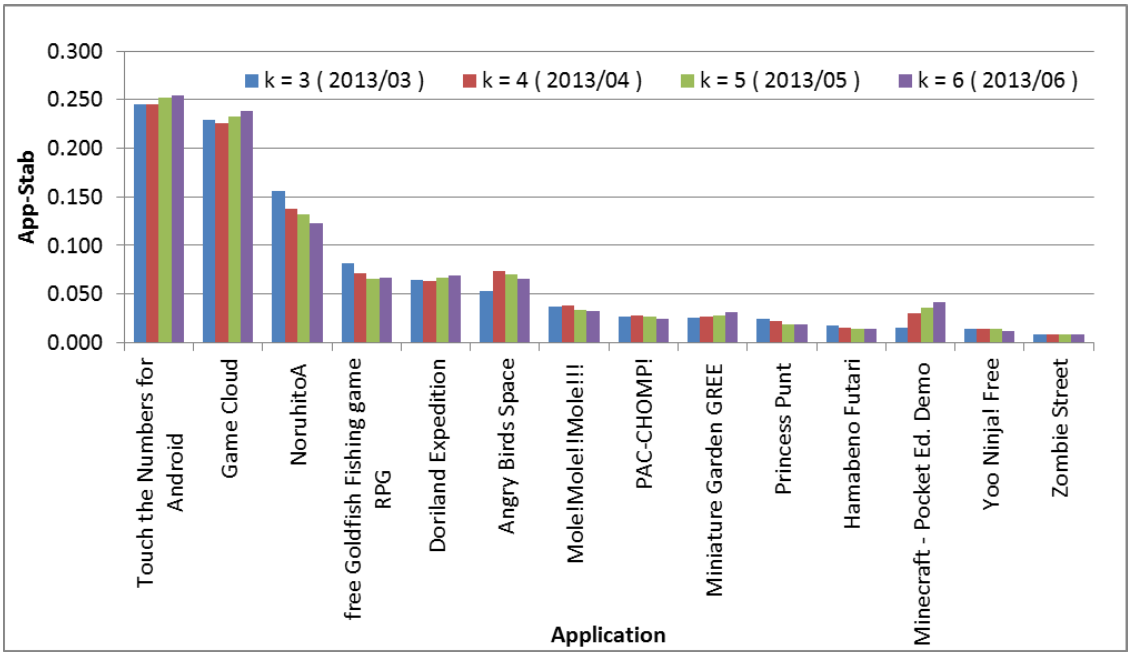


Figure 4. 5 App-Stab(a, k) Values (Sub-category: Arcade)

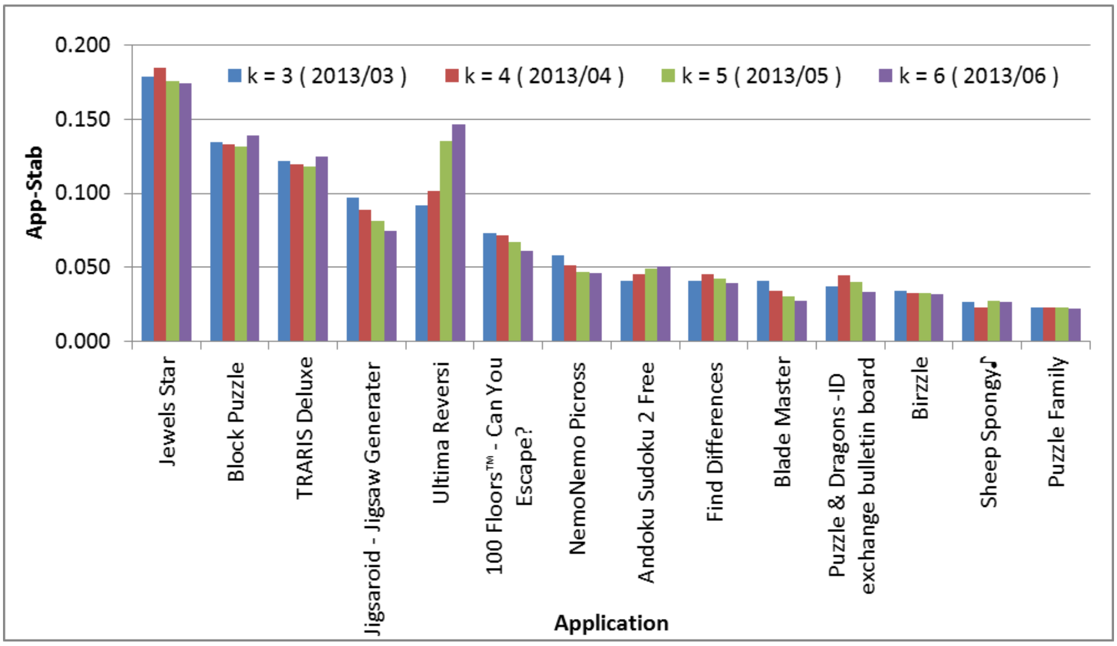


Figure 4. 6 - App-Stab(a, k) Values (Sub-category: Brain)

By comparing the $App\text{-}Stab(a,k)$ distributions with $App\text{-}Dev(a,k)$ distributions of both 'Arcade' and 'Brain' sub-categories, we can observe that the last two applications have remained the same in both cases. However considering the top most applications, the top 6 applications of 'Brain' have remained unchanged across $App\text{-}Stab(a,k)$ and

$App\text{-}Dev(a,k)$ compared to top 4 applications of ‘Arcade.’ Further, one can also note that ‘Ultima Reversi’ of ‘Brain’ sub-category is an application with increasing number of stable users.

4.3 Specific Applications

In the previous section, we determined the values of the two key performance measures of 53 applications chosen over three Game sub-categories of ‘Casual,’ ‘Arcade’ and ‘Brain’ over a six month period from January 2013 through June 2013. From the observations one can understand that the smartphone application ‘Mushroom Garden Deluxe’ of the sub-category ‘Casual’ stands-out the rest of the applications in terms of Application Device Ratio and Application Stability during this period. As of (“App Brain,” 2016), this application has been introduced to the Android smartphone application market four years back in July 2012 and it has enjoyed a very fast growing number of users. It has surpassed 5 million downloads in July 2014, and remain in the range of ‘5 million ~ 10 million’ at present with a considerably high user rating. One can use this application and its features as a reference in introducing a new smartphone game application to the market.

Chapter 5
Application Usage Patterns

Usage of an application on a smartphone device may strongly associate with the lifestyle and the personal behaviors of the user. Therefore, the presence of an application on a smartphone device over a period of time may greatly vary on different devices. By investigating the presence of an application on a certain device for a period of time, once can understand the existence of different application usage patterns.

A sequence of zero-one values representing an application's presence in adjacent individual timeslots can be considered as an application usage pattern. One can denote the presence of a smartphone application on a specific device as '1' while denoting the non-presence of the application on the device as '0.' This binary representation of application's presence in an individual timeslot on a certain device leads to a set of application usage patterns represented by sequence of binary numbers. More specifically, for a certain application, on a certain device during k consecutive timeslots, there exist a 2^k number of possible usage patterns. Table 5.1 illustrates the number of possible patterns associated with a period of k.

Table 5. 1 – Possible application usage patterns for a period of k

k	2^k	**Possible patterns**							
1	2	0				1			
2	4	00		01		10		11	
3	8	000	001	010	011	100	101	110	111
4	16	…							
5	32	…							
6	64	000000 000001	…						111110 111111
…	…								

The length of an individual timeslot can take a wide range values from minutes, hours to months, years and so forth. Application usage patterns can express a widely different information and user behaviors, depending on the length of underlying timeslots. In this work, we set the length of a single timeslot to one month, so as to get an insight of application usage patterns over a longer duration. As the next step, the period of time is to be decided. Employing a too short time period is not appropriate as a shorter period may not be able to properly exhibit the underlying usage patterns. On the other hand, too long time period results a large number of possible binary patterns and may cause the identification of usage pattern too complex. Considering that a period of 6 months is of a sufficient length to properly describe application usage patterns, we set the period of time, k to 6 months. Thereby we define application usage patterns for six month periods, by taking the monthly presence of applications on a set of devices of interest.

5.1 Definition of Usage Patterns

As explained by the Table 5.1, one understands that the number of possible binary patterns for a 6 month period with 1 month timeslots is 2^6 (= 64.) By investigating the common features of this 64 different binary number sequences, we introduce 10 smartphone application usage patterns that each binary number sequence can be grouped into one of them. For the ease of defining these usage patterns, we use the following notation.

We denote the six month period ending in month $t \in T$ by $I(t)$. That is, $I(t) = \{t-5, 5-4, \ldots, t\}$. Let A be the set of applications under consideration and D be the set of devices under consideration. For each $a \in A$ and $I(t)$, we decompose D into separate subgroups based on the 10 application usage patterns. Tables 5.2 through 5.11 present the groups of binary number sequences under the 10 application usage patterns, where those shaded cells with number 1 represent the presence of application.

Table 5. 2 – Usage pattern 'Null'

Month	t-5	t-4	t-3	t-2	t-1	t
Presence	0	0	0	0	0	0

Table 5. 3 – Usage pattern 'Exploring'

Month	t-5	t-4	t-3	t-2	t-1	t
Presence	0	0	0	0	0	1
	0	0	0	0	1	0
	0	0	0	1	0	0
	0	0	0	1	1	0
	0	0	1	1	0	0

Table 5. 4 – Usage pattern 'Rising'

Month	t-5	t-4	t-3	t-2	t-1	t
Presence	0	0	0	0	1	1
	0	0	0	1	1	1
	0	0	1	1	1	1

Table 5. 5 – Usage pattern 'Stable'

Month	t-5	t-4	t-3	t-2	t-1	t
Presence	0	1	1	1	1	1
	1	0	1	1	1	1
	1	1	0	1	1	1

Table 5. 6 – Usage pattern 'Matured'

Month	t-5	t-4	t-3	t-2	t-1	t
Presence	1	1	1	1	1	1

Table 5. 7 – Usage pattern 'Warning'

Month	t-5	t-4	t-3	t-2	t-1	t
Presence	0	1	1	1	0	0
	0	0	1	1	1	0
	1	1	1	1	0	0
	0	1	1	1	1	0
	1	0	1	1	1	0
	1	1	0	1	1	0
	1	1	1	0	1	0
	1	1	1	1	1	0

Table 5. 8 – Usage Pattern 'Ceasing'

Month	t-5	t-4	t-3	t-2	t-1	t
Presence	0	0	1	0	0	0
	0	1	0	0	0	0
	1	0	0	0	0	0
	0	1	1	0	0	0
	1	0	1	0	0	0
	1	1	0	0	0	0
	1	1	1	0	0	0

Table 5. 9 - Usage pattern 'Recovering'

Month	t-5	t-4	t-3	t-2	t-1	t
Presence	0	0	1	0	1	1
	0	1	0	0	1	1
	1	0	0	0	1	1
	0	1	1	1	0	1
	1	1	1	0	0	1
	0	1	0	1	1	1
	0	1	1	0	1	1
	1	0	0	1	1	1
	1	0	1	0	1	1
	1	1	0	0	1	1
	1	1	1	0	1	1
	1	1	1	1	0	1

Table 5. 10 - Usage pattern 'Reactivating'

Month	t-5	t-4	t-3	t-2	t-1	t
Presence	0	0	0	1	0	1
	0	0	1	0	0	1
	0	1	0	0	0	1
	1	0	0	0	0	1
	0	0	1	1	0	1
	0	1	0	1	0	1
	0	1	1	0	0	1
	1	0	0	1	0	1
	1	0	1	0	0	1
	1	1	0	0	0	1
	1	1	0	1	0	1
	1	0	1	1	0	1

Table 5. 11 - Usage pattern 'Fickle'

Month	t-5	t-4	t-3	t-2	t-1	t
Presence	0	0	1	0	1	0
	0	1	0	0	1	0
	1	0	0	0	1	0
	0	1	0	1	0	0
	1	0	0	1	0	0
	0	1	0	1	1	0
	0	1	1	0	1	0
	1	0	0	1	1	0
	1	0	1	0	1	0
	1	1	0	0	1	0
	1	0	1	1	0	0
	1	1	0	1	0	0

Devices separated in to the application usage patterns given in Tables 5.2 through 5.11 are briefly described below:

(1) $Null(a, t)$: Those devices $d \in D$ for which application $a \in A$ was not present over the period $I(t)$

(2) $Exploring(a, t)$: Those devices $d \in D$ for which [application $a \in A$ was not present during the first 5 months of the period $I(t)$ and then present in $t \in I(t)$] or [application $a \in A$ was not present during the first 2 to 4 months of the period $I(t)$, was present subsequently for 1 or 2 months, and then not present in $t \in I(t)$]

(3) $Rising(a, t)$: Those devices $d \in D$ for which application $a \in A$ was not present during the first 2 to 4 months of the period $I(t)$ and then was present thereafter

(4) $Stable(a, t)$: Those devices $d \in D$ for which application $a \in A$ was present for 2 months during the first 3 months of the period $I(t)$, and then was present in each of the last 3 months

(5) $Matured(a, t)$: Those devices $d \in D$ for which application $a \in A$ was present throughout the period $I(t)$

(6) $Warning(a, t)$: Those devices $d \in D$ for which [application $a \in A$ was present

for exactly 3 months consecutively during the first 5 months of the period $I(t)$ or present for 4 to 5 months in the first 5 months] and then was not present in $t \in I(t)$

(7) $Ceasing(a, t)$: Those devices $d \in D$ for which application $a \in A$ was present for at least 1 month during the first 3 months of the period $I(t)$, and was not present throughout the last 3 months of $I(t)$

(8) $Recovering(a, t)$: Those devices $d \in D$ for which application $a \in A$ was present for 1 to 4 months of the period $I(t)$ before getting removed and then present again subsequently

(9) $Reactivating(a, t)$: Those devices $d \in D$ for which application $a \in A$ was present for 1 to 3 months during the first 4 months of the period $I(t)$, removed in the 5th month and present again in the 6th month

(10) $Fickle(a, t)$: Those devices $d \in D$ for which application $a \in A$ was present and removed several times and the usage pattern does not belong to any of 'Warning,' 'Ceasing,' 'Recovering' or 'Reactivating'

For notational convenience, we define $UP(a, t) = \{ UP_1(a, t), \dots, UP_{10}(a, t)\}$, where $UP_j(a, t)$ corresponds to (j) above, that is, $UP_1(a, t) = Null(a, t)$, $UP_2(a, t) = Exploring(a, t)$, and so on. In order to define these device categories more rigorously, let

(5.1) $$b(a, d, k) = \begin{cases} 1 & application\ a\ is\ present\ in\ device\ d\ in\ month\ k \\ 0 & otherwise \end{cases} .$$

The flag $sign(a, d, k)$ for indicating the switch of the status of application a in device d in month k is defined, for $k \neq 1$, as

(5.2) $$sign(a, d, k) = \begin{cases} 1 & if\ b(a, d, k-1) \neq b(a, d, k) \\ 0 & otherwise \end{cases} .$$

We are now in a position to define $UP_j(a,t)$ formally.

(5.3) $d \in UP_1(a,t) \leftrightarrow \sum_{k \epsilon I(t)} b(a,d,k) = 0$.

(5.4) $d \in UP_2(a,t) \Leftrightarrow \left(\sum_{k=3}^{5} b(a,d,t-k) = 0 \text{ } and \text{ } \sum_{k=0}^{2} b(a,d,t-k) = 1\right) or$

$$\begin{pmatrix} \sum_{k=4}^{5} b(a,d,t-k) = 0 \text{ } and \text{ } b(a,d,t) = 0 \text{ } and \\ \sum_{k=1}^{3} b(a,d,t-k) = 2 \text{ } and \\ \sum_{k=0}^{3} sign(a,d,t-k) = 2 \end{pmatrix} .$$

(5.5) $d \in UP_3(a,t) \Leftrightarrow \sum_{k=m}^{5} b(a,d,t-k) = 0 \text{ } and \text{ } \sum_{k=0}^{m-1} b(a,d,t-k) = m,$

$m = 2,3,4$.

(5.6) $d \in UP_4(a,t) \Leftrightarrow \sum_{k=3}^{5} b(a,d,t-k) = 2 \text{ } and \text{ } \sum_{k=0}^{2} b(a,d,t-k) = 3$.

(5.7) $d \in UP_5(a,t) \Leftrightarrow \sum_{k \in I(t)} b(a,d,k) = 6$.

(5.8) $d \in UP_6(a,t) \Leftrightarrow \left(b(a,d,t) = 0 \text{ } and \text{ } \sum_{k=1}^{5} b(a,d,t-k) \geq 4\right) or$

$$\begin{pmatrix} b(a,d,t-5) = b(a,d,t) = 0 \text{ } and \\ \sum_{k=1}^{4} b(a,d,t-k) = 3 \text{ } and \\ \sum_{k=0}^{4} sign(a,d,t-k) = 2 \end{pmatrix}.$$

(5.9) $d \in UP_7(a,t) \Leftrightarrow \sum_{k=0}^{2} b(a,d,t-k) = 0 \text{ } and \text{ } \sum_{k=3}^{5} b(a,d,t-k) \geq 1$.

$$(5.10)\ d \in UP_8(a,t) \Leftrightarrow \begin{pmatrix} b(a,d,t) = b(a,d,t-1) = 1 \ \ and \ \ b(a,d,t-2) = 0 \\ and \\ \sum_{k=3}^{5} b(a,d,t-k) \geq 1 \end{pmatrix} \ or$$

$$\begin{pmatrix} b(a,d,t) = 1 \ and \ b(a,d,t-1) = 0 \ and \\ \sum_{k=2}^{5} b(a,d,t-k) \geq 3 \ \ and \\ \sum_{k=2}^{4} sign(a,d,t-k) \leq 1 \end{pmatrix} \ or$$

$$\begin{pmatrix} b(a,d,t) = b(a,d,t-1) = b(a,d,t-2) = 1 \ \ and \\ b(a,d,t-3) = 0 \ and \\ \sum_{k=4}^{5} b(a,d,t-k) = 1 \end{pmatrix} .$$

$$(5.11)\ d \in UP_9(a,t) \Leftrightarrow \begin{pmatrix} b(a,d,t) = 1 \ and \ b(a,d,t-1) = 0 \ \ and \\ 1 \leq \sum_{k=2}^{5} b(a,d,t-k) \leq 2 \end{pmatrix} or$$

$$\begin{pmatrix} b(a,d,t) = 1 \ and \ b(a,d,t-1) = 0 \ and \\ \sum_{k=2}^{5} b(a,d,t-k) = 3 \ and \\ \sum_{k=2}^{4} sign(a,d,t-k) \geq 2 \end{pmatrix} .$$

$$(5.12)\ \ d \in UP_{10}(a,t) \ \Leftrightarrow \ \sum_{k=0}^{4} sign(a,d,t-k) \ \geq \ 3 \ and$$

$$d \notin \left(UP_6(a,t) \ \cup \ UP_7(a,t) \cup \ UP_8(a,t) \ \cup \ UP_9(a,t)\right) .$$

5.2 Dataset under Consideration

In order to understand the distribution of usage patterns in the dataset, we employ the 'Dataset 2' introduced in Chapter 3. Dataset 2 is consisted of two application sub-categories 'Casual' and 'Puzzle.' We denote the sets of applications and devices in those sub-categories by,

$A(S)$: The set of applications in sub-category $S \in \{ Casual, Puzzle \}$

$D(a,k)$: The set of devices having application a in month $k \in T$

$D(A(S),k)$: The set of devices having at least one application in $A(S)$ in month $k \in T$. i.e.

(5.13) $D(A(S),k) = \bigcup_{a \in S} D(a,k)$.

(5.14) $D(A(S)) = \bigcup_{k \in T} D(A(S),k)$.

Table 5.12 below shows the actual data set for $D(A(S),k)$ and $D(A(S))$ for $S \in \{Casual, Puzzle\}$ and $k \in T$ for the 21 month period from January 2013 through September 2014. According to usage pattern definitions, we reserve the first 5 months $t-k$, where $k = 1,2,\dots,5$ for determining the usage patterns of the month $t = 6$.

Table 5. 12 - Number of Devices Having Application(s) in Considered Dataset

Month	t	Casual		Puzzle	
		|D(S,t)|	%	|D(S,t)|	%
2013-01	1	50733	26.4	78784	29.7
2013-02	2	53974	28.0	80420	30.3
2013-03	3	54776	28.5	79472	29.9
2013-04	4	51286	26.6	77566	29.2
2013-05	5	48737	25.3	74837	28.2
2013-06	6	41926	21.8	69517	26.2
2013-07	7	48305	25.1	73088	27.5
2013-08	8	47954	24.9	80044	30.2
2013-09	9	47954	24.9	76540	28.8
2013-10	10	47062	24.4	72607	27.4
2013-11	11	42778	22.2	67217	25.3
2013-12	12	39936	20.7	61206	23.1
2014-01	13	36435	18.9	56065	21.1
2014-02	14	32119	16.7	80415	30.3
2014-03	15	29057	15.1	50335	19.0
2014-04	16	27933	14.5	46393	17.5
2014-05	17	25719	13.4	41986	15.8
2014-06	18	22577	11.7	38165	14.4
2014-07	19	20985	10.9	36145	13.6
2014-08	20	18879	9.8	33755	12.7
2014-09	21	17743	9.2	31150	11.7
|D(S)|		192529	100.0	265446	100.0

5.3 Distribution of Devices over Smartphone Application Usage Patterns

For the period of 16 months from June 2013 to September 2014, we observe following distribution of devices among the 10 application usage patterns in the considered two sub-categories ‘Casual’ and ‘Puzzle.’ The distribution ‘Casual’ sub-category is presented by Table 5.13 and Figure 5.1:

Table 5. 13 - Number of Devices with Each Usage Pattern (Sub-category: Casual)

	Null	Exploring	Rising	Stable	Matured	Warning	Ceasing	Recovering	Reactivating	Fickle
2013-06	169537	49200	24763	4387	15944	17274	80227	786	497	711
2013-07	141543	49530	22192	5378	17159	16606	76429	1415	881	812
2013-08	105740	50733	12517	3254	10951	15909	65066	867	486	842
2013-09	130213	19759	38374	3333	13834	10456	52021	799	0	750
2013-10	150045	26646	22334	3688	17886	7349	74435	456	480	168
2013-11	145361	28029	17997	3903	13895	15954	44750	615	426	321
2013-12	135477	22911	10902	7424	12957	15262	45783	869	276	434
2014-01	157340	22104	11373	149	15930	11871	31907	680	299	281
2014-02	176828	18153	10275	2862	14247	12196	49879	775	327	585
2014-03	190537	14206	7805	2438	13718	10544	45183	764	359	573
2014-04	201082	12977	6276	1943	12786	9465	40171	669	263	495
2014-05	212995	9907	6163	1874	12014	8078	33800	550	281	465
2014-06	224309	8339	5087	1361	11223	6738	27920	534	230	386
2014-07	233148	6760	3973	1269	10231	6209	23497	440	263	337
2014-08	239888	5413	2958	1433	9165	5852	20321	458	247	392
2014-09	245005	5767	2526	871	8586	5163	17285	401	169	354

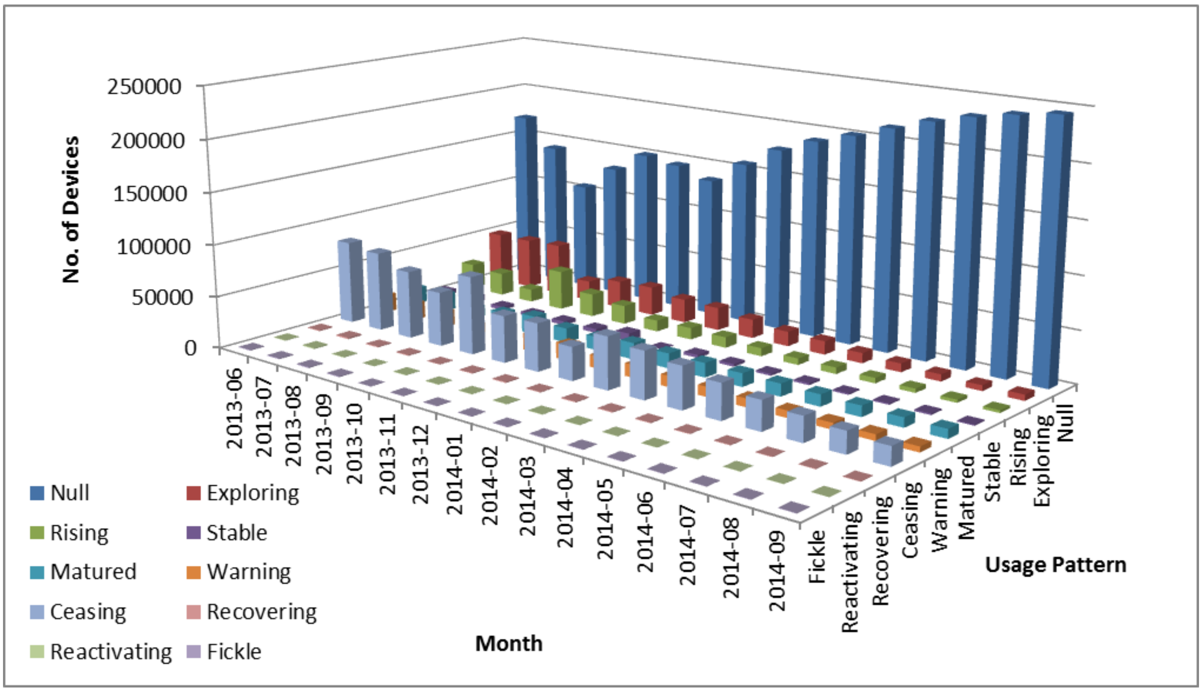


Figure 5. 1- Distribution of Devices According to Usage Patterns (Sub-category: Casual)

The same for sub-category ‘Puzzle’ are exhibited by the Table 5.14 and Figure 5.2.

Table 5. 14 - Number of Devices with Each Usage Pattern (Sub-category: Puzzle)

	Null	Exploring	Rising	Stable	Matured	Warning	Ceasing	Recovering	Reactivating	Fickle
2013-06	144093	47537	25187	4938	8144	12296	53314	422	406	472
2013-07	207251	73595	50633	10622	31042	29904	116017	2622	1823	1549
2013-08	100502	46869	13351	5123	9497	16889	57879	967	624	930
2013-09	120234	13888	35578	4787	14513	11708	49999	1077	0	847
2013-10	123784	26866	19825	3387	15438	6085	59241	466	462	170
2013-11	135828	31836	18961	3495	14911	15798	37524	581	391	361
2013-12	143973	26898	13187	7654	14494	16140	41820	873	358	428
2014-01	194342	26513	15039	166	17642	12866	33754	662	348	298
2014-02	174213	33732	13711	3103	14034	12550	48653	701	360	573
2014-03	180203	24398	21313	3027	13664	11436	46030	639	356	564
2014-04	187667	24580	20875	2393	13262	10346	40957	614	444	492
2014-05	196394	18423	20144	2849	12818	10454	38995	654	428	471
2014-06	205544	13714	12786	7385	12797	10785	37086	664	362	507
2014-07	211162	12801	9912	3841	16738	10184	35173	766	581	472
2014-08	219477	10050	9409	3161	17017	9622	31208	866	329	491
2014-09	226113	10993	8246	2456	16655	9053	26652	663	293	506

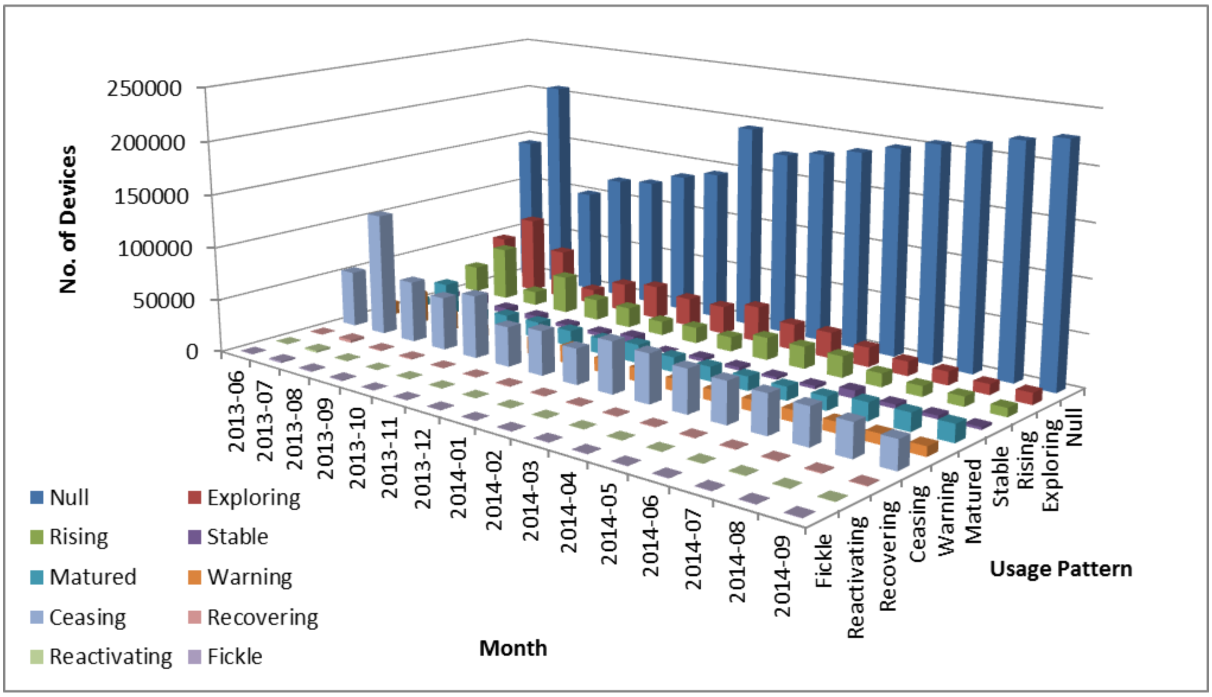


Figure 5. 2 - Distribution of Devices According to Usage Patterns (Sub-category: Puzzle)

By comparing the device distributions across usage patterns for 'Casual' and 'Puzzle' applications, one can understand that the number of users (smartphone devices) for all the applications are decreasing over the period of time, showing that the prevalence of applications does not hold for a longer time. This may be due to the new applications introduced to the market or dissatisfaction of users about the applications. Another possible explanation for this may be that some of the games are introduced to the market in two versions as 'Free' and 'Paid' where the free version consists of only a few levels of the game. Once a user wishes to play beyond these levels, he or she is compelled to convert to the paid version resulting the free version to be discontinued on the device. When it comes to ‘Puzzle’ applications, it is noticeable that the usage pattern ‘Rising’ exhibits several waves of growth and decrease while that is a decreasing trend with time for the ‘Casual’ applications. One can argue ‘Puzzle’ applications are more capable of maintaining the attraction of the users, perhaps by introducing new capabilities and features with periodical updates. Another noticeable point is that the ‘Puzzle’ sub-category possesses a relatively larger number of ‘Stable’ and ‘Matured’ devices compared to ‘Casual’ applications showing the steadiness of ‘Puzzle’ in the market place.

Chapter 6

Key Performance Measures Based on Usage Patterns

In Chapter 4, we introduced two types of key competitive performance measures for smartphone applications based on application downloads. Those measures are defined for a specific month of interest. In this chapter, we try to define the key competitive performance measures for a period of six months so as to reflect the effects of underlying application usage patterns on the performance measures. Further, we expect to reduce the performance measures' susceptibility of abrupt variations during individual months, thereby to reduce the resulting sudden shocks. Accordingly, here, we re-define the two key performance measures introduced in Chapter 4 along with three more new key performance measures based on application usage patterns introduced in Chapter 5. These five performance measures can describe the smartphone application characteristics distinct aspects such as application's market share, stability, popularity, potential and risks of persisting in the marketplace.

6.1 Five Key Competitive Performance Measures

The five key competitive performance measures we define in this work are: (1) Application Device Ratio, (2) Application Stability, (3) Application Popularity, (4) Application Advancement and (5) Application Declination. In order to define these performance measures, we adhere to following notation.

S: Application category or sub-category under consideration

$A(S)$: Set of applications in the considered application category or sub-category

$D(A(S))$: Set of devices having any of the applications in $A(S)$ present

We formally write $D(a,k)$, the set of devices having application $a \in A(S)$ in month $k \in T$ which is previously defined in Chapter 5 as,

(6.1) $$D(a,k) = \{ d \in D(A(S)) | \, b(a,d,k) = 1 \} \quad .$$

Similarly, we define the set of devices having application $a \in A(S)$ in at least one month during the period $I(t) \subseteq T$ as

(6.2) $$D\big(a, I(t)\big) = \cup_{k \in I(t)} D(a,k)$$

$$= \{ d \in D(A(S)) \mid b(a,d,k) = 1 \, for \, some \, k \in I(t) \} \quad .$$

The set of devices having some applications $a \in A(S)$ in month k is defined as

(6.3) $$D(A(S),k) = \cup_{a \in A(S)} D(a,k)$$

$$= \{ d \in D(A(S)) \,\big|\, b(a,d,k) = 1 \, for \, some \, a \in A(S) \} \quad .$$

The set of devices having some application $a \in A(S)$ in at least one month during the period $I(t) \subseteq T$ is given by

(6.4) $$D\big(A(S), I(t)\big) = \cup_{k \in I(t)} D(A(S),k)$$

$$= \{ d \in D\big(A(S)\big) | b(a,d,k) = 1 \, for \, some \, a \in A(S) \, and \, k \in I(t) \} \quad .$$

We are now in a position to introduce the five performance measures mentioned above for each application.

6.1.1 Application Device Ratio

We name the first key performance measure as Application Device Ratio, and it is concerned with the relative size of the user-base of an application and can be consider as an indicator of applications market share. The Application device ratio of application $a \in A(S)$ for the period of $I(t) \subseteq T$ is defined as

$$(6.5) \quad App\text{-}Dev\big(a, I(t)\big) = \frac{|D(a,I(t)|}{\big|D\big(A(S),I(t)\big)\big|} \qquad (defined\ for\ t \in T\backslash\{1,2,3,4,5\}) \quad .$$

6.1.2 Application Stability

The next key performance measure is called Application Stability, and it is concerned with the consistency of the user-base of an application. We denote Application Stability by $App\text{-}Stab(a, I(t))$ and define it as the portion of the number of devices with application usage pattern of either 'Matured' or 'Stable' over $D(a, I(t))$ during the period (t) . More specifically, write it as

$$(6.6) \quad App\text{-}Stab(a, I(t)) \ = \ w(a, I(t),\ Matured(a,t)) \ + \ w(a, I(t),\ Stable(a,t)),$$

where

$$(6.7) \quad w\left(a, I(t), UP_j(a,t)\right) \ = \frac{|D(UP_j(a,t))|}{|D\big(a,I(t)\big)|} \quad (defined\ for\ t \in T\backslash\{1,2,3,4,5\}).$$

6.1.3 Application Popularity

We refer to the third key performance measure as Application Popularity. It is concerned with the popularity of a smartphone application in the application market. We define Application Popularity for an application $a \in A(S)$ during the period $I(t)$ as the portion of the cumulative number of months for $d \in D(a, I(t))$ to possess $a \in A(S)$ over the period $I(t)$. More formally, Application Popularity, denoted by $App\text{-}Pop(a, I(t))$ for $a \in A(S)$ and $I(t) \subseteq T$, is given by

$$(6.8) \quad App\text{-}Pop\big(a, I(t)\big) = \frac{\sum_{d \in D(a,I(t))} \sum_{k \in I(t)} b(a,d,k)}{6 \times |D(a,I(t))|}$$

$$\big(defined\ for\ t \in T \backslash \{1,2,3,4,5\}\big) \quad .$$

The calculation of this measure is illustrated below in Figure 6.1

$I(t)$

(Application: a)

Device		t-5	t-4	t-3	t-2	t-1	t
d_1		1	0	1			0
d_2		0	1	1			1
d_3		0	0	0			1
...							
d_n		0	0	1			1

Numerator: Sum of all 1's

Denominator: 6 x No. of devices in $D(a, I(t))$

Figure 6. 1 - Calculation of Application Popularity, App-Pop(a, I(t))

6.1.4 Application Advancement

The next key performance measure is named as Application Advancement. It is concerned with an application's potential of growing in the market. We define Application Advancement considering the availability of application usage patterns in the positive direction. We write the Application Advancement of application $a \in A(S)$ for the period of $I(t) \subseteq T$ as

$$(6.9) \quad App\text{-}Adv\big(a, I(t)\big) = w(a, I(t),\ Rising) + w(a, I(t),\ Recovery) + w(a, I(t),\ Reactivate)\ .$$

6.1.5 Application Declination

The last key performance measure is referred to as Application Declination. It is concerned with the risk of an application being rejected from the market. Considering the availability of usage patterns in the negative direction, we define the Application Declination of application $a \in A(S)$ for the period of $I(t) \subseteq T$ as

$$(6.10) \quad App\text{-}Dec(a, I(t)) = w(a, I(t), Ceasing) + w(a, I(t), Warning)$$

6.2 Numerical Example

Considering the 'Dataset 2' described in Section 3.3.3, we calculated the values of five performance measures for the applications under consideration. Respective values of five key competitive performance measures of all the applications in 'Dataset 2' are included in the Appendix in Tables A.1 through A.10. Due to the limitations of space, here we discuss about the values of five key competitive performance measures for only two applications each from the 'Casual' and 'Puzzle' sub-categories: 'MindStep,' '1 Million eggs' from 'Casual' sub-category; LINE Hidden Catch,' 'Pop Star for Android' from 'Puzzle' sub-category. Figure 6.2 depicts these values for the four applications, over the three month period from June 2013 ($k = 1$) through August 2013 ($k = 3$).

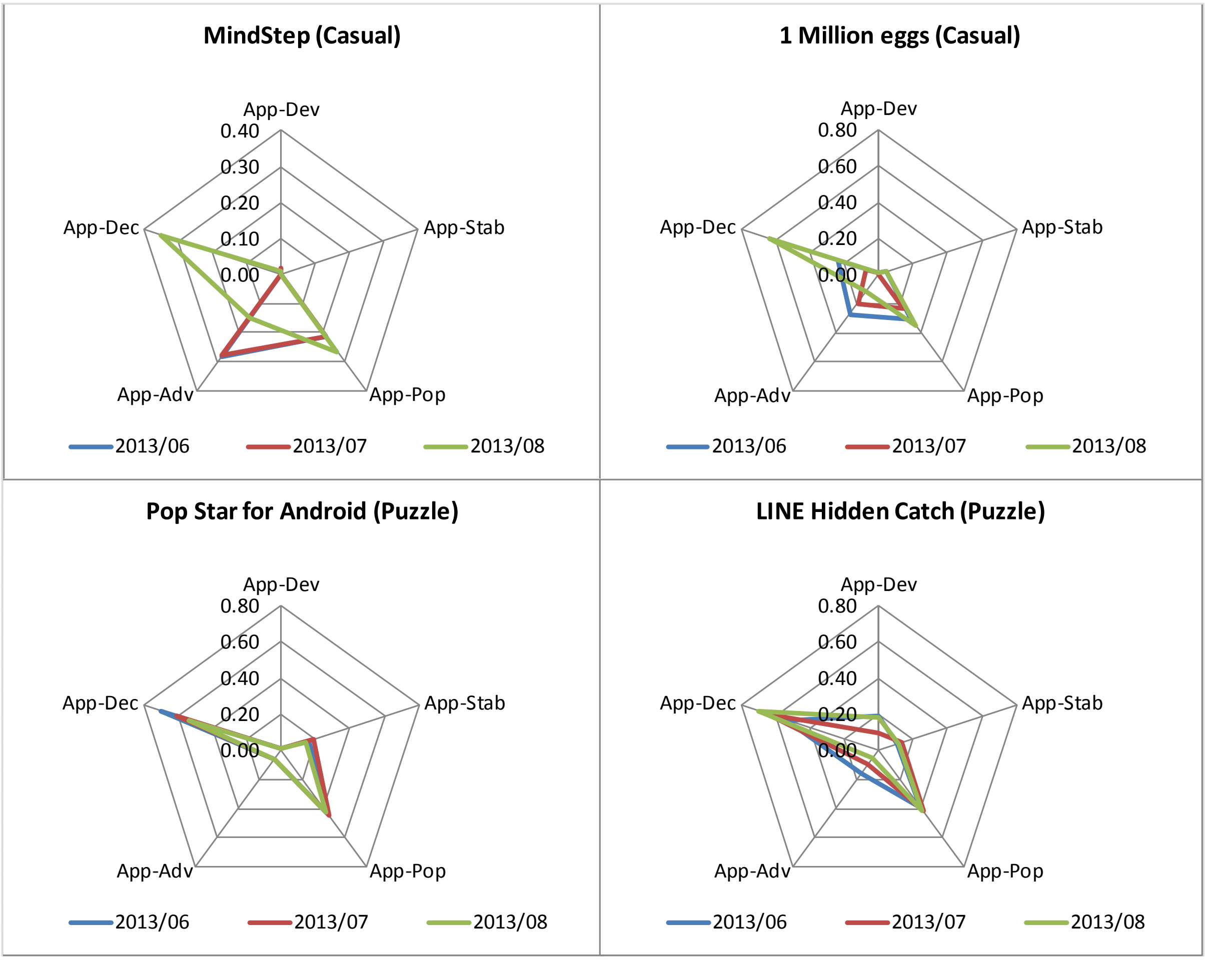


Figure 6. 2 - Five Performance Measures for the Month 2013/06 through 2013/07

In Figure 6.2, one can observe relatively different distributions of five performance measures over three months period. For 'Mind Step' although the $App\text{-}Pop(a, I(t))$ has increased in the third month ($k = 3$), application usage patterns over the 6 month period $I(3)$ indicate that the applications potential of advancing $App\text{-}Adv(a, I(t))$ is decreasing while the risk of decreasing in the market, $App\text{-}Dec(a, I(t))$ is increasing. When it comes to '1 Million eggs,' it shows the gradual decrease in $App\text{-}Adv(a, I(t))$ while increasing $App\text{-}Dec(a, I(t))$ over the time. Relatively high $App\text{-}Pop(a, I(t))$ value and very low $App\text{-}Dev(a, I(t))$ and $App\text{-}Stab(a, I(t))$values in both cases indicate that although the application being installed by the devices, they do not experience any continued use of the application under consideration. The performance measures of the application 'Pop Star for Android' do not show any vicious changes. Accordingly one can observe a slow growth in $App\text{-}Stab(a, I(t))$

and $App\text{-}Pop\big(a, I(t)\big)$ with the reducing risk of being rejected from the market, $App\text{-}Dec\big(a, I(t)\big)$. 'LINE Hidden Catch' exhibits somewhat opposite scenario of this.

6.3 Specific Applications

By observing the applications in all five aspects described by the five key performance measures, throughout the 16 month period from June 2013 to September 2014, we find certain applications hold the strength of maintaining the top position compared to other applications in the dataset. These applications can be suggested as model applications for application developers to refer. 'Mushroom Garden,' 'Nyanko Dai Senso' and 'COLPL' are such applications from the sub-category 'Casual' that maintain relatively high values for Application Device Ratio, Application Stability, Application Popularity and a relatively low value for Application Declination. However, it was difficult to find any Casual applications in the dataset that consistently maintain a high value for Application Advancement. Perhaps, this may be due to the high competition between the applications that no application can keep advancing continuously in the market throughout a long period of time. When it comes to 'Puzzle' sub-category, 'LINE Disney Tsum Tsum' can be considered as a successful application which maintain a relatively high market share (Application Device Ratio), Application Advancement and low Application Declination values during the period. Although the Application Stability and Application Popularity values are not in the top compared to other applications, we notice that these two values increase with time implying that this application show positive trends in all five perspectives. Apart from that, it is worth to point another 'Puzzle' application 'Chokokushi,' which maintains relatively high value for Application Stability with increasing Application-Advancement and decreasing Application Declination values over the period.

Chapter 7
Statistical Estimation of Key Performance Measures

In the previous chapter, we introduced five different key performance measures for smartphone applications. In order to make these performance measures useful in drawing business implications, it is worthwhile to have a mechanism of deriving the future values of performance measures. In this chapter, we develop statistical approaches of estimating the key competitive performance measures defined in Chapter 6.

Algorithms for estimating future values based on past data can be classified into two categories. One is the class of algorithms relying upon past sequential values of the entity to be estimated. The other category is the class of algorithms involving variables expected to have functional relationships with the entity to be estimated. In this work, we use Auto Regressive Integrated Moving Average (ARIMA) models as well as linear regression methods that represent these two classes respectively. It is known that the ARIMA models would work better when the environmental conditions are under a strong inertia of the past. On the other hand, the linear regression model has the advantage when the environmental conditions are subject to substantial changes in a short period. In order to take the advantages and contain the disadvantages of the two approaches, we experiment several approaches by employing them in different stages in different combinations. As a result, we introduce a novel approach developed for estimating future values by combining the ARIMA model with the linear regression model.

7.1 Dataset under Consideration

In this chapter, we perform our analysis on Dataset 2 described in Chapter 3. The dataset is consisted of application and application usage data of 130 smartphone applications from two free game sub-categories ‘Casual’ and ‘Puzzle.’ Application usage data spans over a period of 21 months from January 2013 to September 2014. In order to facilitate our analysis, we divide this 21 month period into 3 sections as depicted in Figure 7.1.

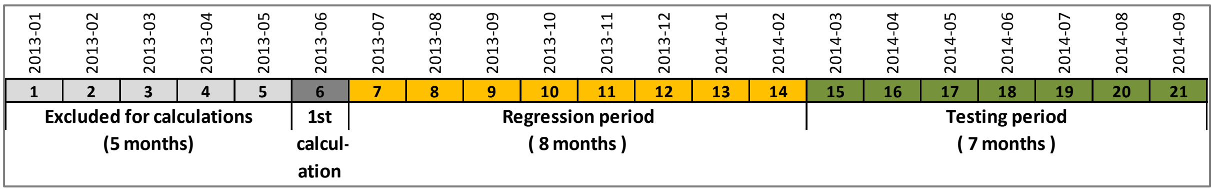


Figure 7. 1 - Division of Data Period

We introduced 10 application usage patterns and 5 key competitive performance measures introduced in Chapter 5 and Chapter 6 respectively. All those are defined based on a six month period $I(t)$ ending in month $t \in T$. Therefore, in order to determine a usage pattern or performance measure in a certain month $t \in T$, it is necessary to have data of 5 month period just prior to month t. Hence in Figure 7.1, the first 5 month period from January 2013 ($k = 1$) to May 2013 ($k = 5$) is kept reserved for the usage pattern/performance measure calculations of June 2013 ($k = 6$). Similarly, the next 5 month period from February 2013 ($k = 2$) to June 2013 ($k = 6$) is kept reserved for the usage pattern/performance measure calculations of July 2013 ($k = 7$) and so on.

Table 7.1 lists the information we extracted from the dataset as independent and dependent variables for the statistical analysis and model generation. Here, the variables 10 through 18 refer to $w(a, I(t),\ UP_j(a, t))$ defined in Chapter 6 for the respective usage pattern UP_j.

Table 7. 1 – List of Variables

#	Variable Name	Description
1	App-Dev(a , I(t))	Application Device Ratio
2	App-Stab(a , I(t))	Application Stability
3	App-Pop(a , I(t))	Application Popularity
4	App-Adv(a , I(t))	Application Advancement
5	App-Dec(a , I(t))	Application Declination
6	log|D(a , t)|	Logarithm of number of devices having application a in month t
7	log(Launch_count(a , t))	Logarithm of total number of launches for application a on devices in D(a , t) in month t
8	log(Act_days(a , t))	Logarithm of total number of active days for application a on devices in D(a , t) in month t
9	log(Removed(a , t))	Logarithm of total number of devices in D(a , t) which removed application a during month t
10	|Exploring(a , t)|	Portion of devices having usage pattern Exploring for application a in month t
11	|Rising(a , t)|	Portion of devices having usage pattern Rising for application a in month t
12	|Stable(a , t) |	Portion of devices having usage pattern Stable for application a in month t
13	|Matured(a , t) |	Portion of devices having usage pattern Matured for application a in month t
14	|Warning(a , t)|	Portion of devices having usage pattern Warning for application a in month t
15	|Ceasing(a , t)|	Portion of devices having usage pattern Ceasing for application a in month t
16	|Recovering(a , t)|	Portion of devices having usage pattern Recovering for application a in month t
17	|Reactivating(a , t)|	Portion of devices having usage pattern Reactivating for application a in month t
18	|Fickle(a , t)|	Portion of devices having usage pattern Fickle for application a in month t

In order to be consistent, we set all the variables to be between 0 and 1 by normalization. This is carried out in the following manner. Let $X(a,t)$ denote the variable under consideration. Let $x(a,t)$ be a single instance of the variable and $x_{MIN}(a,t)$ and $x_{MAX}(a,t)$ be the minimum and maximum values of the variable in the considered data. Then the normalized value of $x(a,t)$ would be,

$$\frac{x(a,t)-x_{MIN}(a,t)}{x_{MAX}(a,t)-x_{MIN}(a,t)} \in [0,1] \quad .$$

7.2 ARIMA Models Approach for Future Value Estimation

Auto Regressive Integrated Moving Average Models, which is well known as ARIMA models are widely used in time-series data to find relationships of a variable based on its historical values. Let $Y(a,k)$ be the value of the variable under consideration in month k for application a. We refer to Autoregressive (AR) models as the models that the value of a variable in a certain time period is related to its past values in the previous time periods. We can define such model by, (7.1) $Y(a,k) = c + \sum_{i=1}^{p} \phi_i Y(a,k-i) + \varepsilon_k$.

Where, c is a constant, ϕ_i is the coefficient for the past variable in time $k-i$ and p is the order of the AR process. We call the error term ε_k the white noise. We refer to Moving Average (MA) models as the models which a variable at a certain time period may possibly have a relationship with its residuals from the past time periods. Moving Average model can be defined by,

(7.2) $Y(a,k) = c + \varepsilon_k + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i}$.

where, θ_i is the coefficient for the past error term in time $k-i$ and q is the order of the MA process. ARIMA model is a combination of differencing with autoregressive model and moving average model. This can be presented by,

(7.3) $Y_d(a,k) - \varepsilon_k = \sum_{i=1}^{p} \phi_i Y_d(a,k-i) + \sum_{j=1}^{q} \theta_j \varepsilon_{k-j}$.

Here, $Y_d(a,k)$ is the d-times difference of $Y(a,k)$, that is, $\Delta^d[Y(a,k)] = \Delta\left[\Delta^{d-1}[Y(a,k)]\right]$ starting with $\Delta^1[Y(a,k)] = Y(a,k) - Y(a,k-1)$. The first term on the right hand side of the formula (7.3) represents the auto-regressive process defined on $Y(a,k-i),\ i = 1,\dots,p$, while the second term describes the moving average of the estimation errors ε_{k-j} , $j = 1,\dots,q$. In order to emphasize the three parameters, we often write ARIMA (*p*, *q*, *d*). The coefficients $\phi_i,\ i = 1,\dots,p$ and $\theta_j,\ j = 1,\dots,q$ are determined by minimizing the sum of the squared errors over a set of learning data. In this research, we employ the ARIMA tool provided by the R software, which provides the best fit set of the three parameters (*p*, *q*, *d*).

7.3 Linear Regression Approach for Future Value Estimation

In the linear regression approach, we try to model the underlying relationship between a dependent variable and a set of independent variables. We expect these independent variables to have a functional relationship with the dependent variable under consideration. We try to construct statistical models for the dependent variables, so that it can be described by the independent variables. We denote the dependent variable by $Y(a,k)$ and the set of independent variables by $x_i(a,k-1), i = 1,\dots,N$. We assume that $Y(a,k)$ is to be described by $x_i(a,k-1)$ in a linear form. More specifically,

(7.4) $Y(a,k) = \beta_0 + \sum_{i=1}^{N} \beta_i x_i(a,k-1) + \varepsilon_k$,

where, $\beta_i,\ i = 0,\dots,N$ are the coefficients and ε_k is the error term. The coefficients are determined by minimizing the sum of the squared errors over $a \in A(S)$. The resulting coefficients are then common for all $a \in A(S)$.

In order to construct successful statistical model by linear regression, it is essential to identify the best set of independent variables for describing the dependent variable. It can be achieved by adhering to following two processes:

(1) Multi-co-linearity check

In a set of independent variables expected to be used to describe a dependent variable, it is possible to exist variables which are correlated to each other. Such pairs of strongly correlated independent variables may introduce unnecessary redundancy to the regression process. This situation which is called ‘multi-co-linearity’ should be avoided by excluding any of the independent variables in the pair. This process should be carried out until all the strong correlations among independent variables are eliminated. In this work, we set the threshold level of judging the strong correlation to the value 0.7 and one variable from the pairs with higher correlation than that are excluded.

(2) Statistical significance check

After excluding the highly correlated variables, it is important to verify that the remaining independent variables are statistically significant. When a certain variable is statistically insignificant, inclusion of that variable could not improve the statistical model any further. In this work, we set the significance level to 0.05 and t-value greater than 2 in order to select a variable. Variables not satisfying those significance level conditions were excluded.

7.3.1 Variable Selection Methods

In linear regression approach, there are several types of variable selection mechanisms. In order to obtain the most successful set of explanatory independent variables, we adopt four types of variable selection methods as explained below:

(1) All Subset Regression

We carry out the regression process by including all the independent variables under consideration at the same time. Then, we derive the best model by extracting only the significant variables. We use SPSS software to perform this process.

(2) Backward Regression

We start the regression process by including all the independent variables under consideration. Then, we compare the R^2 value of this with that obtained by removing one variable. The retained set of independent variables is determined by as the one achieving the maximum R^2 value. We continue to remove one variable at a time in the same manner until the highest R^2 is reached. R^2 is defined by,

$$R^2 = 1 - \frac{SSE}{SST} \quad . \tag{7.5}$$

where SSE and SST stand for Sum Squared Errors and Total Sum of Squares respectively. We use SPSS software to perform this process.

(3) Stepwise Regression

We start the regression process with combinations of two of independent variables under consideration. We compare these R^2 values with those obtained by including another independent variable. Then we pick the top 10 combinations having the highest R^2 values. We continue the same process and finally we select the model with the highest R^2 value. We perform this process using R software.

(4) Optimization of AIC Value

We start the AIC process by including all the independent variables under consideration. We then compare this AIC value with that obtained by removing one of the independent variables. The retained set of independent variables is determined by as the one achieving the minimum AIC value. We repeat this process until the AIC value can no longer be reduced. AIC value define by,

$$(7.6) \quad AIC = n \times ln\left(\frac{SSR}{n}\right) + 2 \times K \quad .$$

Here, n, SSR and K represent the sample size, Sum of Squared Residuals and sample size respectively. We perform this process using R software.

7.3.2 Statistical Model Generation

In order to construct statistical models for the five dependent variables listed from (1) to (5) in Table 7.1, we carry out linear regression. Each regression process is consisted of several steps. As the first step, we decide the dependent variable, and we consider the remaining 17 variables as the set of independent variables. Then, we eliminate highly correlated variables by performing multi-co-linearity check as described above. Next we separately practise the four variable selection criteria for each dependent variable. Since it is likely to have a difference between different game sub-categories, we perform these steps separately on the two free game sub-categories 'Casual' and 'Puzzle' of each of those dependent variables. Therefore, in this work we perform 40 separate linear regression processes as shown in Figure 7.2.

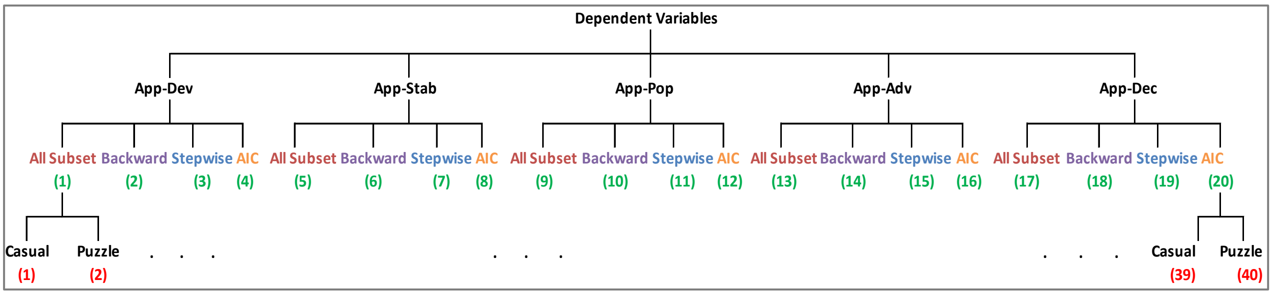


Figure 7. 2 - Linear Regression Processes

By each linear regression process, we try to model the relationships between the considered dependent variable and the set of remaining independent variables. As explained in formula (7.4), we start with calculating the independent variables involving $I(t)$ of month $k = 6$ and consider the dependent variable to be from month $k = 7$. We repeat the same for all the 40 scenarios given in Figure 7.2. Thereafter, we calculate these values through the rolling horizon approach by shifting the period $I(t)$ by one month. Through the steps described above, regression analysis is conducted over $a \in A(S)$ for the months of $k = 7, \ldots, 14$. Regression analysis performed during the Regression period for a single scenario is depicted in Figure 7.3.

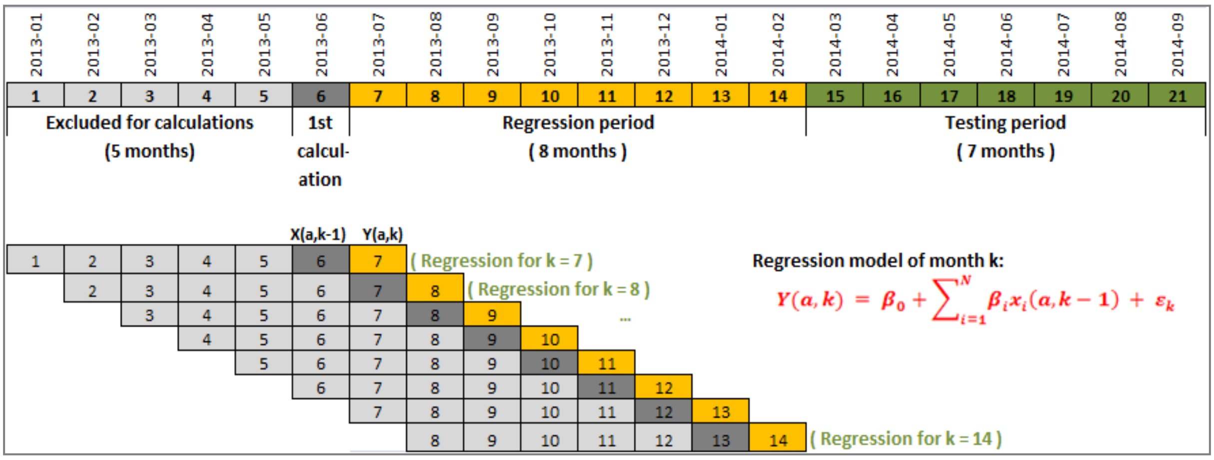


Figure 7. 3 - Regression Analysis of a Single Scenario

7.3.3 Construction of the Unified Model

After performing regression analysis for the period from $k = 7$ through $k = 14$, one obtains 8 different regression models respective to each month $k = 7, 8, \ldots, 14$. These monthly models may consist of different sets of independent variables and different values of coefficients associated to them as listed in Table 7.2.

Table 7. 2 - Monthly Regression Models

	k=7	k=8	k=9	k=10	k=11	k=12	k=13	k=14
Adj R2	0.296	0.409	0.971	0.972	0.952	0.924	0.933	0.869
Intercept	-0.060	-0.092	0.036	-0.175	-0.126	-0.006	-0.184	-0.239
App-Dev(a , I(t))			0.257		-0.786			
App-Pop(a , I(t))	0.448	0.391		0.889	0.716		1.005	0.739
App-Adv(a , I(t))								
App-Dec(a , I(t))								
log|D(a , t)|		0.063			0.117			
log(Launch_count(a , t))			-0.039	-0.018		0.038		0.060
log(Act_days(a , t))								
log(Removed(a , t))					-0.078	-0.049		
|Exploring(a , t)|								
|Rising(a , t)|				-0.111	-0.303	0.130	-0.205	
|Stable(a , t) |			1.610		-0.301	0.311	-0.712	
|Matured(a , t) |			1.359			0.779		
|Warning(a , t)|					0.767	0.670		
|Ceasing(a , t)|								
|Recovering(a , t)|				1.371		-1.603		
|Reactivating(a , t)|			-1.451			1.799		1.468
|Fickle(a , t)|				-2.261	8.154			-1.970

Dependent Variable: App-Stab(a , I(t))

Variable Selection Method: Optimization of AIC Value

Game Sub-Category: Casual

Legend :	
	Variable is excluded due to multi-collinearity
	Variable is not chosen by the regression analysis
x.xx	Variable with a positive coefficient in the regression model
x.xx	Variable with a negative coefficient in the regression model

In this table, the 8 columns $k = 7, 8, \ldots, 14$ represent the regression model of the respective month. The values in the cells represent the coefficient of the respective independent variable. We term the number of times an independent variable appears in the regression models of the considered 8 periods as 'Appearance.' We also term the number of changes of the sign of the coefficient from an appearance to the next appearance of an independent variable as 'Sign change.' Nevertheless, we do not consider a non-appearance as a sign change.

However, in order to move forward with future value estimations and validation of the constructed models, it is necessary to obtain a single model which can reasonably represent the different statistical models of months $k = 7, 8, \ldots, 14$. Therefore, we construct a single unified model by considering all the monthly models. In order to determine the set of independent variables in the unified model, we introduce the following variable selection criteria given by (V1) and (V2). Only those independent variables satisfying the conditions are chosen for the unified model.

(V1) Independent variables with 4 or more appearances in the 8 regression models of the months $k = 7, 8, \dots, 14$

(V2) Independent variables with 2 or less sign changes in the 8 regression models of the months $k = 7, 8, \dots, 14$

Once the independent variables of the unified model are determined by the variable selection rules, we calculate the intercept and variable coefficients of the unified model by adhering to following mechanism given from (M1) through (M3):

(M1) Calculation of the intercept of the unified model

Average of intercept values of the monthly regression models in which at least one variable is selected for the unified model

(M2) Determination of the sign of the coefficient of a variable

Majority sign of the coefficients of the variable. In the case of having similar number of positive and negative coefficients, the average values of positive and negative coefficients are considered separately. The respective sign of the average value with highest absolute value is chosen as the sign of the independent variable in the unified model

(M3) Calculation of the value of the coefficient of a variable

Average of coefficient values of the variable with the selected sign

In the Table 7.3, we present the application of variable selection rules and unified model construction for the regression models given in Table 7.2. The numbers highlighted by red font represents the satisfactory cases when variable selection rules are applied.

Table 7. 3 - Construction of Unified Model

	k=7	k=8	k=9	k=10	k=11	k=12	k=13	k=14	No. of Appearan-ces	No. of sign changes	Variable Chosen for Unified Model	Coefficient Values of Unified Model
Adj R2	0.296	0.409	0.971	0.972	0.952	0.924	0.933	0.869				
Intercept	-0.060	-0.092	0.036	-0.175	-0.126	-0.006	-0.184	-0.239				-0.126
App-Dev(α, I(t))			0.257		-0.786				2	1	NO	
App-Pop(α, I(t))	0.448	0.391		0.889	0.716		1.005	0.739	6	0	YES	0.698
App-Adv(α, I(t))									0	0	NO	
App-Dec(α, I(t))									0	0	NO	
log|D(α, t)|		0.063			0.117				2	0	NO	
log(Launch_count(α, t))			-0.039	-0.018		0.038		0.060	4	1	YES	0.049
log(Act_days(α, t))									0	0	NO	
log(Removed(α, t))					-0.078	-0.049			2	0	NO	
|Exploring(α, t)|									0	0	NO	
|Rising(α, t)|				-0.111	-0.303	0.130	-0.205		4	2	YES	-0.207
|Stable(α, t) |			1.610		-0.301	0.311	-0.712		4	3	NO	
|Matured(α, t) |			1.359			0.779			2	0	NO	
|Warning(α, t)|					0.767	0.670			2	0	NO	
|Ceasing(α, t)|									0	0	NO	
|Recovering(α, t)|				1.371		-1.603			2	1	NO	
|Reactivating(α, t)|			-1.451			1.799		1.468	3	1	NO	
|Fickle(α, t)|				-2.261	8.154			-1.970	3	2	NO	

The Tables 7.4 and 7.5 lists respectively the unified models of Casual and Puzzle sub-categories constructed using above described mechanism. We use these linear regression models for estimation of five key performance measures over the period $k = 14, 15, \dots, 21$.

Table 7. 4 - Unified Linear Regression Models (Sub-category: Casual)

<table>
<tr><th>Sub-category</th><th>Performance Measure</th><th>Regression Method</th><th>Unified Linear Regression Model</th></tr>
<tr><td rowspan="24">**Casual**</td><td rowspan="4">App-Dev(a, I(t))</td><td>All subset</td><td>-0.003 + 0.361 App-Pop(a, I(t-1)) + 0.874 log|D(a, t-1)|</td></tr>
<tr><td>Backward</td><td>-0.003 + 0.365 App-Pop(a, I(t-1)) + 0.869 log|D(a, t-1)| + 0.474 log(Removed(a, t-1))</td></tr>
<tr><td>Stepwise</td><td>-0.004 + 0.028 App-Pop(a, I(t-1)) + 0.051 log|D(a, t-1)| + 0.017 log(Removed(a, t-1))</td></tr>
<tr><td>AIC</td><td>-0.004 + 0.031 App-Pop(a, I(t-1)) + 0.05 log|D(a, t-1)| + 0.017 log(Removed(a, t-1))</td></tr>
<tr><td colspan="3"></td></tr>
<tr><td rowspan="4">App-Stab(a, I(t))</td><td>All subset</td><td>-0.144 + 0.756 App-Pop(a, I(t-1)) - 0.344 |Rising(a, t-1)|</td></tr>
<tr><td>Backward</td><td>-0.128 + 0.752 App-Pop(a, I(t-1)) + 0.12 log(Launch_count(a, t-1))
- 0.335 |Rising(a, t-1)|</td></tr>
<tr><td>Stepwise</td><td>-0.126 + 0.703 App-Pop(a, I(t-1)) + 0.044 log(Launch_count(a, t-1))
- 0.204 |Rising(a, t-1)|</td></tr>
<tr><td>AIC</td><td>-0.126 + 0.698 App-Pop(a, I(t-1)) + 0.049 log(Launch_count(a, t-1))
- 0.207 |Rising(a, t-1)|</td></tr>
<tr><td colspan="3"></td></tr>
<tr><td rowspan="4">App-Pop(a, I(t))</td><td>All subset</td><td>0.267 + 0.785 |Matured(a, t-1)|</td></tr>
<tr><td>Backward</td><td>0.268 + 0.791 |Matured(a, t-1)|</td></tr>
<tr><td>Stepwise</td><td>0.267 + 0.857 |Matured(a, t-1)| + 0.646 |Stable(a, t-1)|</td></tr>
<tr><td>AIC</td><td>0.27 + 0.847 |Matured(a, t-1)| + 0.646 |Stable(a, t-1)|</td></tr>
<tr><td colspan="3"></td></tr>
<tr><td rowspan="4">App-Adv(a, I(t))</td><td>All subset</td><td>0.179 - 0.278 App-Pop(a, I(t-1)) + 0.17 log(Launch_count(a, t-1))
+ 0.493 |Rising(a, t-1)|</td></tr>
<tr><td>Backward</td><td>0.182 - 0.291 App-Pop(a, I(t-1)) - 0.261 App-Dev(a, I(t-1))
+ 0.175 log(Launch_count(a, t-1)) + 0.492 |Rising(a, t-1)|</td></tr>
<tr><td>Stepwise</td><td>0.181 - 0.232 App-Pop(a, I(t-1)) + 0.168 log(Launch_count(a, t-1))
+ 0.291 |Rising(a, t-1)|</td></tr>
<tr><td>AIC</td><td>0.198 - 0.238 App-Pop(a, I(t-1)) - 3.084 App-Dev(a, I(t-1))
+ 0.167 log(Launch_count(a, t-1)) + 0.283 |Rising(a, t-1)|</td></tr>
<tr><td colspan="3"></td></tr>
<tr><td rowspan="4">App-Dec(a, I(t))</td><td>All subset</td><td>0.304 + 0.296 App-Dev(a, I(t-1)) + 0.186 |Fickle(a, t-1)|</td></tr>
<tr><td>Backward</td><td>0.401 + 0.249 App-Dev(a, I(t-1)) - 0.281 log|D(a, t-1)|
- 0.24 log(Launch_count(a, t-1))</td></tr>
<tr><td>Stepwise</td><td>0.245 + 6.045 App-Dev(a, I(t-1)) - 0.252 log(Launch_count(a, t-1))</td></tr>
<tr><td>AIC</td><td>0.401 + 4.998 App-Dev(a, I(t-1)) - 0.345 log|D(a, t-1)|
- 0.235 log(Launch_count(a, t-1))</td></tr>
</table>

Table 7. 5 - Unified Linear Regression Models (Sub-category: Puzzle)

Sub-category	Performance Measure	Regression Method	Unified Linear Regression Model
Puzzle	App-Dev(a, I(t))	All subset	-0.02 + 0.7 log|D(a, t-1)|
		Backward	-0.028 + 0.702 log|D(a, t-1)|
		Stepwise	-0.028 + 0.114 log|D(a, t-1)|
		AIC	-0.026 + 0.113 log|D(a, t-1)|
	App-Stab(a, I(t))	All subset	-0.146 + 0.947 App-Pop(a, I(t-1)) + 0.149 |Stable(a, t-1)|
		Backward	-0.146 + 0.949 App-Pop(a, I(t-1)) + 0.143 |Stable(a, t-1)|
		Stepwise	-0.146 + 0.903 App-Pop(a, I(t-1)) + 1.892 |Stable(a, t-1)|
		AIC	-0.145 + 0.903 App-Pop(a, I(t-1)) + 1.467 |Stable(a, t-1)|
	App-Pop(a, I(t))	All subset	0.224 + 0.901 |Matured(a, t-1)| + 0.21 |Stable(a, t-1)| + 0.276 |Rising(a, t-1)|
		Backward	0.225 + 0.128 App-Dev(a, I(t-1)) + 0.898 |Matured(a, t-1)| + 0.215 |Stable(a, t-1)| + 0.278 |Rising(a, t-1)|
		Stepwise	0.225 + 0.433 App-Dev(a, I(t-1)) + 0.973 |Matured(a, t-1)| + 0.93 |Stable(a, t-1)| + 0.253 |Rising(a, t-1)|
		AIC	0.225 + 0.417 App-Dev(a, I(t-1)) + 0.974 |Matured(a, t-1)| + 0.934 |Stable(a, t-1)| + 0.254 |Rising(a, t-1)|
	App-Adv(a, I(t))	All subset	0.244 - 0.232 |Stable(a, t-1)|
		Backward	0.336 - 0.236 |Stable(a, t-1)| - 0.414 |Ceasing(a, t-1)|
		Stepwise	0.182 - 0.206 |Ceasing(a, t-1)|
		AIC	0.327 - 3.252 |Stable(a, t-1)| - 0.222 |Ceasing(a, t-1)|
	App-Dec(a, I(t))	All subset	-0.013 + 0.377 |Stable(a, t-1)| + 0.765 |Ceasing(a, t-1)|
		Backward	-0.014 + 0.253 App-Pop(a, I(t-1)) + 0.175 App-Dev(a, I(t-1)) + 0.172 |Rising(a, t-1)| + 0.761 |Ceasing(a, t-1)|
		Stepwise	-0.002 + 0.528 App-Pop(a, I(t-1)) + 1.246 App-Dev(a, I(t-1)) + 3.132 |Stable(a, t-1)| + 0.478 |Rising(a, t-1)| + 0.887 |Ceasing(a, t-1)|
		AIC	-0.002 + 0.534 App-Pop(a, I(t-1)) + 1.232 App-Dev(a, I(t-1)) + 3.534 |Stable(a, t-1)| + 0.909 |Ceasing(a, t-1)|

7.4 Future Value Estimation of Key Competitive Performance Measures

In this research, we carry out experiments on four mechanisms of future value estimation for the five competitive performance measures introduced in Chapter 6. In these, we employ the ARIMA approach and linear regression approach separately as well as in different combinations to estimate the values of performance measures in near future. By each attempt, we expect to achieve the highest possible accuracy in estimation of future values. The four approaches of statistical estimation of future values of competitive measures are introduced in the following sections.

7.4.1 Future Value Estimation by Using Only ARIMA Models

In this, we perform the future value estimation in the standard manner for time series, by exclusively based on the ARIMA models. In the process of future value prediction with ARIMA models approach, it considers a series of past values of the variable to predict a number of future values of it in the near future. Here, we employ ARIMA approach to forecast performance measures relying upon its past values. More specifically, we consider the actual past values of the performance measure involving $I(t)$ in the months $k = 6, 7, \dots, 14$ to predict the value of that in the months $k = 15, 16, \dots, 21$ as depicted in Figure 7.4.

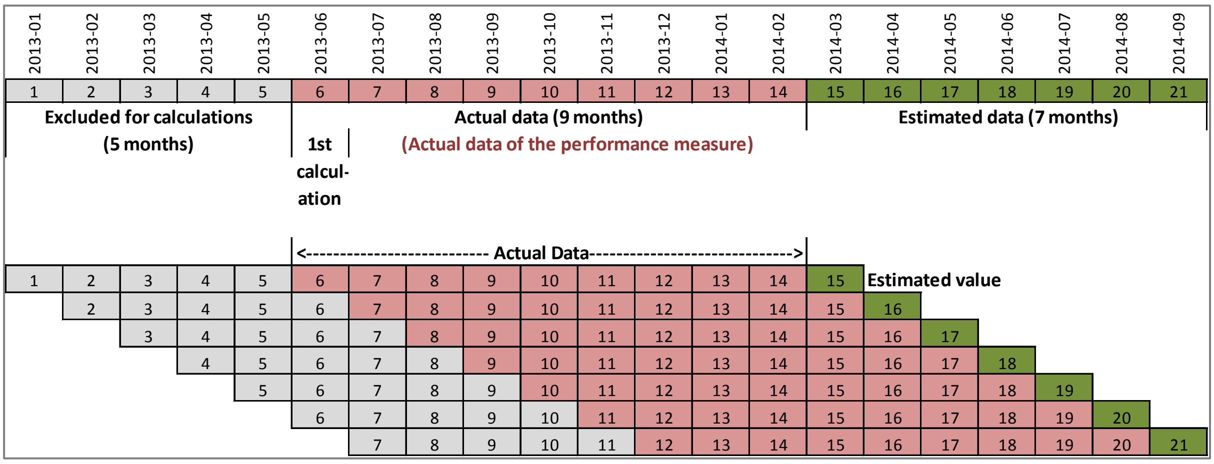


Figure 7. 4- Future Value Estimation Based on ARIMA Models

We validate the estimation, by comparing the estimated values with the actual values in the testing period.

7.4.2 Future Value Estimation by Using Only Linear Regression Method

In this approach, we use the unified linear models constructed at the end of the regression period to determine the values of performance measures over the testing period from $k = 15, 16, \ldots, 21$. We assign the values of the necessary independent variables of month $k-1$ in the linear regression models listed in tables 7.4 and 7.5 to estimate the value of the respective performance measure of month k. This is further described by the Figure 7.5. The approach is validated by comparing the estimated values with the real values in the testing period.

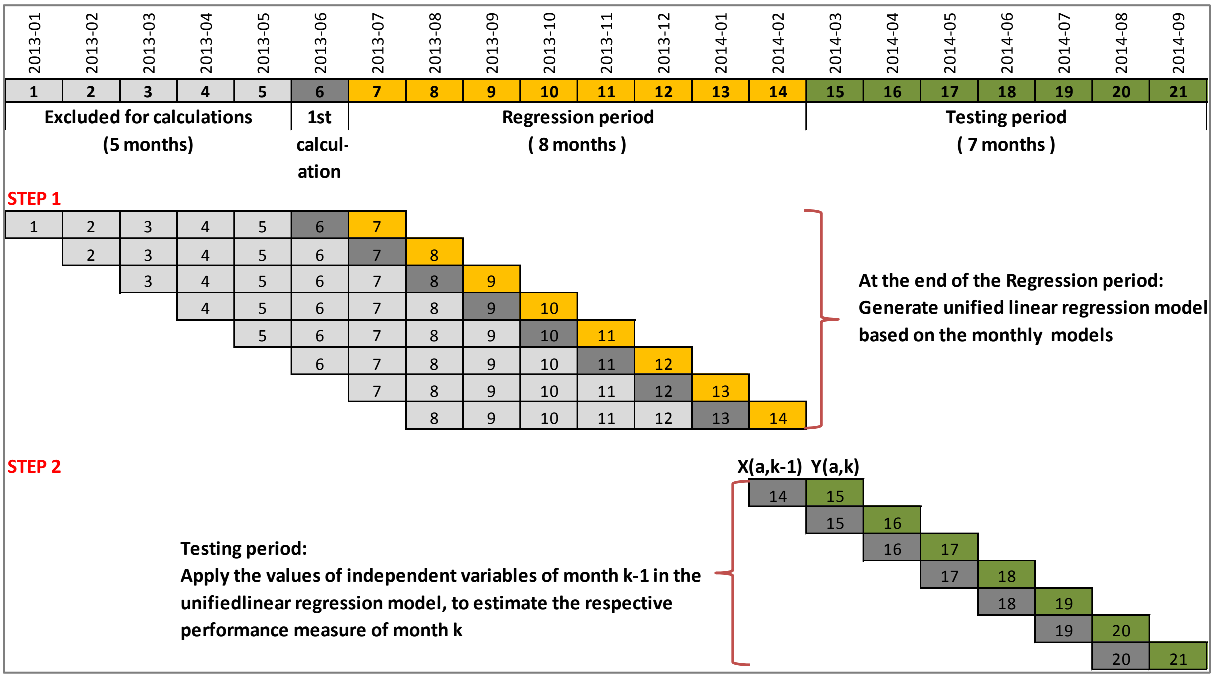


Figure 7. 5 - Future Value Estimation Using Linear Regression

7.4.3 Future Value Estimation by ARIMA Prediction of Linear Regression Models

In this approach as well, we first determine the unified linear regression models using the monthly linear regression models during the regression period. Next, we estimate the values of the independent variables appearing in the linear regression model using the ARIMA approach for the testing period. For this purpose we use the actual values of those independent variables involving $I(t)$ in the months $k = 6, 7, \ldots, 13$ and based on those, estimate the values for the months $k = 14, 15, \ldots, 20$. Then, we assign the estimated independent variable values in the linear regression model to determine the values of respective performance measures in the months $k = 15, 16, \ldots, 21$. To validate the approach, we compare the estimated values with the real values in the testing period. This mechanism is depicted in the Figure 7.6 below.

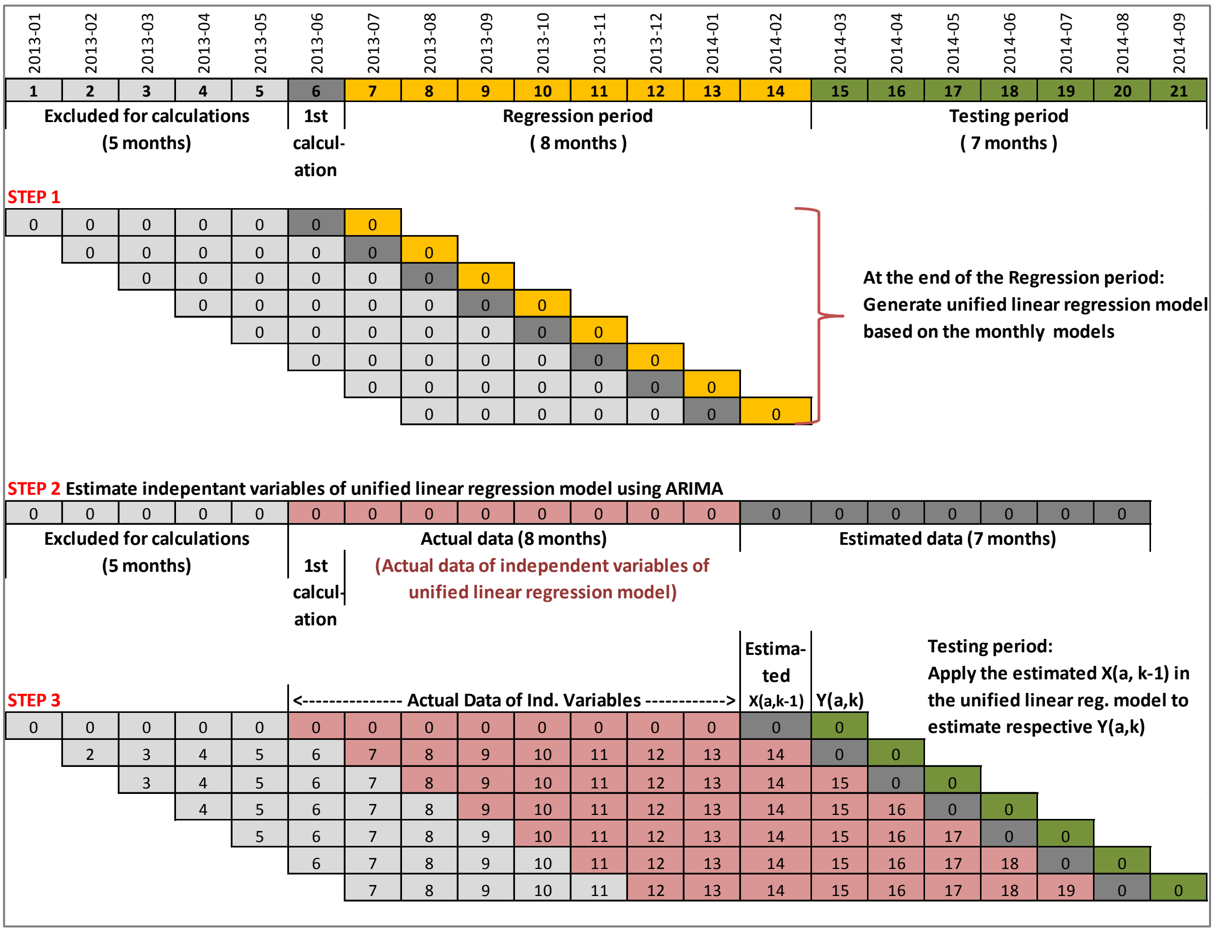


Figure 7. 6 - Future Value Estimation by Applying ARIMA Estimation on Linear Regression Model

7.4.4 Future Value Estimation by Combining Predictions of the Linear Regression Approach and ARIMA Approach

In this approach, we try to achieve highest possible accuracy in estimations by taking the advantages and containing the disadvantages of the ARIMA and linear regression approaches. For this, we introduce an optimum convex combination of the five estimated values by ARIMA and the linear regression approaches (all subset, backward, Stepwise and AIC). More specifically, suppose that these ARIMA model and the linear regression models are established at time k. Furthermore, the values $Y(a,k)$ are known for $a \in A(S)$. Then the challenge is to estimate $\hat{Y}(a,k+1)$, the value of that for the next month.

Let's denote the estimated values in month k obtained by the ARIMA model and the linear regression model of All subset, Backward, Stepwise and AIC as $Y_{ARI}(a,k)$, $Y_{LIN\_ALS}(a,k)$, $Y_{LIN\_BKW}(a,k)$, $Y_{LIN\_STP}(a,k)$ and $Y_{LIN\_AIC}(a,k)$ respectively. Then, we define,

(7.6) $Y_{MAX}(a,k) =$

$$max\,\{Y_{ARI}(a,k), Y_{LIN\_ALS}(a,k), Y_{LIN\_BKW}(a,k), Y_{LIN\_STP}(a,k)\;\; Y_{LIN\_AIC}(a,k)\}\;\; ;$$

(7.7) $Y_{MIN}(a,k) =$

$$min\,\{Y_{ARI}(a,k), Y_{LIN\_ALS}(a,k), Y_{LIN\_BKW}(a,k), Y_{LIN\_STP}(a,k)\;\; Y_{LIN\_AIC}(a,k)\}\quad .$$

We then seek for an optimal convex combination of $Y_{MAX}(a,k)$ and $Y_{MIN}(a,k)$. For this purpose, let

(7.8) $$g(\alpha) = \{\alpha Y_{MAX}(a,k) + (1-\alpha)Y_{MIN}(a,k) - Y(a,k)\}^2\;\; ,$$

where, $0 \leq \alpha \leq 1$.

It can be readily seen that $g(\alpha)$ is convex in α and achieves the unique minimum without regard to $0 \leq \alpha \leq 1$ at

(7.9) $$\alpha_{min}(a,k) = \frac{\{Y(a,k)-Y_{MIN}(a,k)\}\{Y_{MAX}(a,k)-Y_{MIN}(a,k)\}}{\{Y_{MAX}(a,k)-Y_{MIN}(a,k)\}^2}\quad .$$

Consequently, the optimal convex combination with $0 \leq \alpha \leq 1$ can be achieved at

$$(7.10) \qquad \alpha^*(a,k) = \begin{cases} 0 & \alpha_{min}(a,k) \leq 0 \\ \alpha_{min}(a,k) & if \ 0 < \alpha_{min}(a,k) < 1 \\ 1 & if \ 1 \leq \alpha_{min}(a,k) \end{cases} .$$

Finally we obtain the estimated value for the month $k+1$,

$$(7.11) \qquad \hat{Y}(a,k+1) = \alpha^*(a,k)Y_{MAX}(a,k) + (1-\alpha^*(a,k))Y_{MIN}(a,k) \ .$$

We first implemented the ARIMA model so as to obtain $Y_{ARI}(a,k)$ based on the past values $Y(a,k-i), i = 1,2,\ldots,9$ for $k = 15,16,\ldots,20$. For linear regression model, the values of $Y_{LIN}(a,k)$ for months $k = 15,16,\ldots,20$ are obtained according to unified linear regression model. Finally, we evaluate $\hat{Y}(a,k+1)$ according to the equation (7.11) for the months, $k = 15,16,\ldots,20$. This process is further explained by the Figure 7.7. When carrying-out the Step 1 of this process, we perform linear regression analysis using the four types of variable selection methods discussed in section 7.3.1. Then validation of the approach is done by comparing the final estimated value with the actual value in the testing period.

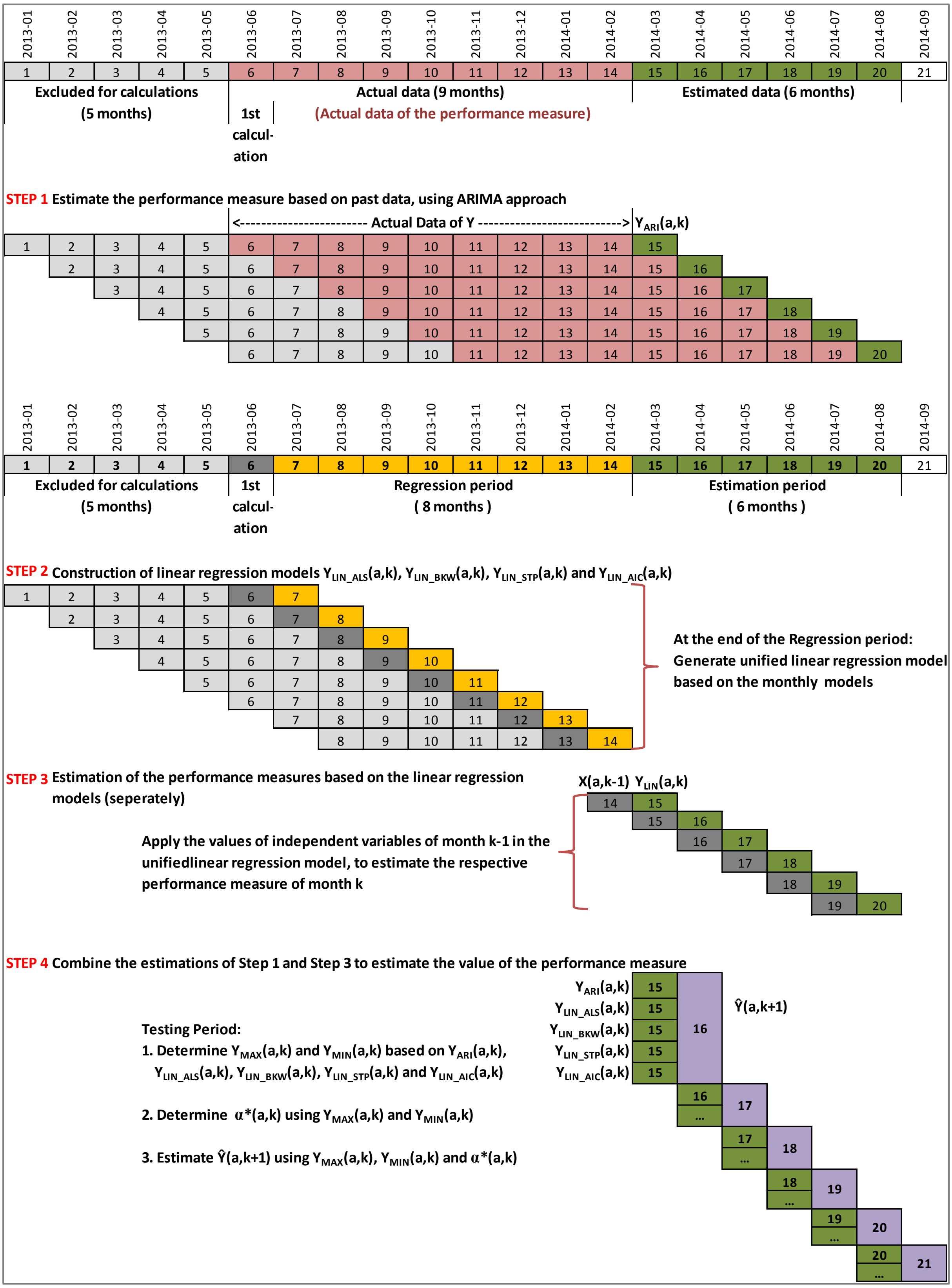


Figure 7. 7 - Future Value Estimation by Combining Estimations of Linear Regression Approach and ARIMA Approach

7.5 Numerical Examples

In this section, we present estimation results using four approaches and validate that against the actual values. For each $a \in A(S)$ and for each variable selection method we calculate relative error of the estimation for each testing month based on the following formula.

$$(7.12) \quad Relative\ Error = \frac{|Y_{ACT} - Y_{EST}|}{c + |Y_{ACT}|} \quad \text{where,} \quad c = \begin{cases} 0 & when\ Y_{ACT} \neq 0 \\ 1 & when\ Y_{ACT} = 0 \end{cases} .$$

Here, the actual value and the estimated value are represented by Y_{ACT} and Y_{EST} respectively and c is a constant. By this, one can calculate relative error of the estimation at each testing month. However, this relative error may vary from one testing month to another. Therefore, in order to obtain individual values of relative error representing the 7 relative error values relevant to each testing month, we determine the mean, the median, the maximum, the minimum and (Max-Min) values of the relative error as shown in Table 7.6.

Table 7. 6 – Five Types of Relative Errors of Estimations

Application_ID	Relative Error of the Estimation							Single Rel. Error Value for All Testing Months				
	k = 15	k = 16	k = 17	k = 18	k = 19	k = 20	k = 21	Average	Median	Max	Min	Max-Min
air.au.com.metro.DumbWaysToDie	28%	27%	28%	29%	30%	31%	33%	29%	29%	33%	27%	5%
air.com.muumuu.neko	36%	38%	40%	41%	39%	39%	39%	39%	39%	41%	36%	5%
air.jp.co.dcArchives.MitchiriNekoMix	3%	1%	3%	5%	6%	9%	13%	6%	5%	13%	1%	12%
...												

To determine the successfulness of the estimated value, we introduce following three acceptance levels:

(1) Acceptable: If relative error of estimation is less than 10%

(2) Medium: If relative error of estimation is between 10% and 35%

(3) Non-Acceptable: If relative error of estimation is above 35%

Using the above mentioned acceptance levels, we present the estimation results of the five key performance measures for 'Casual' and 'Puzzle' sub-categories by the following Figures 7.8 through 7.17.

Figure 7. 8 - Estimation Results of App-Dev (Sub-category: Casual)

Figure 7. 9 - Estimation Results of App-Dev (Sub-category: Puzzle)

Figure 7. 10 - Estimation Results of App-Stab (Sub-category: Casual)

Figure 7. 11- Estimation Results of App-Stab (Sub-category: Puzzle)

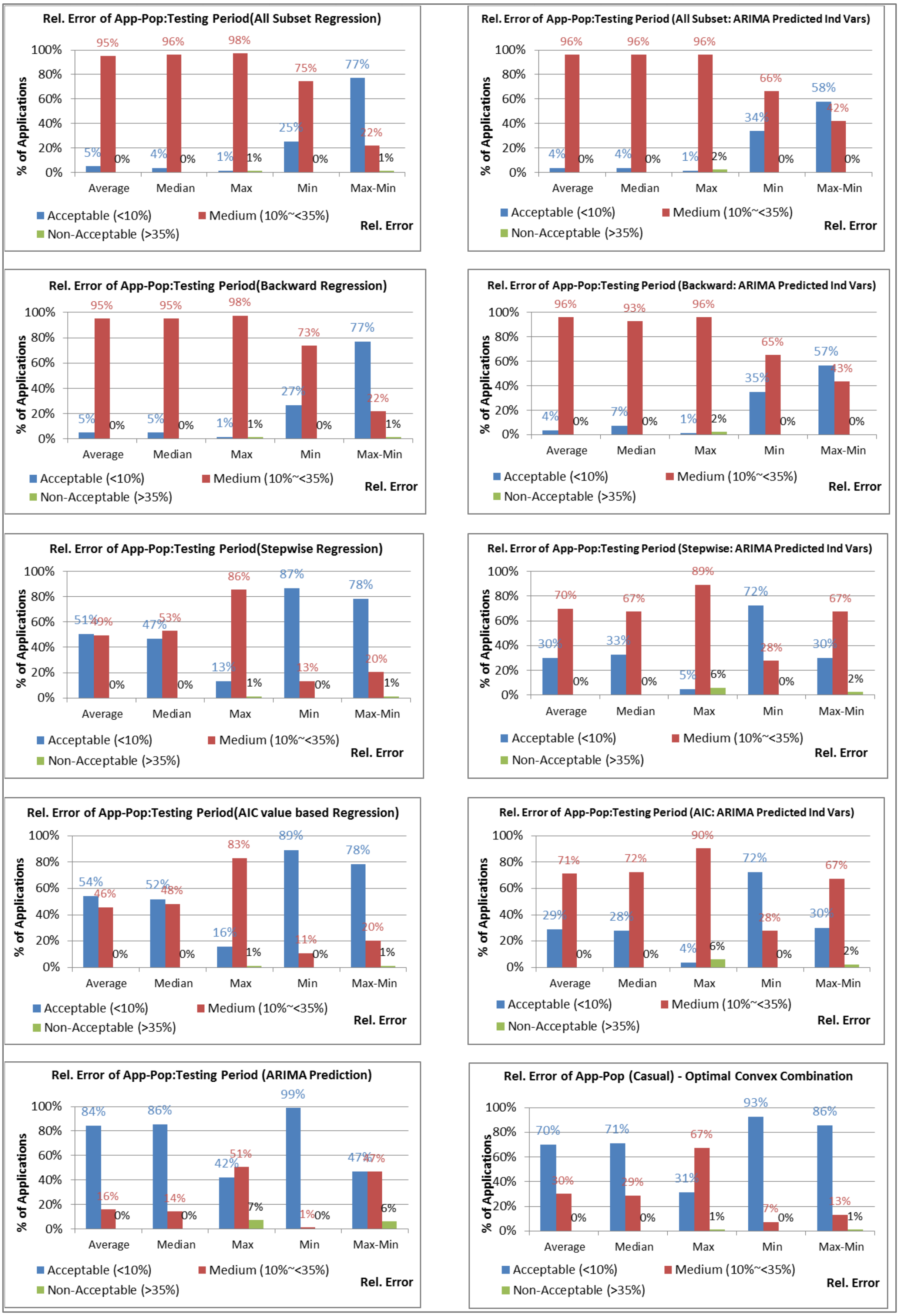


Figure 7. 12 - Estimation Results of App-Pop (Sub-category: Casual)

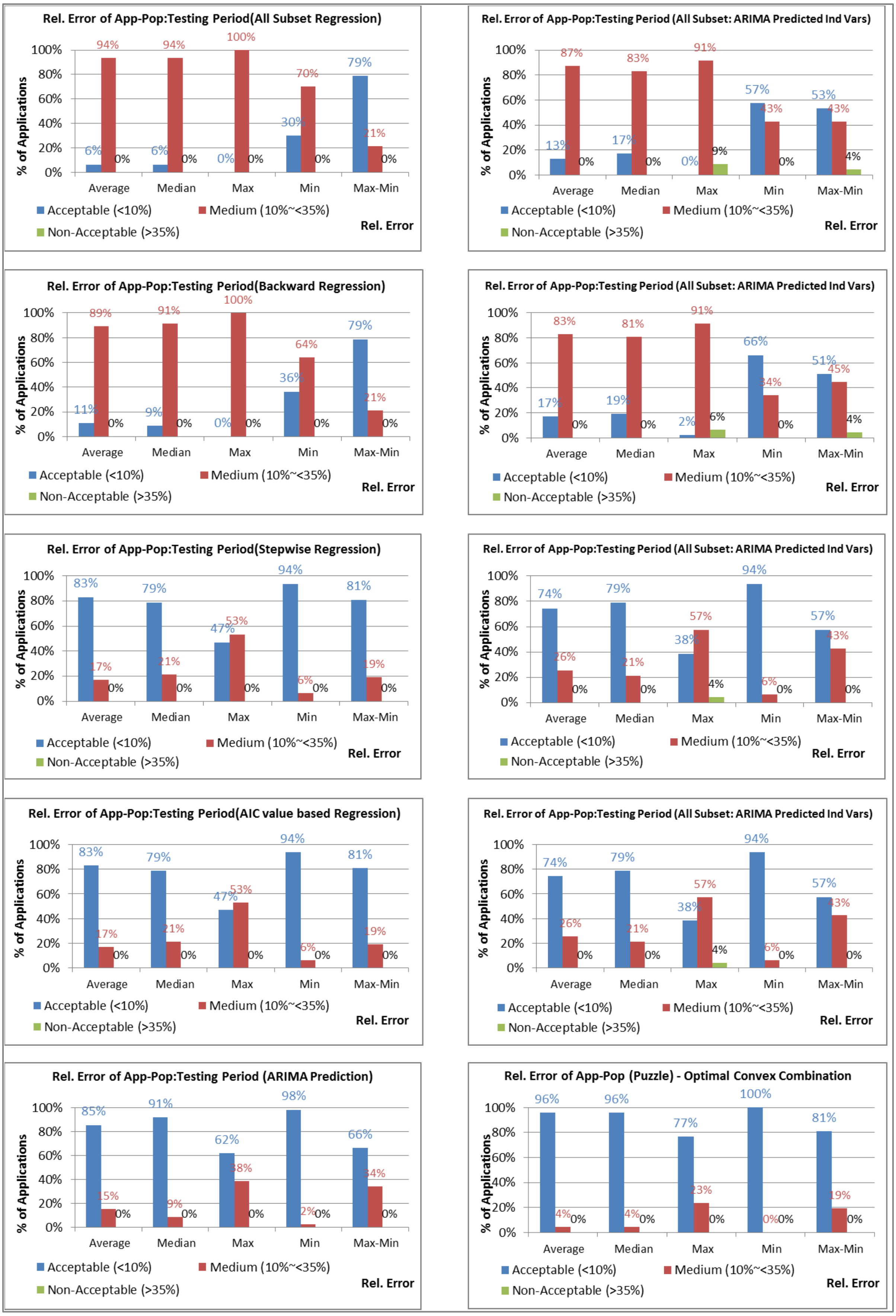


Figure 7. 13 - Estimation Results of App-Pop (Sub-category: Puzzle)

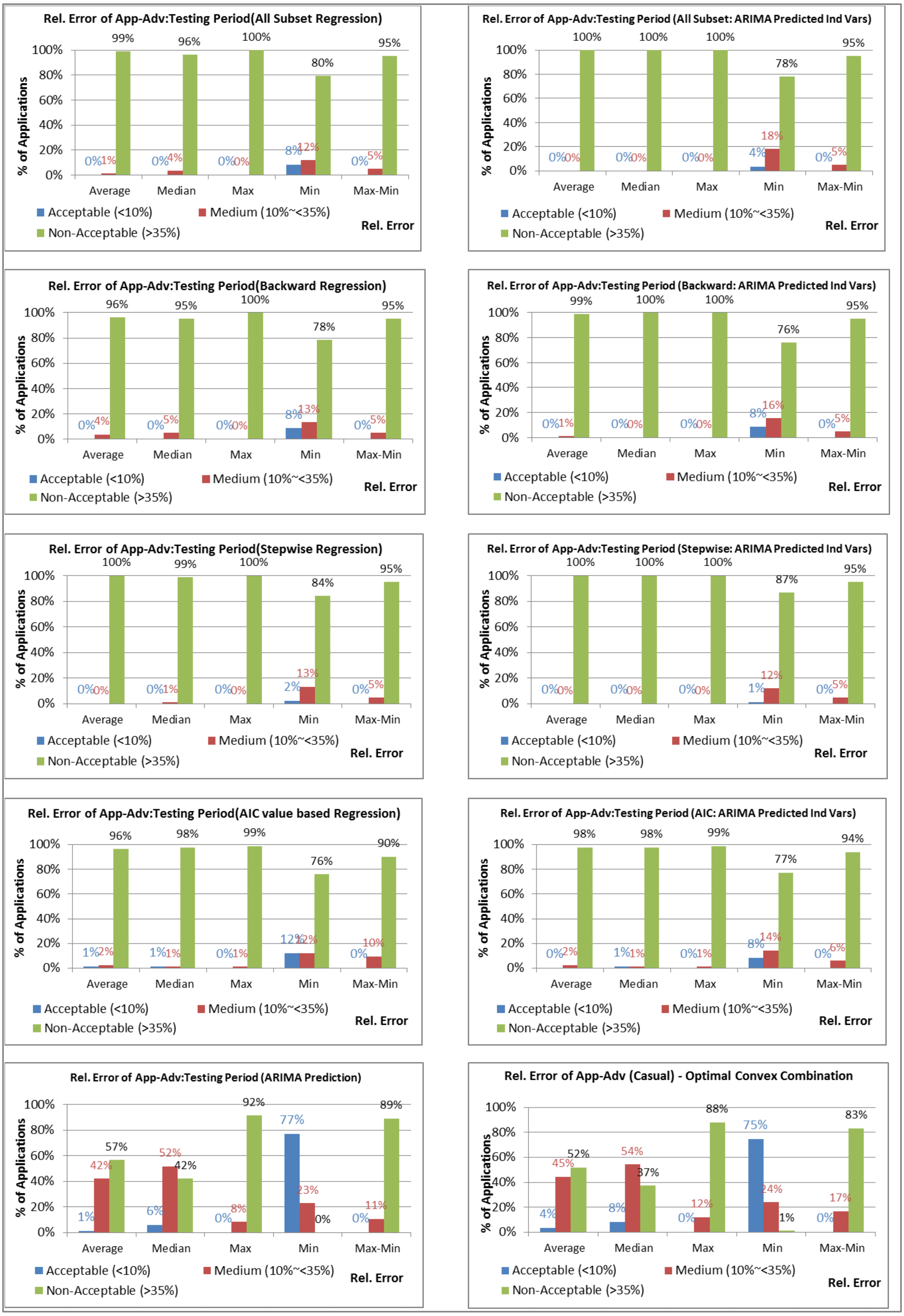


Figure 7. 14 - Estimation Results of App-Adv (Sub-category: Casual)

Figure 7. 15 - Estimation Results of App-Adv (Sub-category: Puzzle)

Figure 7. 16 - Estimation Results of App-Dec (Sub-category: Casual)

Figure 7. 17 - Estimation Results of App-Dec (Sub-category: Puzzle)

By observing the Figures 7.8 and 7.9 concerning the estimation results of $App\text{-}Dev\big(a, I(t)\big)$, once can notice a large amount of non-acceptable estimations relevant to linear regression approach, while ARIMA models approach results relatively successful estimations. Possible explanation for this may be that, in reality the value of the performance measure $App\text{-}Dev\big(a, I(t)\big)$ may not hold strong functional relationships with the independent explanatory variables considered in the study. It might rather rely on its own historical values. However, it is worth noting that the proposed approach of considering optimal convex combination of the estimations by ARIMA approach and linear regression approach has improved accuracy of individual approaches to a considerable level. We can notice the same in rest of the cases presented by Figures 7.9 through 7.17, except for rare situation presented in Figure 7.12. Figure 7.10 displays that the $App\text{-}Stab\big(a, I(t)\big)$ estimations of 'Casual' applications comprises of acceptable estimations to some level. By comparing the left and right graphs in the first 4 lines, one can notice that in this case, predicting linear regression model's independent variables via ARIMA approach does not necessarily improve the accuracy of exclusive linear regression approach. From Figure 7.11 concerned with $App\text{-}Stab\big(a, I(t)\big)$ estimations of 'Puzzle' sub-category, one can understand the difficulty of measuring the $App\text{-}Stab\big(a, I(t)\big)$ value accurately and it may be also possible to say that the $App\text{-}Stab\big(a, I(t)\big)$ values may not rely on its past values. $App\text{-}Pop\big(a, I(t)\big)$ depicted in the Figures 7.12 and 7.13 enjoy the least number of rejected estimations showing the predictability of the performance measure. Further, we also note that among the two categories, the puzzle applications are more predictable. Figures 7.14 through 7.17 concerned with $App\text{-}Adv\big(a, I(t)\big)$ and $App\text{-}Dec\big(a, I(t)\big)$ indicate that linear regression methods alone may not be the most suitable for estimating the two performance measures. When the four linear regression approaches are considered, all the Figures 7.8 through 7.17 indicate that the two linear regression methods stepwise regression and AIC value based regression perform relatively better over the other linear regression methods with respect to the considered dataset.

Chapter 8

Application Lifecycles

In the field of marketing, the product lifecycle consists of four stages as in (Kotler & Keller, 2012):

(1) Introduction

During this period, the product is introduced in the market. Sales growth is slow and profits are non-existent due to the heavy expenses of product introduction.

(2) Growth

During this period, market acceptance may increase rapidly and substantial improvement of profits can be observed.

(3) Maturity

By this time, the product may have achieved the acceptance by most potential buyers. Therefore, slowdown in sales growth can be observed. Competition increases and due to that, profits get stabilize or may start to decline.

(4) Decline

During this period, one can observe a downward drift in sales and the profits get eroded.

For typical products and services, the stage of the product lifecycle may be determined by tracing the number of products sold during a unit of time, along the time axis. This could be a day, a week, a month or a year. The conventional product lifecycle takes the form as depicted in Figure 8.1

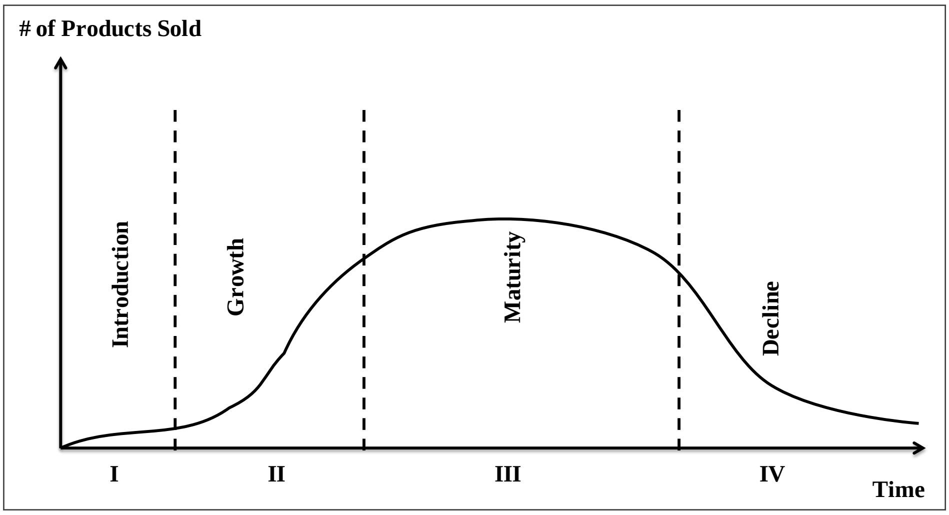


Figure 8. 1 – A Typical Product Lifecycle

Smartphone applications are different from such ordinary products and services which stay in use for a long period once they are purchased. In contrast, smartphone applications may be installed and uninstalled dynamically over time in a repeated manner. Accordingly, it may not be appropriate to describe the product lifecycle of a smartphone application in terms of the number of devices that install it in the underlying unit of time. Considering this feature of dynamic installation and uninstallation of smartphone applications, we try to employ the application usage patterns introduced in Chapter 5, to determine the lifecycle stage of a smartphone application. Application usage patterns of a certain smartphone application on a certain device rely on the six month record of installation and uninstallation of that application by the considered device. Therefore, for determining the stage of the lifecycle of individual smartphone applications, it may be more appropriate to use the distribution of devices over different usage patterns than the distribution of devices that installed that application during the underlying time unit. In this chapter, we develop segmentation criteria for classifying smartphone applications into three groups based on the usage patterns and the concepts of the product lifecycle and Markov chains.

8.1 Dataset under Consideration

To carry-out the work discussed in this chapter, 'Dataset 3' described in Chapter 3 is utilized. The dataset is consisted of 20 applications from 7 game sub-categories. Those applications are listed in Table 8.1 below.

Table 8. 1 - Applications under Consideration

#	Group	Sub-category	Application	Introduction to the Market	Number of Devices
1	Development Data	Puzzle	LINE Disney TSUM TSUM	Jan-2014	126006
2			LINE Pokopang	May-2013	103293
3			LINE Jelly	Apr-2013	42382
4			LINE Hidden Catch	Feb-2013	39863
5			LINE Bubble!	Dec-2012	116986
6			Puzzle & Dragons	Nov-2012	132659
7			LINE POP	Nov-2012	132192
8		Casual	Nyanko Dai Sensō	Dec-2012	37754
9			Candy Crush Saga	Nov-2012	67135
10			Mushroom Garden Deluxe	Jul-2012	41966
11			Mushroom Garden Seasons	Mar-2012	29539
12			Mushroom Garden	Dec-2011	19311
13		Arcade	LINE Cookie Run	Jan-2014	32954
14			LINE Wind runner	Feb-2013	65566
15			Temple Run	Mar-2012	25202
16	Test Data	Action	Monster Strike	Dec-2013	50897
17			Temple Run 2	Jan-2013	48952
18		Card	Scratch de Coupon	Nov-2012	29954
19		Casino	LINE Dozer	Nov-2013	20432
20		Roleplaying	Quiz RPG Witch and the Blak Cat Wiz	Mar-2013	31282

We develop the application classification mechanism based on the 15 applications from 'Arcade,' 'Casual' and 'Puzzle' sub-categories in the development data. And the remaining 5 applications from 'Action,' 'Card,' 'Casino' and 'Roleplaying' sub-categories reserved as test data are used for validating the proposed mechanism.

8.2 The Concept of Markov Chains

The concept of Markov chains is referred to as stochastic process where the state of the next period relies only on the current state. More specifically, (see (Sumita, 2011)) a stochastic process $N(k)$ on state space $\mathcal{N} = \{0, 1, \dots, N\}$ is said to be a Markov chain in discrete time if,

(8.1) $$P[N(k+r) = n_{k+r} | N(0) = n_0, N(1) = n_1, \dots, N(k) = n_k]$$

$$= P[N(k+r) = n_{k+r} | N(k) = n_k] \quad \text{for all } r \geq 1 \quad .$$

We denote the one step transition probability matrix at time k as,

(8.2) $\underline{\underline{a}}(k) = [a_{ij}(k)]$.

This can be defined by,

(8.3) $a_{ij}(k) = P[N(k) = j | N(k-1) = i] \quad where\ i, j \in \mathcal{N}$.

Then, clearly one can write the k-step transition probability matrix at time k as,

(8.4) $\underline{\underline{b}}(k) = \underline{\underline{a}}(0)\underline{\underline{a}}(2) \ldots \underline{\underline{a}}(k-1)$.

For a temporally homogeneous Markov chain in discrete time, the one step transition probability matrix and the k-step transition probability matrix at time k becomes $\underline{\underline{a}}$ and $\underline{\underline{a}}^k$ respectively. Then, the distribution of $N(k)$ is given by the probability vector,

(8.5) $\underline{p}^T(k) = [p_0(k), p_1(k), \ldots]\ where\ p_j(k) = P[N(k) = j]$.

Thereby, we can write the distribution of $N(k+1)$ as,

(8.6) $\underline{p}^T(k+1) = \underline{p}^T(k)\underline{\underline{a}}(k+1) = \underline{p}^T(0)\underline{\underline{a}}(0)\underline{\underline{a}}(1) \ldots \underline{\underline{a}}(k)$.

We employ this concept to develop the classification algorithm in next sections.

8.3 Three Classes of Applications based on Lifecycles

In order to classify the smartphone applications into meaningful groups based on the concept of product lifecycle, we first consider the information on application usage patterns of the 15 applications in the development data, over the period of 35 months from January 2013 to April 2016. The Figures 8.2 through 8.16 illustrate the distribution of devices over the 9 application usage patterns excluding 'Null.' The Figures 8.2 through 8.16 are listed in an order such that applications with similar distributions to be adjacent.

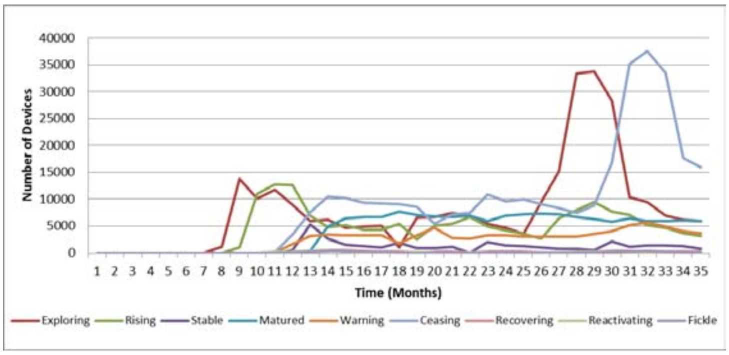


Figure 8. 2 – Distribution of Devices over the Usage Patterns ('LINE Disney TSUM TSUM')

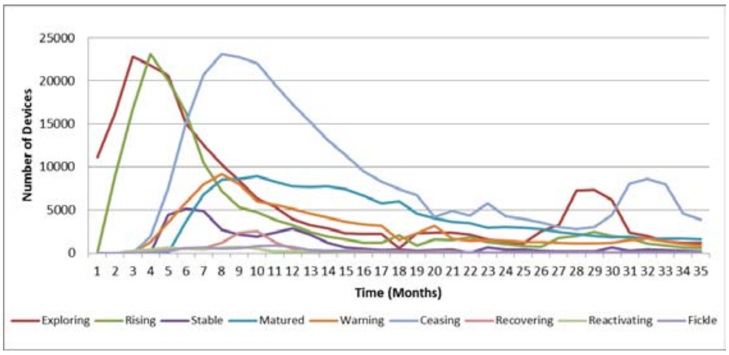


Figure 8. 3 – Distribution of Devices over the Usage Patterns ('LINE Pokopang')

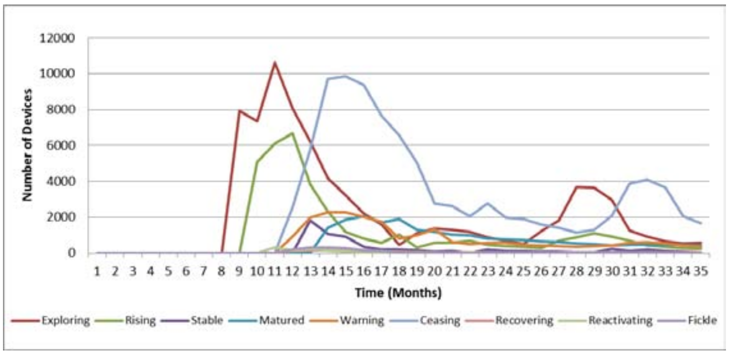


Figure 8. 4 – Distribution of Devices over the Usage Patterns ('LINE Cookie Run')

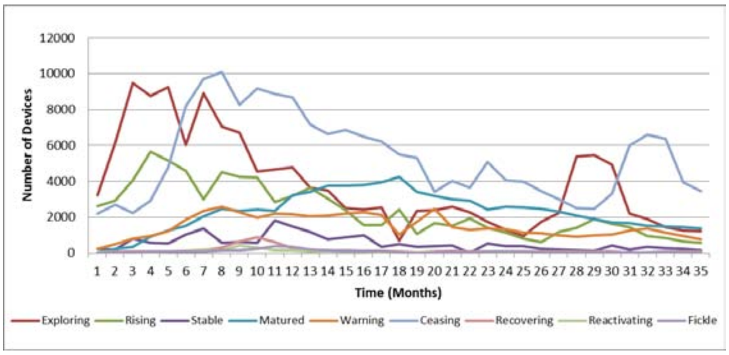


Figure 8. 5 – Distribution of Devices over the Usage Patterns ('Candy Crush Saga')

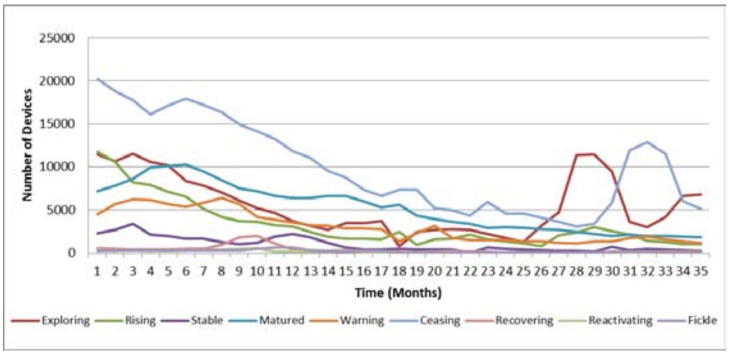


Figure 8. 6 – Distribution of Devices over the Usage Patterns ('Puzzle & Dragons')

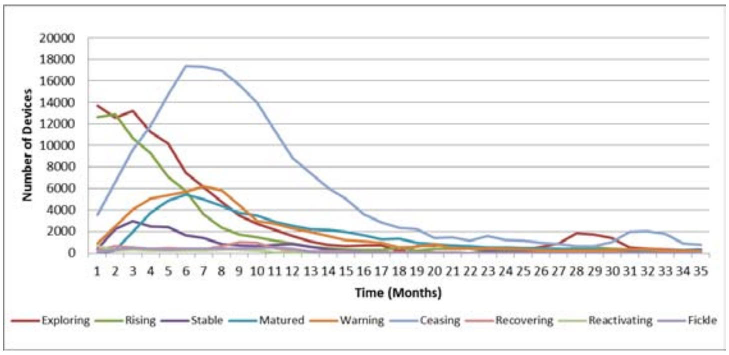


Figure 8. 7 – Distribution of Devices over the Usage Patterns ('LINE Wind Runner')

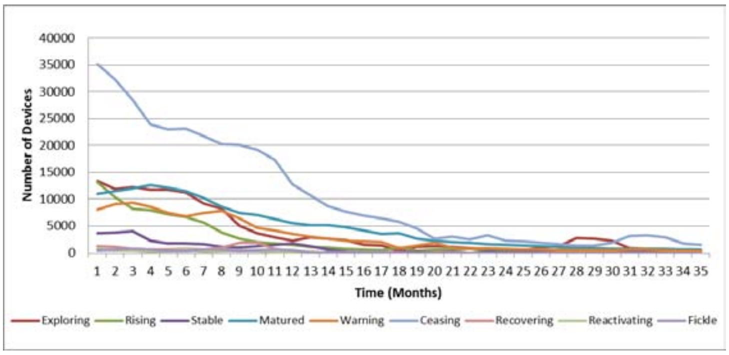


Figure 8. 8 – Distribution of Devices over the Usage Patterns ('LINE POP')

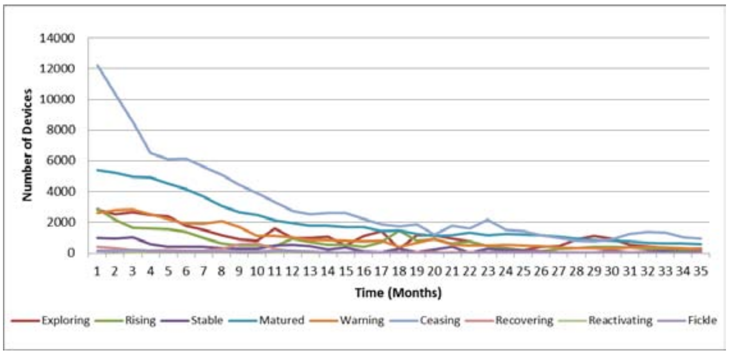


Figure 8. 9 – Distribution of Devices over the Usage Patterns ('Mushroom Garden Deluxe')

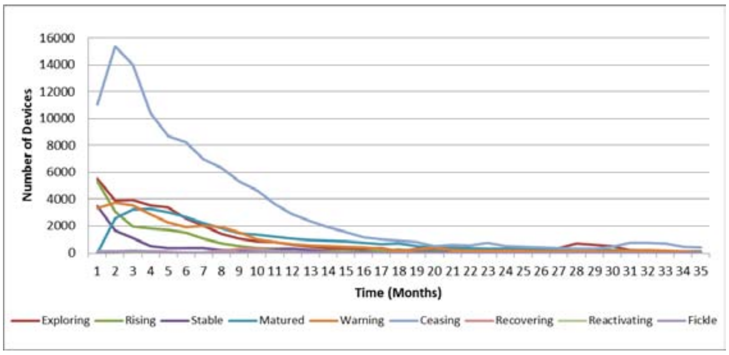


Figure 8. 10 – Distribution of Devices over the Usage Patterns ('LINE Hidden Catch')

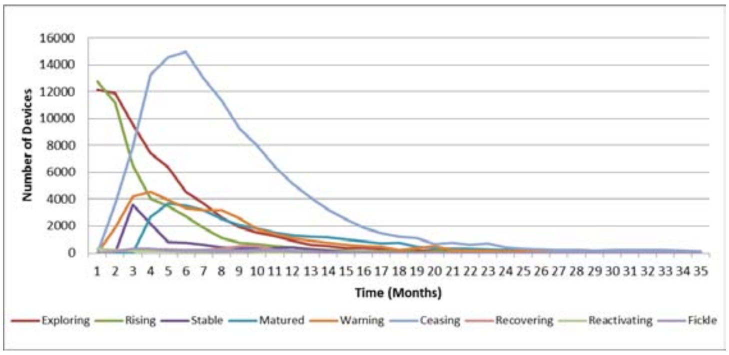


Figure 8. 11 – Distribution of Devices over the Usage Patterns ('LINE Jelly')

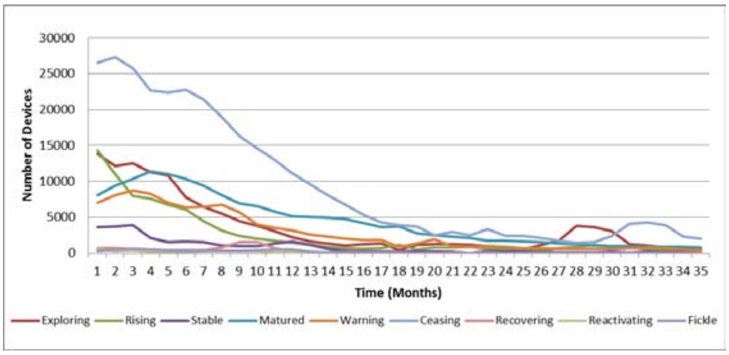


Figure 8. 12 – Distribution of Devices over the Usage Patterns ('LINE Bubble!')

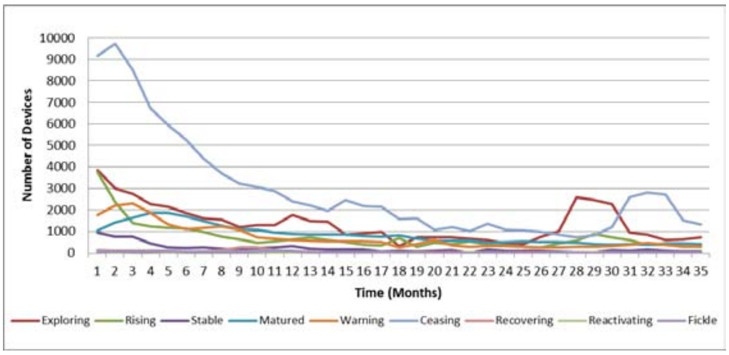


Figure 8. 13 – Distribution of Devices over the Usage Patterns ('Nyanko Dai Sensō')

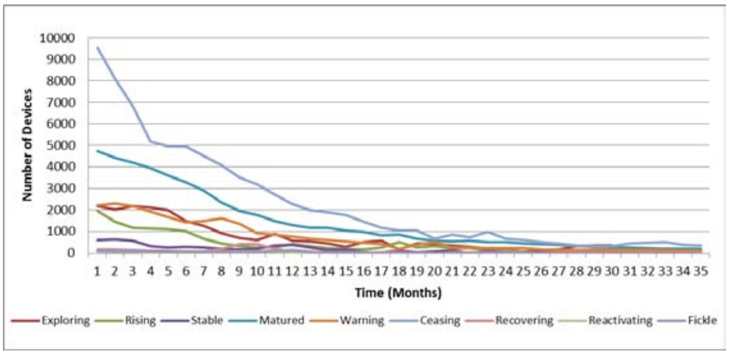


Figure 8. 14 – Distribution of Devices over the Usage Patterns ('Mushroom Garden Seasons')

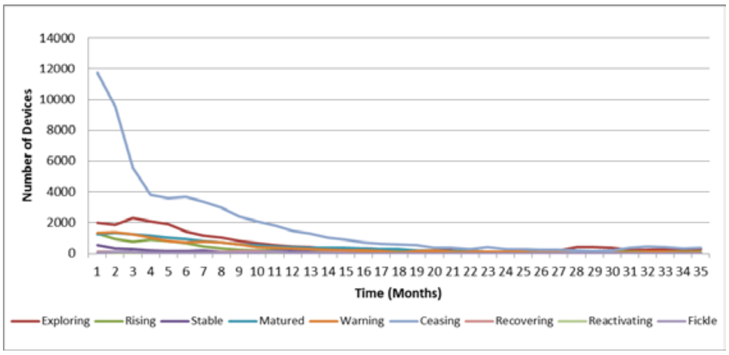


Figure 8. 15 – Distribution of Devices over the Usage Patterns ('Temple Run')

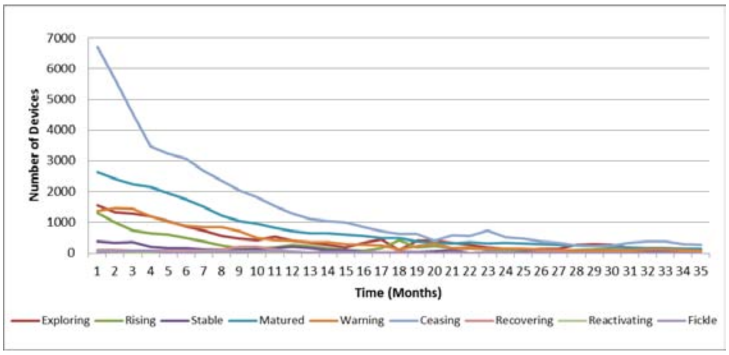


Figure 8. 16 – Distribution of Devices over the Usage Patterns ('Mushroom Garden')

Observing the Figures 8.2 through 8.16, we understand that certain applications have widely dispersed distributions of devices over the nine usage patterns. Hence, in order to describe the product lifecycle consisting of four stages in terms of ten usage patterns, we combine the ten usage patterns into the following four groups, representing the four stages of product lifecycle.

Group (a): Exploring and Fickle

Group (b): Rising, Stable, Matured, Recovering, and Reactivating

Group (c): Warning and Ceasing

Group (d): Null

It is expected that the changes over time of the distribution of devices across the four groups would enable one to identify in which stage of the product lifecycle the underlying smartphone application is.

In Figure 8.17.a, the number of devices having "LINE Disney TSUM TSUM" is plotted, with comparison of the portions of Group (a) through (c) depicted in Figure 8.17.b. Similar graphs are exhibited for the remaining 14 smartphone applications of the development data in Figures 8.18.a and 8.18.b through Figures 8.31.a and 8.31.b. We note that the data period is over 35 months since the introduction of the underlying smartphone application into the market, except for "LINE Disney TSUM TSUM" and "LINE Cookie Run" with 28 months.

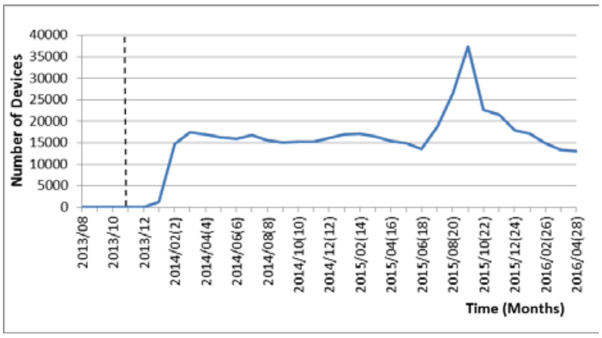


Figure 8.17. a - Number of Devices ('LINE Disney TSUM TSUM')

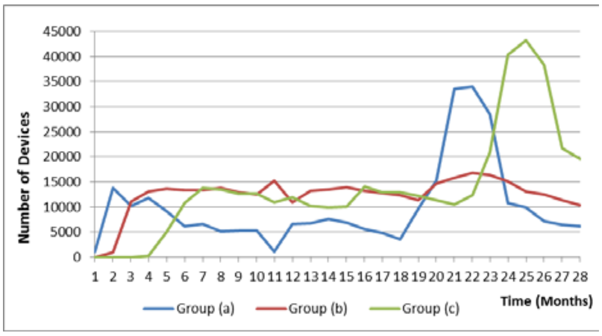


Figure 8.17. b - Usage Pattern Groups ('LINE Disney TSUM TSUM')

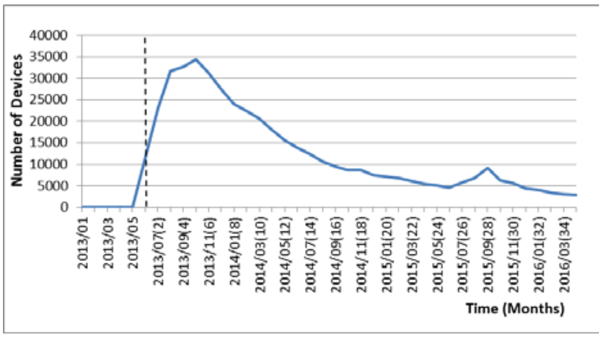


Figure 8.18. a - Number of Devices ('LINE Pokopang')

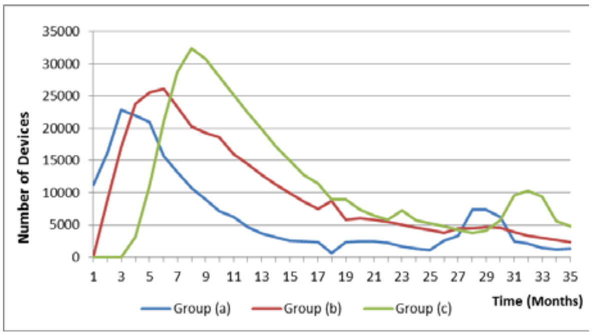


Figure 8.18. b - Usage Pattern Groups ('LINE Pokopang')

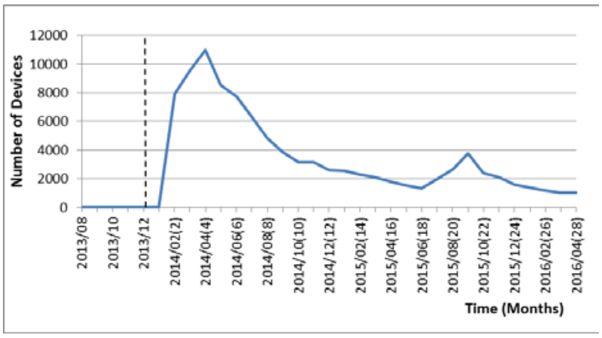


Figure 8.19. a - Number of Devices ('LINE Cookie Run')

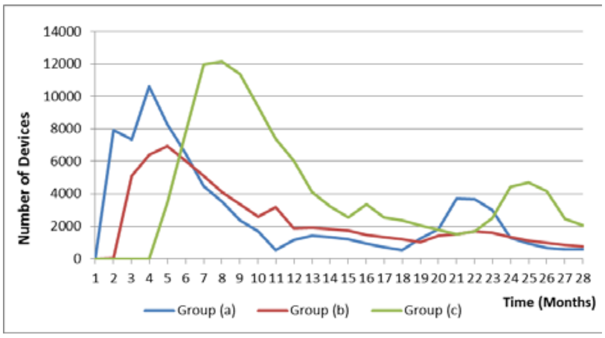


Figure 8.19. b - Usage Pattern Groups ('LINE Cookie Run')

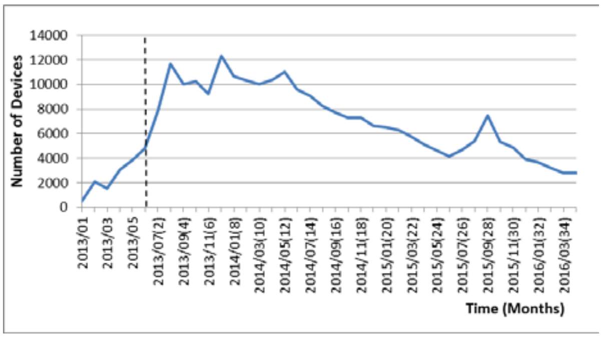


Figure 8.20. a - Number of Devices ('Candy Crush Saga')

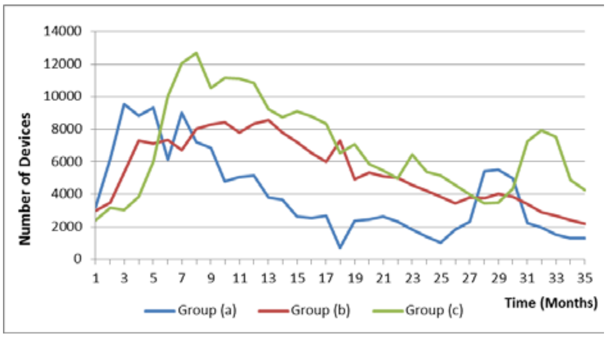


Figure 8.20. b - Usage Pattern Groups ('Candy Crush Saga')

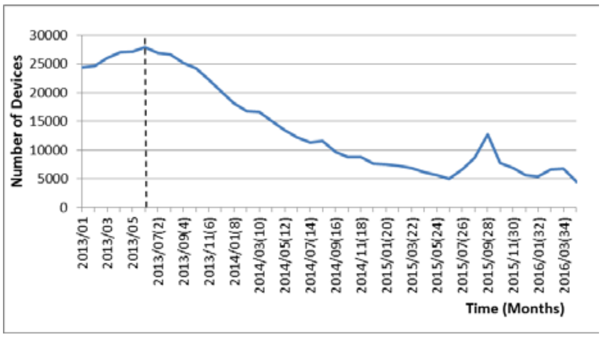


Figure 8.21. a - Number of Devices ('Puzzle & Dragons')

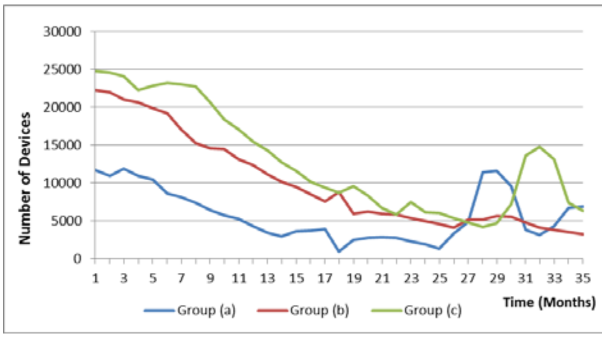


Figure 8.21. b - Usage Pattern Groups ('Puzzle & Dragons')

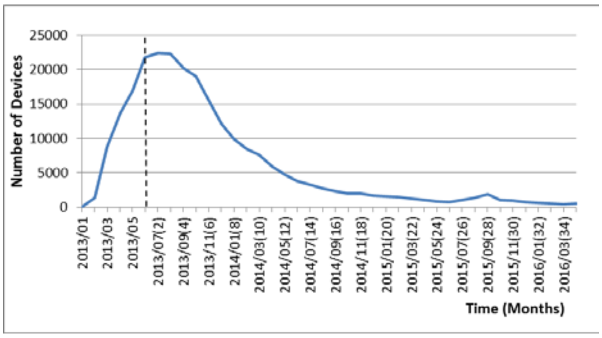


Figure 8.22. a - Number of Devices ('LINE WIND Runner')

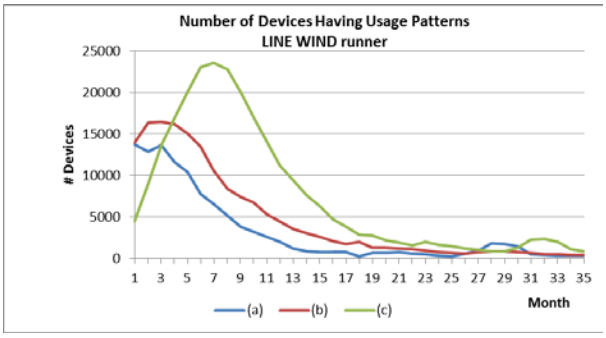


Figure 8.22. b - Usage Pattern Groups ('LINE WIND Runner')

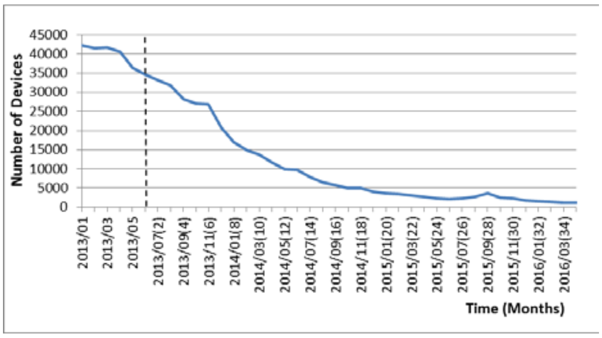


Figure 8.23. a - Number of Devices ('LINE POP')

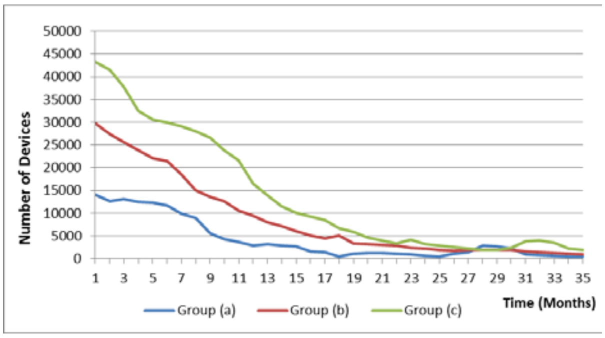


Figure 8.23. b - Usage Pattern Groups ('LINE POP')

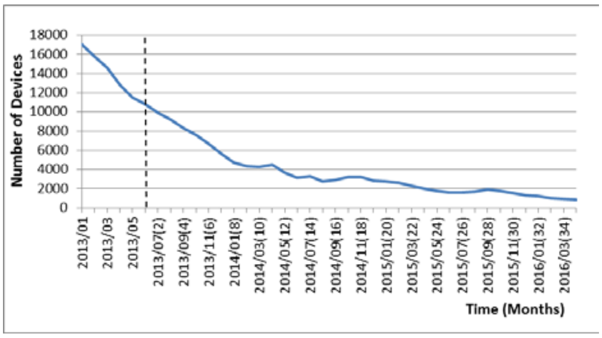


Figure 8.24. a - Number of Devices ('Mushroom Garden Deluxe')

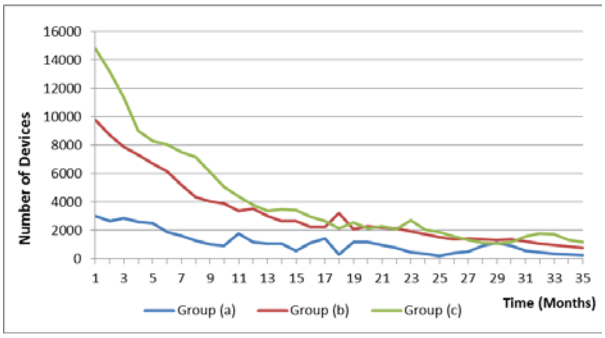
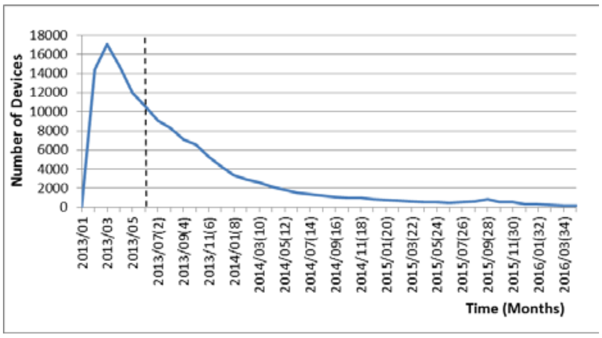


Figure 8.24. b - Usage Pattern Groups ('Mushroom Garden Deluxe')



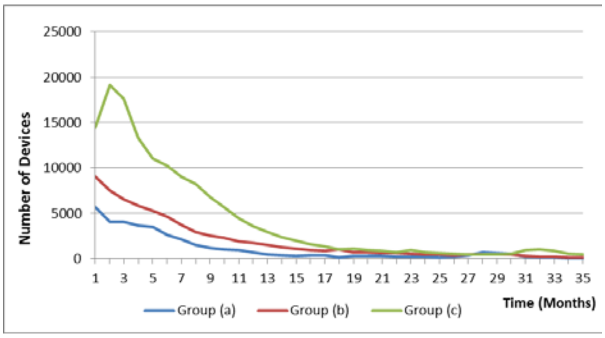
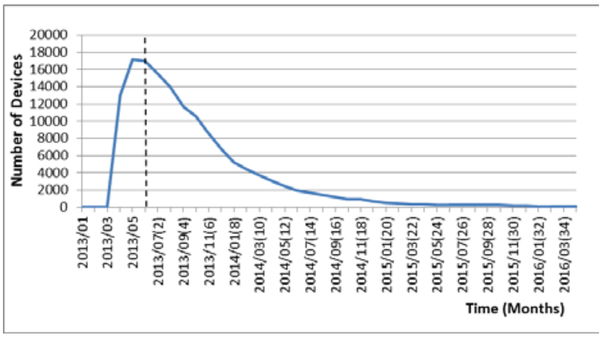


Figure 8.25. a - Number of Devices ('LINE Hidden Catch')

Figure 8.25. b - Usage Pattern Groups ('LINE Hidden Catch')



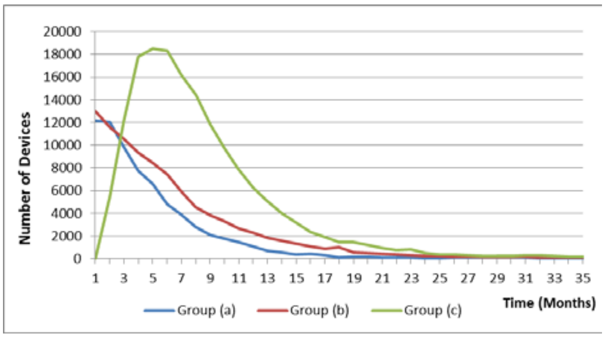
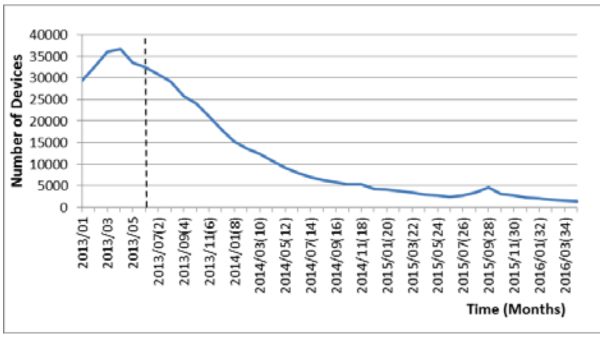


Figure 8.26. a - Number of Devices ('LINE Jelly')

Figure 8.26. b - Usage Pattern Groups ('LINE Jelly')



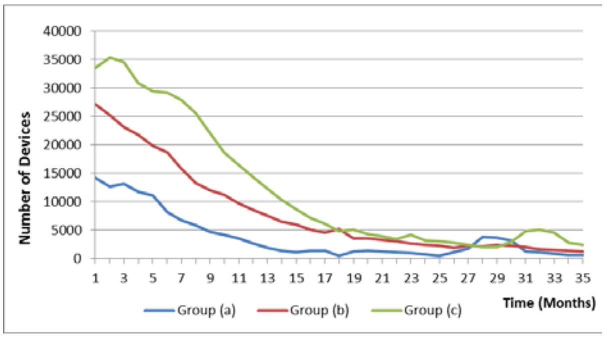
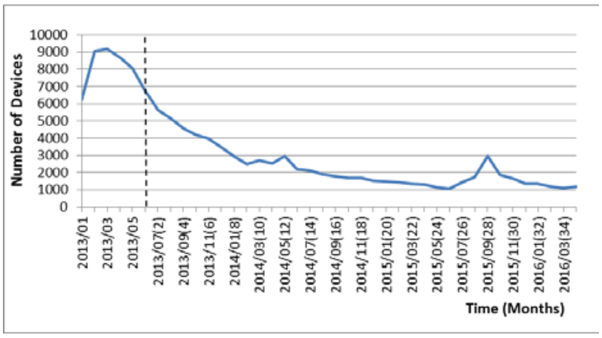


Figure 8.27. a - Number of Devices ('LINE Bubble!')

Figure 8.27. b - Usage Pattern Groups ('LINE Bubble!')



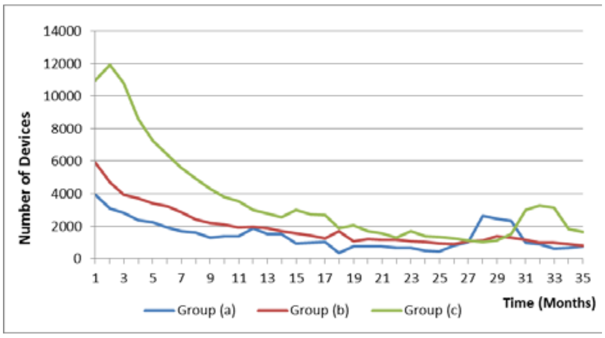


Figure 8.28. a - Number of Devices ('Nyanko Dai Sensō')

Figure 8.28. b - Usage Pattern Groups ('Nyanko Dai Sensō')

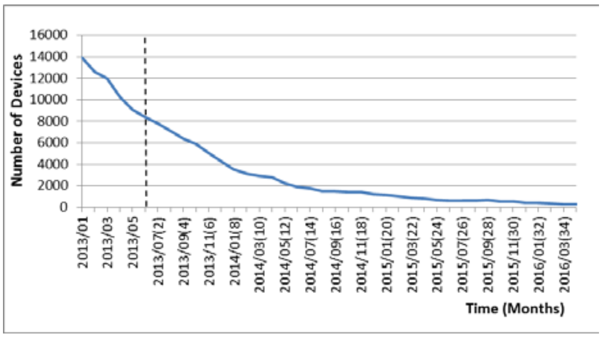


Figure 8.29. a - Number of Devices ('Mushroom Garden Seasons')

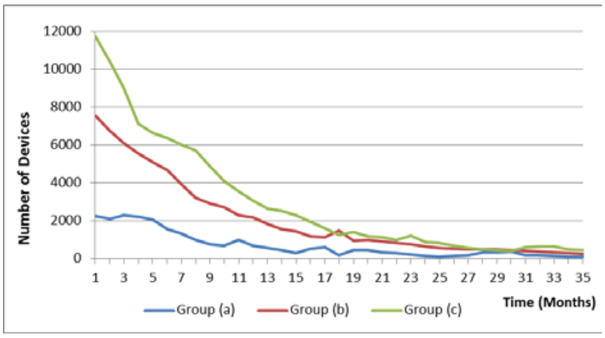


Figure 8.29. b - Usage Pattern Groups ('Mushroom Garden Seasons')

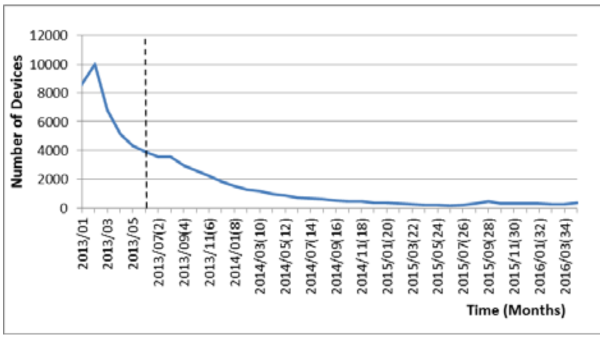


Figure 8.30. a - Number of Devices ('Temple Run')

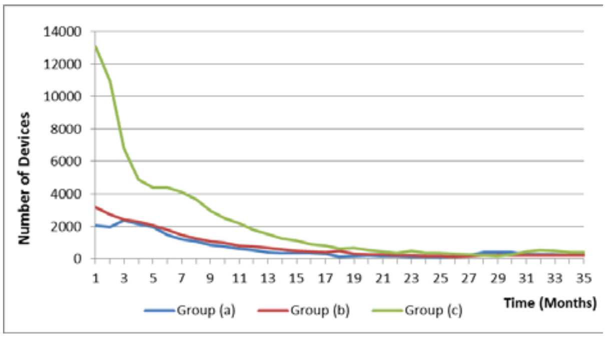


Figure 8.30. b - Usage Pattern Groups ('Temple Run')

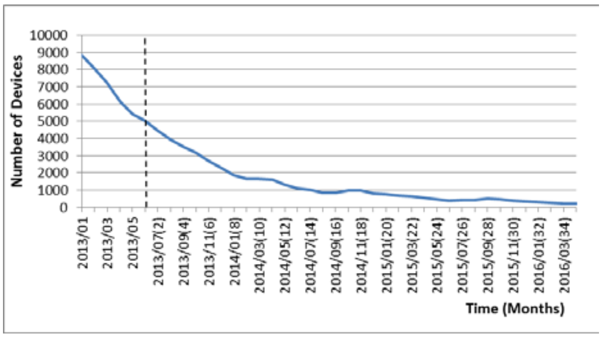


Figure 8.31. a - Number of Devices ('Mushroom Garden')

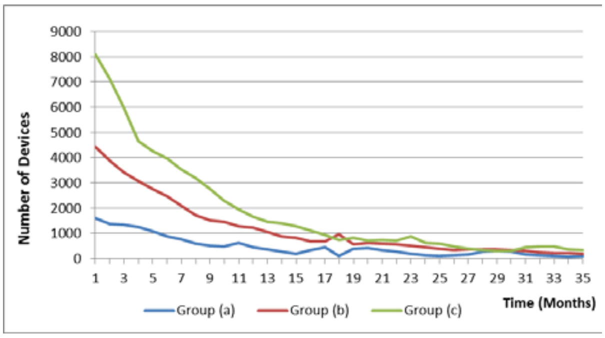


Figure 8.31. b - Usage Pattern Groups ('Mushroom Garden')

It can be noticed from the Figures 8.17.b through 8.31.b shapes of the graphs relevant to Group (a), Group (b) and Group (c) has in general shifted with the time, showing the devices moving through the lifecycle stages of 'Introduction,' 'Growth/Maturity' and 'Decline.' It is also worth noting that sudden surge after about two years since the introduction into the market in Figure 8.17.a can be explained by the same surge of the portion for Group (a) followed by the shifted surge for Group (c) in Figure 8.17.b. This means that those devices contributed to the later surge in Figure 8.17.b uninstalled the application after only a few months, without making any contribution to increase the portion for Group (b). Similar observations can be made for other applications with a secondary surge, demonstrating the usefulness of the introduction of Groups (a) through (c).

By observing the graphs in Figures 8.17.a and 8.17.b through 8.31.a and 8.31.b, the fifteen smartphone applications may be classified into three groups as shown below.

Class (A): Applications which are active in lifecycle at the end of the data period

Class (B): Applications which are in the tail of the lifecycle at the end of the data period

Class (C): Applications which are diminished at the end of the data period

Table 8.2 presents the classification of 15 applications into three groups, Class (A) through (C) based on the concept of the product lifecycle.

Table 8. 2 - Classification of Applications

Group	Applications
Class (A)	Disney TSUM TSUM
	LINE Pokopang
Class (B)	LINE cookie Run
	LINE Wind Runner
	Candy Crush Saga
	Puzzle & Dragons
	LINE POP
	Mushroom Garden Deluxe
Class (C)	LINE Hidden Catch
	LINE Jelly
	LINE Bubble!
	Nyanko Dai Senso
	Mushroom Garden Seasons
	Temple Run
	Mushroom Garden

8.4 Classification Algorithm

During the lifecycle of an application on a certain device, the device may move between the states represented by the usage pattern groups, Group (a) though (d) in various manners with time as depicted in Figure 8.32 below.

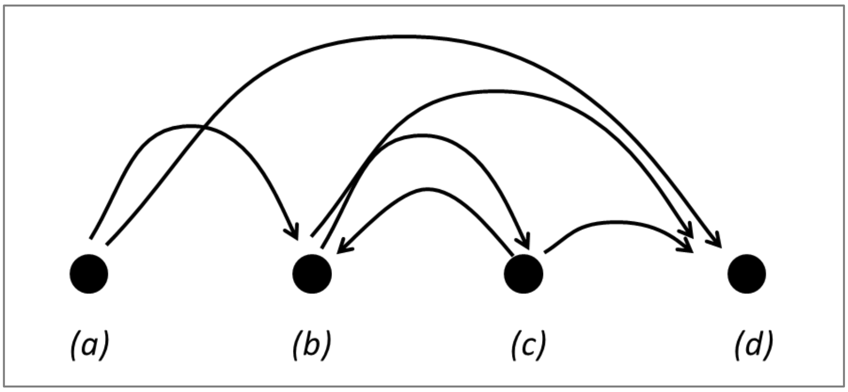


Figure 8.32 – Device's Move between the States a, b, c and d

In this section, we establish an algorithmic procedure for classifying any smartphone application into one of the three classes, Class (A) through (C) based on the time for a device to move from states a, b or c to state d for the first time. Since smartphone applications are dynamically installed and uninstalled by the individual devices, the characteristics of the three classes Class (A) through Class (C) may be captured by assessing the staying power of each application. More specifically, each application's power of staying beyond a certain level of the portion of devices with usage patterns in Group (b). For this purpose, given a smartphone application, we construct a Markov chain $N(k)$ on $\mathcal{N} = \{a, b, c, d\}$ where $w \in \mathcal{N}$ means a device is in Group (w) for $w = a, b,$ c or d. We name the states $\{a, b, c\}$ as 'Good states,' which represent the application to be alive in the lifecycle. That is,

$$G = \{\, a, b, c \,\} \ .$$

In order to estimate the stochastic matrix $A = [a_{ij}]_{i,j \in \mathcal{N}}$, we extract the following statistics from the sequential data for the underlying application.

n_{ij}: The number of devices moving from i to j in one month over the data period, $i, j \in \mathcal{N}$.

The number of devices in month i is given by,

(8.7) $\quad N_i = \sum_{j \in \mathcal{N}} n_{ij} \quad .$

Now we determine the elements a_{ij} in matrix $A = [a_{ij}]_{i,j \in \mathcal{N}}$ by,

$$(8.8) \qquad a_{ij} = \frac{n_{ij}}{N_i} \quad .$$

We denote the minimum time taken to move from state $w \in G = \{a, b, c, \}$ to state d (representing 'Null'), which is known as first passage time as,

$$(8.9) \qquad T_{G\to d} = \min\{k: N(k) = d | N(0) = w\} \quad .$$

Then by using the Markov chain approach discussed in 8.3, we can write the the probability of moving to state d for the first time in month $k+1$ for an application as,

$$(8.10) \qquad p_G^T(k+1) = \underline{p}^T(0)\underline{\underline{a}}(0)\underline{\underline{a}}(1)\ldots\underline{\underline{a}}(k) \quad .$$

Therefore,

$$(8.11) \qquad p_G^T(k+1)\underline{1} = \underline{p}^T(0)\underline{\underline{a}}(0)\underline{\underline{a}}(1)\ldots\underline{\underline{a}}(k)\underline{1} \quad .$$

Then, the survival function of $T_{G\to d}$, denoted by $\overline{S}_{G\to d}(k)$ can be written as,

$$(8.12) \qquad \overline{S}_{G\to d}(k) = P[T_{Gd} > k] = p_G^T(k+1)\underline{1} = \underline{p}^T(0)\underline{\underline{a}}(0)\underline{\underline{a}}(1)\ldots\underline{\underline{a}}(k)\underline{1} \quad .$$

$\overline{S}_{G\to d}(k) = P[T_{Gd} > k]$, the survival function of $T_{G\to d}$ plays an important role in many applications as an indicator of end of application's lifecycle. By applying (8.12) for $k = 1,2,\ldots,35$, we develop a numerical procedure for computing the survival functions of the first passage times of Groups (a) and (b) of the 15 applications. Table 8.3 lists the distributions of survival functions of $T_{A\to d}$, $T_{B\to d}$ and $T_{C\to d}$ for the months $k = 10, 15, 20$ and 25.

Table 8. 3 – Distribution of Survival Functions

Application	P[T_ad > k] = α				P[T_bd > k] = α				P[T_cd > k] = α			
	k = 10	k = 15	k = 20	k = 25	k = 10	k = 15	k = 20	k = 25	k = 10	k = 15	k = 20	k = 25
Disney TSUM TSUM	0.63	0.38	0.18	0.11	0.63	0.38	0.18	0.11	0.38	0.16	0.06	0.04
LINE Pokopang	0.57	0.23	0.10	0.05	0.57	0.23	0.10	0.05	0.37	0.12	0.04	0.02
LINE cookie Run	0.45	0.15	0.05	0.02	0.45	0.15	0.05	0.02	0.30	0.06	0.02	0.01
Candy Crush Saga	0.38	0.17	0.08	0.04	0.42	0.19	0.09	0.04	0.10	0.03	0.01	0.01
Puzzle & Dragons	0.32	0.15	0.07	0.03	0.39	0.18	0.08	0.04	0.08	0.03	0.01	0.01
LINE Wind Runner	0.30	0.09	0.03	0.01	0.33	0.10	0.03	0.01	0.14	0.03	0.01	0.00
LINE POP	0.26	0.09	0.04	0.02	0.35	0.13	0.05	0.02	0.07	0.02	0.01	0.00
Mushroom Garden Deluxe	0.25	0.12	0.06	0.03	0.36	0.18	0.08	0.04	0.06	0.02	0.01	0.00
LINE Hidden Catch	0.15	0.05	0.02	0.01	0.24	0.08	0.03	0.01	0.05	0.01	0.00	0.00
LINE Jelly	0.21	0.05	0.02	0.01	0.25	0.07	0.02	0.01	0.12	0.03	0.01	0.00
LINE Bubble!	0.24	0.09	0.04	0.02	0.32	0.13	0.06	0.03	0.07	0.02	0.01	0.00
Nyanko Dai Senso	0.17	0.07	0.03	0.01	0.26	0.11	0.04	0.02	0.05	0.02	0.01	0.00
Mushroom Garden Seasons	0.24	0.10	0.04	0.02	0.35	0.16	0.07	0.03	0.05	0.02	0.01	0.00
Temple Run	0.17	0.05	0.02	0.01	0.23	0.08	0.03	0.01	0.03	0.01	0.00	0.00
Mushroom Garden	0.22	0.09	0.04	0.02	0.32	0.14	0.06	0.03	0.05	0.02	0.01	0.00

By mapping the above distributions with the three classes of applications given in Table 8.2, we understand that the limit values of 0.5 and 0.3 satisfy the expected segmentation. We present the following conditions (CR1) through (CR3) as quantifiable criteria for classifying smartphone applications into Class A, Class B and Class C.

(CR1) An application is said to be Class (A) if,

$$P[T_{a\to d} > 10] > 0.5\ OR\ P[T_{b\to d} > 10] > 0.5$$

(CR2) An application is said to be in Class (B) if,

$$0.3 <= P[T_{a\to d} > 10] <= 0.5\ AND\ 0.3 <= P[T_{bd} \to > 10] <= 0.5$$

(CR 3) An application is said to be in Class (C) if, none of the above two.

8.5 Classification Results

In order to validate the smartphone application classification criteria introduced in the previous section, we apply it on the 5 applications in the test data. In Table 8.3, we list the classification of those applications according to the proposed criteria.

Table 8. 4 - Classification of the Applications in the Test Data

Sub-category	Applications	Introduction to the Market	P[T_ad > 10]	P[T_bd >10]	Classification
Action	Temple Run 2	Jan-13	0.2	0.2	C
	Monster Strike	Dec-13	0.6	0.6	A
Card	Scratch de coupon	Nov-12	0.4	0.4	B
Casino	LINE Dozer	Nov-13	0.3	0.3	C
Roleplaying	Quiz RPG Witch and the Blak Cat Wiz	Mar-13	0.4	0.5	B

The distributions of the portions of Group (a) through (c) for the applications classified as Class (A), Class (B) and Class (C) are illustrated in Figure 8.33 through 8.35 respectively. We note that the data period is over 35 months period since the introduction of the underlying smartphone application into the market, except for "Monster Strike" and "LINE Dozer" with 29 and 30 months respectively.

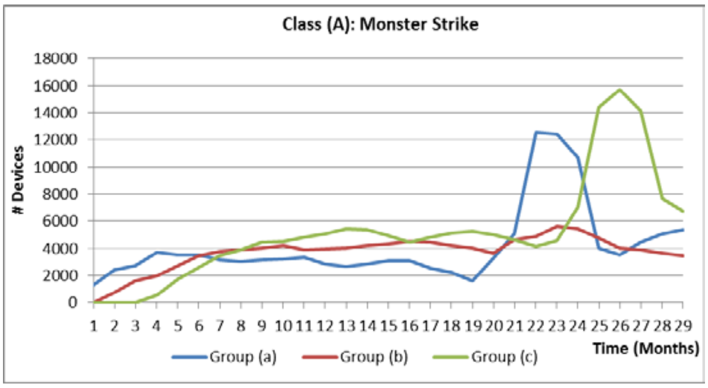


Figure 8.33 – Distribution of Usage Patter Groups of Class (A) Applications

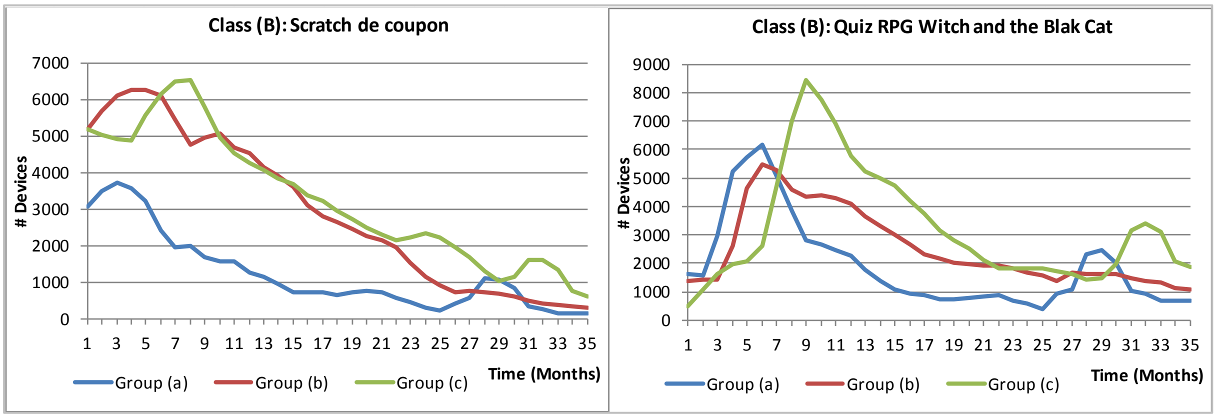


Figure 8.34 – Distribution of Usage Patter Groups of Class (B) Applications

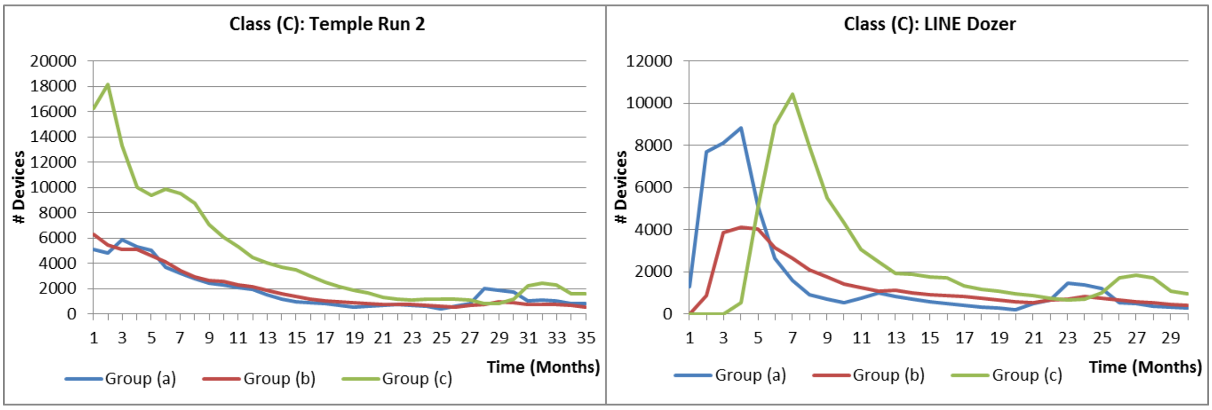


Figure 8.35 – Distribution of Usage Patter Groups of Class (C) Applications

The shape of the Group (b) towards the end of the time period in Figure 8.33 is at a considerable value for number of devices, showing that the application is still maintaining a considerable portion of devices with usage patterns 'Rising,' 'Stable,' 'Matured,' 'Recovering,' and 'Reactivating,' which indicates that the application is still active in the lifecycle. The shape of the Group (b) graphs in Figure 8.34 shows a declining trend, which may mean that the applications is losing the Group (b) devices and is in the tail of the life cycle. The close zero value of Group (b) graphs in Figure 8.35 indicates the application is already diminished from the market. The above observations demonstrate the appropriateness of the proposed approach for classifying applications based on the concept of the product lifecycle.

Chapter 9
Concluding Remarks

Extensive literature exists on the smartphone application usage patterns and popularity where different areas of interest are discussed; contextual user behaviors, network traffic, energy drain and application ranking based on App-store indicators such as number of downloads are to name some. However, to the best knowledge of the authors, literature does not address the problem of analyzing smartphone application characteristics based on the application usage patterns, performance indicators and application lifecycle. In this research we attempted to address this issue, taking advantage of a rare opportunity of employing a large volume of actual data on smartphone application usage.

Smartphone applications are different from the ordinary products and services in that they may be installed and uninstalled dynamically over time in a repeated manner, while ordinary products and services stay in use for a long period once they are purchased. By considering smartphone applications from three free game sub categories, 'Casual,' 'Puzzle' and 'Arcade' we studied about various patterns concerning application usage, such as application download patterns, application usage and their behavior in the product lifecycle. Investigating the monthly smartphone application usage information, we introduced a set of smartphone application usage patterns spanning over a period of six months: 'Exploring,' 'Rising,' 'Stable,' 'Matured,' 'Warning,' 'Ceasing,' 'Recovering,' 'Reactivating' and 'Fickle.' We introduced key competitive performance measures for smartphone applications based on monthly application downloads and as well as based on the six-month usage patterns. Owing to dynamic nature of smartphone application usage, we understand the appropriateness of defining the application performance measures based on the usage patterns over monthly downloads.

The five key competitive performance measures we introduced in this work are: Application device ratio, Application stability, Application popularity, Application advancement and Application declination. These five performance measures can be used for understanding smartphone application characteristics in different aspects. Application device ratio can be considered as an indicator of relative market share while Application stability describes the steadiness of the user-base in the market. As the name indicates Application popularity measure the application's level of popularity in the market and Application advancement is concerned with the potential of growing in the market. Finally, Application declination indicates the risk of being rejected from the market. These performance measures might be useful in assessing smartphone applications in the market.

We then experimented on four approaches of estimating the future values of performance measures by employing linear regression and ARIMA models. We performed future value estimation by 1) exclusively with ARIMA models approach, 2) exclusively with linear regression methods, 3) employing ARIMA approach for predicting explanatory variables of linear regression model and 4) combining ARIMA estimation and linear regression estimation by optimal convex combination. By numerical examples, we showed that the last approach of optimal convex combination of ARIMA estimation and linear regression estimation provides the future estimations with highest accuracy.

Finally, we performed lifecycle analysis of smartphone applications and proposed segmentation criteria for classifying smartphone applications based on the concept of the product lifecycle. In order to acknowledge the distinct dynamic usage patterns in smartphone applications compared to typical products and services, we incorporated six-month usage patterns for the segmentation criteria. We grouped the ten usage patterns above into four groups to represent the stages of product lifecycle:

Introduction stage → Group (a): Exploring and Fickle

Growth and Maturity stages → Group (b): Rising, Stable, Matured, Recovering, and Reactivating

Decline stage → Group (c): Warning and Ceasing

Diminished state → Group (d): Null

We developed discrete time Markov chain model where the Markov chain has the state space {a, b, c, d} relevant to four groups above. A numerical procedure is developed for computing the survival functions of the first passage times $T_{a \to d}$ and $T_{b \to d}$. Quantifiable criteria for classifying smartphone applications into Class A, Class B and Class C are then developed in terms of $P[T_{a \to d} > 10]$ and $P[T_{b \to d} > 10]$. We verified the suitability of the developed criteria by applying it on a distinct set of applications.

9.1 Limitations

Typically, the contract between Japanese mobile service providers and the smartphone users is set for a period of two years. During this two year period, users are offered with various valued discounts and services and however, any change in the mobile plan or the smartphone device before the end of the two year contract period, costs additional fee for the user. By the end of the two year period, the discounts and special offers become ceased, and the contract is extended for another two years without the discounts, unless specified by the user. However, at this point, a user has the opportunity of changing the mobile plan and/or the smartphone device without paying any changing fee. Due to this nature, many users tend to change their smartphone device to a new one at the end of two years. In our dataset, the device information contains no information on the smartphone user and therefore, it is impossible to capture such situations like device changes. Although the smartphone user continues to use the same set of applications in the same manner on the new smartphone device, it may be interpreted as two independent cases: the applications in the previous device are being removed and another device started to use a set of applications. This unavoidable situation may have a certain effect on the results of this work.

9.2 Future Work

In this study, we employed monthly application download, installation, uninstallation information of smartphone applications to perform various tasks such as statistical analysis, model generation, estimations and so on. However, it may be useful to compare the applicability of the proposed approaches on more fine-grained usage information such as daily usage data or application launch information. It may provide one with an insight on the unseen behaviours of the smartphone application users during a shorter period of time and one can project such insight to draw useful business implications.

Throughout this work, the period of time for the application usage patterns and the key competitive usage patterns were set to six months. However, one can perform the research varying the length of time period, so that the most optimal time period could be verified.

Further, it may be also be worth to verify the applicability of the proposed approaches on the smartphone applications from other categories without limiting to free Games. It may help one to understand the conditions under which the proposed approaches and algorithms work best. However, we did not address these areas by this research, so as not to limit the research scope by the time limitations imposed by the massive amount of row data and the extensive time taken for data processing.

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Appendix

Table A. 1 - App-Dev Values (Sub-Category: Casual)

Application	2013/06	2013/07	2013/08	2013/09	2013/10	2013/11	2013/12	2014/01	2014/02	2014/03	2014/04	2014/05	2014/06	2014/07	2014/08	2014/09
Dumb Ways to Die			0.00				0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.02	0.02
Come Here... Cat!	0.02		0.01	0.01	0.01		0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Matchless Cat Sew	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
STICK NINJA	0.02	0.01	0.01	0.01	0.01	0.01			0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Star Vanisher	0.01	0.01	0.02	0.02	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00
Star Vanisher -S		0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Kamehame Beam!				0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Alpaca Evolution	0.06	0.07	0.08	0.07	0.04	0.04	0.03	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.01
Alpaca Evolution Begins	0.00	0.01	0.02	0.02	0.02	0.02	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Star Chef			0.00	0.00	0.00	0.00	0.00		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Bound Monsters	0.01	0.02	0.02	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Snoopy Street	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Boiling OSSAN Eggs!		0.01	0.02	0.02	0.02	0.02	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.02	0.02
Ossan Train Stuffing!						0.01	0.01	0.02	0.02	0.02	0.03	0.03	0.03	0.03	0.03	0.03
Hamster Life	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Naughty Little Devil			0.00	0.01	0.01	0.01		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Constellation game		0.00	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Jewels Star	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Despicable Me	0.01	0.02														
Garden - escape game			0.00	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Pocket Dairy		0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Ah! Monster		0.00		0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00
Candy Crush Saga	0.04															
Pet Rescue Saga	0.00	0.00	0.01	0.01	0.01	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
Burger	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
My principal	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Scary Stories		0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00
Kumatomo							0.00	0.01	0.01	0.02	0.02	0.03	0.03	0.03	0.02	0.02
Kamen rider Raidabout!	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Dots			0.00	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Kanji ability diagnosis	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
My Talking Tom						0.00	0.01	0.01	0.01	0.01	0.02	0.02	0.02	0.02	0.02	0.02
Escape from the subway	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00
Sudden Kiss of Vow						0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Brain power + payment	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Poop Rearing Simulator	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Fire a match	0.02	0.02	0.02		0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Match DE Rhythm	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00
Please reply			0.01	0.01	0.01	0.01	0.01	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Homo-tampering	0.04	0.03	0.03		0.02	0.02	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Mushroom Garden	0.07	0.06	0.07	0.06	0.05	0.05	0.05	0.04	0.04	0.04	0.04	0.04	0.04	0.05	0.05	0.05
Mushroom Garden Seaso	0.11				0.08											
Touche Investigator Gard		0.00	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Ultra! Gourmet life form	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Kotatsu cat	0.01	0.13	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Nyanko Dai Senso	0.11	0.01	0.11	0.10	0.07	0.07			0.06	0.07	0.07	0.08	0.08	0.09	0.09	0.10
COLOPL	0.02	0.10	0.03	0.02	0.02	0.02	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
Sparkle Drop!	0.01	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Rhythm coin!	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Rhythm coin 2	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01

(Cont.)

(Cont. from the previous page)

Application	2013/06	2013/07	2013/08	2013/09	2013/10	2013/11	2013/12	2014/01	2014/02	2014/03	2014/04	2014/05	2014/06	2014/07	2014/08	2014/09
Disney Magician	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Go home!	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00
Game Gift		0.01		0.00	0.00	0.01	0.01	0.01	0.01	0.02	0.02	0.03	0.03	0.03	0.04	0.04
Crazy Tower	0.03		0.02	0.02	0.02	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Abepyon	0.00	0.02		0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00
Don't step the white tile	0.02	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.01	0.02	0.02	0.02	0.03	0.03
Tokimeki Restaurant	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Big Cat in Advance		0.01	0.00	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00
Angry Yoshida & Hungr						0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00
Nyanko dai bōsō							0.01	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
LINE Theater Town	0.03			0.04	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.02	0.02	0.02	0.02
LINE quiz		0.03	0.01	0.01	0.02		0.04	0.04	0.04	0.04	0.04	0.04	0.03	0.03	0.03	0.02
LINE ranch life	0.01	0.01	0.02	0.03	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
Runaway Let's tiger's	0.01	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
SkySwings		0.01			0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00
Wall Jump	0.00		0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Touch the Numbers for	0.03	0.00	0.03	0.03	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
Nyanko Hazard		0.03	0.01	0.01	0.01	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Pou	0.02	0.01	0.02	0.01	0.01	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
iStair+		0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00
ForbiddenEgg	0.01		0.02	0.02	0.02	0.03	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00
MindStep	0.01	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00
PressZombie		0.01	0.00	0.00	0.00	0.01	0.01		0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ReacheeE	0.02		0.03	0.03	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
ShotZombie	0.00	0.03		0.03	0.03	0.03	0.04	0.03	0.03	0.03	0.02	0.02	0.02	0.02	0.01	0.01
ShotZombieHalloween		0.01			0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00
YureteruBose			0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Your dream story	0.02	0.00	0.02	0.02	0.01	0.02	0.02	0.02	0.01	0.02	0.02	0.02	0.02	0.02	0.02	0.02
Creative pastry chef		0.02			0.00	0.01	0.01	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
Pancake Tower	0.00		0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Spoon mania		0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
More than 700 stories!			0.00		0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Busu diagnosis			0.01		0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Pesoguin Battery 3D Pe	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Hardest Game Ever 2	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
One million eggs	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table A. 2 - App-Stab Values (Sub-Category: Casual)

Application	2013/06	2013/07	2013/08	2013/09	2013/10	2013/11	2013/12	2014/01	2014/02	2014/03	2014/04	2014/05	2014/06	2014/07	2014/08	2014/09
Dumb Ways to Die			0.00				0.05	0.03	0.08	0.08	0.09	0.10	0.10	0.09	0.10	0.12
Come Here... Cat!	0.11		0.13	0.19	0.21		0.26	0.30	0.28	0.30	0.31	0.31	0.32	0.32	0.34	0.35
Matchless Cat Sew	0.06	0.11	0.10	0.16	0.16	0.17	0.18	0.19	0.17	0.18	0.18	0.21	0.21	0.22	0.21	0.21
STICK NINJA	0.07	0.10	0.09	0.15	0.17	0.19			0.22	0.24	0.24	0.27	0.26	0.28	0.29	0.29
Star Vanisher	0.00	0.00	0.00	0.00	0.09	0.08	0.12	0.19	0.16	0.17	0.20	0.24	0.29	0.32	0.32	0.34
Star Vanisher -S		0.00	0.00	0.00	0.00	0.09	0.12	0.15	0.12	0.13	0.15	0.22	0.21	0.24	0.22	0.22
Kamehame Beam!				0.00	0.00	0.00	0.16	0.11	0.11	0.14	0.18	0.20	0.23	0.24	0.24	0.25
Alpaca Evolution	0.01	0.06	0.05	0.09	0.11	0.13	0.17	0.22	0.19	0.19	0.19	0.20	0.22	0.24	0.24	0.25
Alpaca Evolution Begins	0.00	0.00	0.00	0.00	0.04	0.11	0.13	0.18	0.16	0.16	0.18	0.18	0.21	0.23	0.23	0.23
Star Chef			0.00	0.00	0.00	0.00	0.09		0.06	0.08	0.14	0.16	0.15	0.14	0.17	0.14
Bound Monsters	0.00	0.00	0.05	0.09	0.10	0.11	0.16	0.18	0.15	0.16	0.17	0.18	0.17	0.18	0.18	0.19
Snoopy Street	0.16	0.18	0.15	0.22	0.21	0.21	0.25	0.25	0.23	0.25	0.27	0.30	0.31	0.31	0.32	0.30
Boiling OSSAN Eggs!		0.00	0.00	0.00	0.00	0.07	0.14	0.13	0.10	0.11	0.11	0.13	0.15	0.17	0.18	0.19
Ossan Train Stuffing!						0.00	0.00	0.00	0.00	0.06	0.07	0.07	0.09	0.10	0.13	0.15
Hamster Life	0.08	0.10	0.08	0.12	0.11	0.11	0.16	0.15	0.15	0.17	0.17	0.18	0.19	0.20	0.20	0.19
Naughty Little Devil			0.00	0.00	0.00	0.06		0.15	0.13	0.12	0.12	0.16	0.17	0.16	0.13	0.11
Constellation game		0.00	0.00	0.01	0.02	0.05	0.12	0.10	0.11	0.12	0.13	0.15	0.15	0.14	0.15	0.15
Jewels Star	0.13	0.14	0.15	0.21	0.20	0.22	0.26	0.27	0.26	0.26	0.27	0.28	0.30	0.31	0.32	0.30
Despicable Me	0.00	0.00														
Garden - escape game			0.00	0.00	0.00	0.00	0.12	0.08	0.07	0.08	0.09	0.11	0.12	0.12	0.12	0.10
Pocket Dairy		0.00	0.00	0.00	0.00	0.12	0.18	0.19	0.17	0.19	0.20	0.22	0.25	0.26	0.25	0.27
Ah! Monster		0.00		0.00	0.00	0.04	0.16	0.14	0.12	0.13	0.15	0.17	0.20	0.21	0.23	0.25
Candy Crush Saga	0.03															
Pet Rescue Saga	0.00	0.00	0.00	0.00	0.03	0.09	0.16	0.15	0.15	0.16	0.17	0.19	0.19	0.20	0.20	0.18
Burger	0.09	0.11	0.11	0.19	0.20	0.21	0.22	0.23	0.21	0.21	0.21	0.24	0.27	0.29	0.29	0.30
My principal	0.00	0.00	0.01	0.06	0.06	0.10	0.14	0.16	0.13	0.12	0.14	0.16	0.20	0.21	0.23	0.23
Scary Stories		0.00	0.00	0.00	0.00	0.03	0.22	0.19	0.17	0.20	0.19	0.21	0.21	0.23	0.24	0.26
Kumatomo							0.00	0.00	0.00	0.00	0.03	0.07	0.09	0.12	0.13	0.16
Kamen rider Raidabout!	0.15	0.17	0.14	0.20	0.20	0.22	0.25	0.24	0.22	0.21	0.21	0.22	0.22	0.26	0.26	0.27
Dots			0.00	0.00	0.00	0.00	0.22	0.16	0.13	0.13	0.15	0.14	0.15	0.17	0.16	0.17
Kanji ability diagnosis	0.16	0.20	0.18	0.27	0.29	0.30	0.31	0.32	0.30	0.31	0.32	0.33	0.34	0.36	0.35	0.34
My Talking Tom						0.00	0.00	0.00	0.00	0.04	0.07	0.09	0.09	0.09	0.09	0.08
Escape from the subway	0.00	0.00	0.00	0.01	0.12	0.10	0.15	0.18	0.16	0.18	0.20	0.22	0.25	0.27	0.26	0.24
Sudden Kiss of Vow						0.00	0.00	0.00	0.09	0.10	0.11	0.14	0.16	0.17	0.18	0.18
Brain power + payment	0.02	0.12	0.11	0.19	0.21	0.25	0.27	0.27	0.25	0.25	0.24	0.29	0.30	0.32	0.30	0.29
Poop Rearing Simulator	0.20	0.21	0.18	0.24	0.23	0.24	0.28	0.30	0.26	0.27	0.27	0.28	0.27	0.27	0.33	0.29
Fire a match	0.09	0.11	0.09		0.15	0.19	0.22	0.23	0.19	0.21	0.21	0.23	0.22	0.24	0.27	0.26
Match DE Rhythm	0.00	0.00	0.03	0.07	0.08	0.11	0.16	0.17	0.15	0.17	0.19	0.22	0.23	0.23	0.25	0.24
Please reply			0.00	0.00	0.00	0.00	0.11	0.07	0.05	0.06	0.08	0.08	0.11	0.12	0.13	0.12
Homo-tampering	0.11	0.12	0.11		0.17	0.19	0.21	0.23	0.20	0.21	0.20	0.21	0.22	0.22	0.22	0.23
Mushroom Garden	0.19	0.22	0.20	0.28	0.28	0.31	0.32	0.35	0.31	0.31	0.30	0.30	0.30	0.30	0.32	0.29
Mushroom Garden Seaso	0.23				0.30											
Touche Investigator Gard		0.01	0.00	0.02	0.01	0.07	0.15	0.13	0.12	0.14	0.14	0.15	0.17	0.18	0.17	0.18
Ultra! Gourmet life form	0.00	0.00	0.00	0.00	0.15	0.20	0.26	0.29	0.27	0.27	0.21	0.22	0.23	0.22	0.30	0.27
Kotatsu cat	0.10	0.24	0.14	0.21	0.21	0.24	0.27	0.30	0.27	0.30	0.31	0.31	0.32	0.34	0.35	0.34
Nyanko Dai Senso	0.08	0.15	0.11	0.17	0.18	0.20			0.22	0.23	0.22	0.22	0.21	0.22	0.23	0.24
COLOPL	0.26	0.11	0.24	0.30	0.29	0.30	0.39	0.38	0.35	0.36	0.34	0.35	0.37	0.37	0.38	0.38
Sparkle Drop!	0.14	0.28	0.12	0.18	0.18	0.18	0.25	0.25	0.25	0.27	0.29	0.30	0.32	0.33	0.37	0.32
Rhythm coin!	0.19	0.15	0.18	0.26	0.25	0.25	0.28	0.28	0.26	0.27	0.29	0.32	0.36	0.35	0.36	0.30
Rhythm coin 2	0.13	0.22	0.14	0.20	0.22	0.24	0.24	0.25	0.22	0.22	0.22	0.24	0.26	0.26	0.26	0.25
Disney Magician	0.12	0.17	0.12	0.17	0.17	0.17	0.21	0.23	0.22	0.25	0.26	0.29	0.31	0.30	0.31	0.31
Go home!	0.00	0.14	0.00	0.08	0.10	0.13	0.21	0.24	0.20	0.22	0.24	0.28	0.32	0.34	0.32	0.34
Game Gift		0.00		0.00	0.00	0.00	0.07	0.04	0.06	0.09	0.11	0.15	0.18	0.23	0.25	0.26
Crazy Tower	0.12		0.12	0.18	0.18	0.20	0.23	0.25	0.23	0.24	0.24	0.25	0.26	0.27	0.25	0.25
Abepyon	0.00	0.14		0.00	0.04	0.13	0.14	0.21	0.19	0.18	0.20	0.24	0.26	0.26	0.29	0.29
Don't step the white tile	0.11	0.00	0.12	0.19	0.20	0.24	0.26	0.30	0.25	0.18	0.08	0.07	0.06	0.08	0.15	0.16
Tokimeki Restaurant	0.00	0.14	0.13	0.19	0.19	0.20	0.23	0.22	0.22	0.23	0.23	0.23	0.25	0.26	0.26	0.24
Big Cat in Advance		0.09	0.00	0.00	0.00	0.00	0.15	0.10	0.08	0.10	0.12	0.17	0.18	0.17	0.16	0.15
Angry Yoshida & Hungr						0.00	0.00	0.00	0.00	0.09	0.09	0.11	0.16	0.18	0.22	0.21
Nyanko dai bōsō							0.00	0.00	0.00	0.07	0.09	0.13	0.15	0.18	0.20	0.22

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Application	2013/06	2013/07	2013/08	2013/09	2013/10	2013/11	2013/12	2014/01	2014/02	2014/03	2014/04	2014/05	2014/06	2014/07	2014/08	2014/09
LINE Theater Town	0.00			0.15	0.16	0.18	0.21	0.22	0.20	0.22	0.22	0.23	0.24	0.24	0.26	0.26
LINE quiz		0.09	0.00	0.00	0.00		0.07	0.06	0.05	0.10	0.10	0.13	0.15	0.17	0.18	0.21
LINE ranch life	0.00	0.00	0.05	0.08	0.10	0.11	0.16	0.15	0.15	0.15	0.16	0.17	0.19	0.21	0.24	0.23
Runaway Let's tiger's	0.16	0.00	0.18	0.24	0.24	0.26	0.29	0.30	0.26	0.27	0.27	0.27	0.28	0.29	0.31	0.33
SkySwings		0.19			0.00	0.00	0.00	0.00	0.02	0.09	0.09	0.11	0.15	0.18	0.22	0.22
Wall Jump	0.00		0.00	0.00	0.10	0.10	0.13	0.14	0.12	0.13	0.16	0.19	0.19	0.21	0.20	0.20
Touch the Numbers for A	0.16	0.00	0.18	0.25	0.27	0.31	0.34	0.37	0.31	0.32	0.32	0.33	0.34	0.37	0.38	0.38
Nyanko Hazard		0.18	0.00	0.00	0.00	0.09	0.10	0.15	0.14	0.14	0.16	0.18	0.16	0.13	0.12	0.09
Pou	0.02	0.00	0.03	0.07	0.07	0.06	0.10	0.08	0.08	0.08	0.09	0.10	0.11	0.12	0.12	0.12
iStair+		0.03	0.00	0.00	0.00	0.00	0.16	0.11	0.09	0.11	0.11	0.13	0.14	0.14	0.17	0.19
ForbiddenEgg	0.00		0.00	0.00	0.05	0.06	0.10	0.14	0.11	0.12	0.13	0.14	0.15	0.18	0.17	0.18
MindStep	0.00	0.00	0.00	0.05	0.07	0.09	0.14	0.17	0.13	0.15	0.16	0.19	0.19	0.19	0.23	0.25
PressZombie		0.00	0.00	0.00	0.00	0.00	0.12		0.07	0.10	0.12	0.15	0.16	0.17	0.15	0.18
ReacheeE	0.03		0.07	0.12	0.12	0.14	0.19	0.21	0.17	0.18	0.19	0.22	0.22	0.22	0.21	0.21
ShotZombie	0.00	0.07		0.01	0.01	0.06	0.12	0.12	0.10	0.11	0.12	0.13	0.15	0.14	0.15	0.15
ShotZombieHalloween		0.00			0.00	0.00	0.00	0.00	0.06	0.07	0.07	0.10	0.14	0.15	0.17	0.17
YureteruBose			0.00	0.00	0.00	0.04	0.20	0.15	0.13	0.14	0.17	0.22	0.20	0.22	0.21	0.19
Your dream story	0.13	0.00	0.12	0.17	0.16	0.18	0.21	0.20	0.17	0.17	0.18	0.19	0.21	0.22	0.22	0.20
Creative pastry chef		0.14			0.00	0.00	0.00	0.00	0.09	0.12	0.13	0.17	0.17	0.19	0.20	0.20
Pancake Tower	0.00		0.00	0.00	0.02	0.03	0.11	0.06	0.07	0.08	0.08	0.14	0.15	0.19	0.20	0.22
Spoon mania		0.00	0.00	0.00	0.00	0.00	0.10	0.06	0.07	0.08	0.09	0.14	0.16	0.19	0.21	0.21
More than 700 stories!			0.00		0.00	0.00	0.00	0.00	0.12	0.12	0.14	0.18	0.18	0.19	0.17	0.17
Busu diagnosis			0.00		0.08	0.07	0.12	0.16	0.12	0.10	0.11	0.13	0.18	0.18	0.17	0.18
Pesoguin Battery 3D Pe	0.08	0.00	0.06	0.08	0.07	0.09	0.21	0.18	0.20	0.22	0.24	0.26	0.30	0.31	0.30	0.34
Hardest Game Ever 2	0.00	0.09	0.01	0.04	0.04	0.06	0.11	0.12	0.11	0.12	0.13	0.15	0.15	0.14	0.13	0.12
One million eggs	0.05	0.00	0.05	0.14	0.14	0.18	0.21	0.27	0.21	0.23	0.19	0.21	0.21	0.25	0.23	0.21

Table A. 3 - App-Pop Values (Sub-Category: Casual)

Application	2013/06	2013/07	2013/08	2013/09	2013/10	2013/11	2013/12	2014/01	2014/02	2014/03	2014/04	2014/05	2014/06	2014/07	2014/08	2014/09
Dumb Ways to Die			0.17				0.32	0.35	0.35	0.37	0.37	0.37	0.37	0.38	0.38	0.40
Come Here... Cat!	0.39		0.42	0.47	0.47		0.50	0.60	0.53	0.55	0.56	0.58	0.59	0.57	0.57	0.57
Matchless Cat Sew	0.34	0.37	0.37	0.43	0.44	0.46	0.45	0.53	0.46	0.48	0.48	0.49	0.49	0.51	0.51	0.50
STICK NINJA	0.34	0.37	0.38	0.43	0.43	0.46			0.51	0.53	0.52	0.53	0.54	0.58	0.56	0.57
Star Vanisher	0.17	0.24	0.26	0.30	0.32	0.34	0.37	0.56	0.46	0.47	0.49	0.50	0.53	0.56	0.57	0.60
Star Vanisher -S		0.17	0.20	0.28	0.32	0.35	0.38	0.50	0.41	0.43	0.44	0.45	0.48	0.51	0.50	0.49
Kamehame Beam!				0.33	0.36	0.41	0.44	0.47	0.37	0.41	0.45	0.47	0.49	0.52	0.52	0.52
Alpaca Evolution	0.30	0.32	0.33	0.37	0.38	0.42	0.42	0.56	0.47	0.48	0.47	0.47	0.50	0.52	0.52	0.53
Alpaca Evolution Begins	0.17	0.19	0.23	0.31	0.34	0.37	0.39	0.53	0.44	0.44	0.46	0.46	0.49	0.50	0.51	0.51
Star Chef			0.17	0.33	0.33	0.37	0.39		0.32	0.34	0.39	0.45	0.47	0.48	0.46	0.44
Bound Monsters	0.27	0.31	0.32	0.36	0.38	0.41	0.42	0.52	0.43	0.44	0.46	0.46	0.47	0.47	0.48	0.48
Snoopy Street	0.45	0.46	0.44	0.50	0.48	0.50	0.50	0.56	0.50	0.51	0.53	0.55	0.58	0.57	0.57	0.55
Boiling OSSAN Eggs!		0.17	0.19	0.30	0.34	0.38	0.39	0.46	0.38	0.39	0.39	0.40	0.42	0.43	0.45	0.47
Ossan Train Stuffing!						0.17	0.22	0.26	0.29	0.32	0.32	0.33	0.36	0.38	0.40	0.42
Hamster Life	0.35	0.35	0.33	0.39	0.39	0.41	0.42	0.47	0.41	0.42	0.44	0.45	0.46	0.46	0.46	0.46
Naughty Little Devil			0.20	0.24	0.27	0.29		0.53	0.42	0.44	0.42	0.43	0.42	0.41	0.41	0.39
Constellation game		0.19	0.20	0.32	0.33	0.35	0.38	0.42	0.38	0.39	0.42	0.43	0.44	0.43	0.43	0.42
Jewels Star	0.38	0.38	0.41	0.47	0.47	0.50	0.50	0.59	0.51	0.53	0.53	0.54	0.55	0.56	0.56	0.55
Despicable Me	0.17	0.20														
Garden - escape game			0.17	0.33	0.37	0.39	0.40	0.41	0.32	0.33	0.34	0.35	0.37	0.38	0.39	0.38
Pocket Dairy		0.17	0.21	0.32	0.37	0.41	0.44	0.54	0.45	0.47	0.48	0.49	0.52	0.54	0.53	0.54
Ah! Monster		0.17		0.31	0.36	0.39	0.42	0.49	0.40	0.43	0.44	0.46	0.47	0.50	0.54	0.55
Candy Crush Saga	0.30															
Pet Rescue Saga	0.17	0.19	0.21	0.32	0.35	0.39	0.42	0.48	0.43	0.44	0.46	0.46	0.47	0.47	0.47	0.43
Burger	0.35	0.36	0.38	0.45	0.46	0.49	0.47	0.53	0.46	0.47	0.48	0.50	0.52	0.54	0.53	0.53

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Application	2013/06	2013/07	2013/08	2013/09	2013/10	2013/11	2013/12	2014/01	2014/02	2014/03	2014/04	2014/05	2014/06	2014/07	2014/08	2014/09
My principal	0.22	0.25	0.26	0.29	0.31	0.38	0.39	0.50	0.42	0.41	0.43	0.43	0.45	0.45	0.49	0.47
Scary Stories		0.17	0.18	0.32	0.38	0.43	0.47	0.53	0.45	0.48	0.49	0.50	0.49	0.49	0.49	0.53
Kumatomo							0.17	0.20	0.24	0.29	0.31	0.34	0.37	0.39	0.40	0.43
Kamen rider Raidabout!	0.42	0.43	0.42	0.47	0.48	0.49	0.49	0.54	0.49	0.47	0.48	0.48	0.49	0.50	0.52	0.53
Dots			0.17	0.33	0.39	0.44	0.47	0.50	0.41	0.42	0.44	0.44	0.42	0.42	0.44	0.43
Kanji ability diagnosis	0.43	0.45	0.46	0.53	0.53	0.56	0.56	0.62	0.56	0.55	0.56	0.58	0.59	0.61	0.60	0.59
My Talking Tom						0.17	0.21	0.25	0.29	0.32	0.34	0.35	0.36	0.35	0.36	0.35
Escape from the subway	0.17	0.25	0.27	0.32	0.35	0.38	0.41	0.53	0.45	0.47	0.49	0.51	0.53	0.55	0.55	0.55
Sudden Kiss of Vow						0.24	0.28	0.31	0.35	0.37	0.38	0.40	0.43	0.44	0.45	0.46
Brain power + payment	0.34	0.37	0.39	0.45	0.47	0.52	0.52	0.57	0.51	0.52	0.53	0.52	0.55	0.55	0.54	0.52
Poop Rearing Simulator	0.50	0.49	0.47	0.51	0.49	0.52	0.52	0.60	0.52	0.54	0.54	0.54	0.55	0.54	0.58	0.56
Fire a match	0.38	0.38	0.37		0.42	0.47	0.46	0.55	0.47	0.49	0.48	0.49	0.50	0.52	0.51	0.52
Match DE Rhythm	0.25	0.28	0.28	0.33	0.37	0.42	0.43	0.51	0.42	0.44	0.45	0.46	0.49	0.51	0.51	0.51
Please reply			0.17	0.33	0.38	0.41	0.41	0.42	0.31	0.33	0.33	0.33	0.35	0.36	0.38	0.38
Homo-tampering	0.40	0.41	0.41		0.44	0.47	0.47	0.57	0.49	0.49	0.49	0.50	0.51	0.51	0.50	0.51
Mushroom Garden	0.48	0.49	0.48	0.53	0.53	0.57	0.56	0.65	0.57	0.57	0.55	0.56	0.57	0.57	0.57	0.55
Mushroom Garden Seaso	0.51				0.55											
Touche Investigator Gard		0.21	0.21	0.32	0.34	0.38	0.40	0.45	0.39	0.40	0.42	0.42	0.43	0.45	0.44	0.46
Ultra! Gourmet life form	0.17	0.24	0.28	0.37	0.43	0.47	0.50	0.59	0.53	0.54	0.47	0.49	0.51	0.52	0.54	0.53
Kotatsu cat	0.37	0.51	0.42	0.48	0.47	0.51	0.52	0.61	0.53	0.55	0.56	0.57	0.58	0.61	0.61	0.59
Nyanko Dai Senso	0.38	0.42	0.40	0.45	0.45	0.49			0.50	0.50	0.49	0.48	0.49	0.50	0.51	0.51
COLOPL	0.51	0.39	0.49	0.56	0.56	0.60	0.59	0.66	0.59	0.61	0.60	0.60	0.62	0.62	0.62	0.62
Sparkle Drop!	0.41	0.52	0.39	0.46	0.46	0.49	0.50	0.57	0.51	0.53	0.54	0.55	0.58	0.59	0.60	0.58
Rhythm coin!	0.46	0.42	0.46	0.52	0.51	0.54	0.52	0.60	0.53	0.53	0.54	0.56	0.61	0.62	0.62	0.59
Rhythm coin 2	0.41	0.48	0.42	0.47	0.48	0.51	0.49	0.56	0.49	0.48	0.48	0.50	0.52	0.53	0.53	0.52
Disney Magician	0.39	0.44	0.40	0.44	0.44	0.46	0.47	0.54	0.47	0.50	0.51	0.53	0.54	0.55	0.55	0.56
Go home!	0.21	0.42	0.28	0.33	0.36	0.42	0.46	0.57	0.48	0.50	0.51	0.53	0.56	0.58	0.59	0.59
Game Gift		0.27		0.33	0.30	0.30	0.33	0.34	0.34	0.37	0.39	0.42	0.46	0.48	0.49	0.51
Crazy Tower	0.42		0.42	0.47	0.46	0.49	0.49	0.59	0.50	0.52	0.52	0.53	0.54	0.53	0.54	0.54
Abepyon	0.17	0.42		0.30	0.34	0.37	0.40	0.55	0.46	0.47	0.50	0.51	0.53	0.54	0.57	0.57
Don't step the white tile	0.40	0.19	0.42	0.47	0.46	0.50	0.51	0.61	0.53	0.42	0.29	0.33	0.36	0.39	0.40	0.42
Tokimeki Restaurant	0.35	0.42	0.40	0.46	0.46	0.49	0.49	0.54	0.48	0.49	0.50	0.51	0.53	0.53	0.53	0.52
Big Cat in Advance		0.39	0.17	0.33	0.38	0.42	0.45	0.47	0.35	0.40	0.41	0.44	0.43	0.44	0.43	0.43
Angry Yoshida & Hungr						0.17	0.24	0.28	0.31	0.34	0.36	0.40	0.45	0.47	0.47	0.49
Nyanko dai bōsō							0.22	0.27	0.31	0.34	0.36	0.39	0.43	0.46	0.48	0.50
LINE Theater Town	0.32			0.43	0.43	0.47	0.47	0.55	0.48	0.49	0.49	0.51	0.52	0.53	0.53	0.53
LINE quiz		0.36	0.20	0.29	0.32		0.32	0.38	0.35	0.37	0.38	0.40	0.42	0.45	0.47	0.48
LINE ranch life	0.27	0.17	0.30	0.36	0.38	0.41	0.42	0.48	0.42	0.43	0.44	0.45	0.47	0.48	0.50	0.51
Runaway Let's tiger's	0.45	0.30	0.45	0.51	0.49	0.52	0.53	0.58	0.52	0.53	0.53	0.54	0.55	0.55	0.56	0.56
SkySwings		0.46			0.17	0.18	0.25	0.29	0.32	0.34	0.36	0.38	0.42	0.45	0.48	0.48
Wall Jump	0.17		0.26	0.33	0.36	0.38	0.39	0.48	0.41	0.42	0.42	0.45	0.47	0.50	0.49	0.48
Touch the Numbers for A	0.44	0.23	0.46	0.51	0.52	0.56	0.57	0.66	0.58	0.58	0.58	0.58	0.58	0.60	0.62	0.63
Nyanko Hazard		0.46	0.21	0.27	0.30	0.33	0.35	0.51	0.41	0.44	0.44	0.45	0.40	0.39	0.39	0.37
Pou	0.24	0.17	0.26	0.35	0.35	0.36	0.36	0.40	0.34	0.35	0.36	0.36	0.37	0.37	0.38	0.37
iStair+		0.26	0.17	0.33	0.37	0.41	0.44	0.46	0.36	0.39	0.41	0.43	0.43	0.43	0.46	0.48
ForbiddenEgg	0.17		0.24	0.28	0.30	0.31	0.35	0.50	0.40	0.41	0.42	0.42	0.43	0.44	0.46	0.45
MindStep	0.21	0.21	0.27	0.32	0.34	0.38	0.40	0.50	0.41	0.43	0.45	0.47	0.47	0.48	0.48	0.50
PressZombie		0.26	0.17	0.33	0.36	0.39	0.42		0.33	0.37	0.40	0.43	0.45	0.46	0.48	0.47
ReacheeE	0.31		0.33	0.39	0.40	0.44	0.45	0.53	0.45	0.46	0.47	0.48	0.48	0.49	0.48	0.48
ShotZombie	0.20	0.33		0.30	0.33	0.36	0.38	0.46	0.38	0.39	0.41	0.42	0.42	0.43	0.43	0.44
ShotZombieHalloween		0.19			0.17	0.22	0.27	0.30	0.33	0.35	0.35	0.38	0.40	0.43	0.45	0.47
YureteruBose			0.18	0.32	0.38	0.41	0.46	0.50	0.42	0.44	0.45	0.47	0.48	0.51	0.53	0.52
Your dream story	0.40	0.17	0.40	0.44	0.44	0.45	0.45	0.51	0.45	0.44	0.44	0.45	0.46	0.47	0.47	0.47
Creative pastry chef		0.41			0.17	0.22	0.28	0.32	0.36	0.39	0.40	0.44	0.46	0.47	0.48	0.48
Pancake Tower	0.17		0.19	0.33	0.33	0.36	0.39	0.36	0.33	0.35	0.38	0.41	0.43	0.45	0.46	0.49
Spoon mania		0.21	0.17	0.33	0.32	0.36	0.38	0.36	0.33	0.36	0.38	0.41	0.43	0.46	0.49	0.50
More than 700 stories!			0.17		0.18	0.24	0.29	0.34	0.37	0.39	0.43	0.46	0.48	0.48	0.46	0.45
Busu diagnosis			0.24		0.29	0.31	0.36	0.53	0.42	0.41	0.41	0.38	0.41	0.43	0.46	0.49
Pesoguin Battery 3D Per	0.39	0.22	0.29	0.39	0.39	0.43	0.45	0.51	0.45	0.49	0.52	0.53	0.56	0.56	0.56	0.59
Hardest Game Ever 2	0.23	0.35	0.24	0.31	0.34	0.37	0.38	0.47	0.38	0.40	0.41	0.42	0.42	0.42	0.42	0.42
One million eggs	0.31	0.23	0.35	0.38	0.40	0.46	0.47	0.60	0.51	0.51	0.50	0.50	0.49	0.51	0.48	0.49

Table A. 4 - App-Adv Values (Sub-Category: Casual)

Application	2013/06	2013/07	2013/08	2013/09	2013/10	2013/11	2013/12	2014/01	2014/02	2014/03	2014/04	2014/05	2014/06	2014/07	2014/08	2014/09
Dumb Ways to Die			0.00				0.29	0.25	0.15	0.15	0.12	0.14	0.14	0.16	0.13	0.11
Come Here... Cat!	0.08		0.04	0.13	0.09		0.07	0.11	0.10	0.09	0.07	0.08	0.06	0.04	0.04	0.06
Matchless Cat Sew	0.16	0.13	0.08	0.19	0.12	0.13	0.11	0.15	0.09	0.08	0.06	0.08	0.08	0.07	0.07	0.07
STICK NINJA	0.05	0.06	0.04	0.10	0.07	0.09			0.06	0.08	0.04	0.06	0.05	0.04	0.04	0.07
Star Vanisher	0.01	0.41	0.17	0.23	0.07	0.05	0.04	0.08	0.06	0.07	0.06	0.06	0.05	0.05	0.06	0.09
Star Vanisher -S		0.00	0.23	0.48	0.28	0.12	0.04	0.06	0.07	0.07	0.11	0.08	0.09	0.10	0.09	0.10
Kamehame Beam!				1.00	0.40	0.33	0.09	0.07	0.04	0.04	0.06	0.07	0.06	0.07	0.08	0.06
Alpaca Evolution	0.21	0.12	0.04	0.08	0.07	0.08	0.05	0.09	0.06	0.05	0.06	0.07	0.07	0.07	0.05	0.06
Alpaca Evolution Begins	0.00	0.16	0.27	0.43	0.23	0.10	0.05	0.09	0.06	0.05	0.08	0.08	0.07	0.06	0.04	0.06
Star Chef			0.00	1.00	0.32	0.28	0.08		0.03	0.03	0.01	0.07	0.07	0.10	0.10	0.04
Bound Monsters	0.30	0.26	0.08	0.12	0.08	0.10	0.05	0.08	0.06	0.06	0.05	0.06	0.07	0.09	0.08	0.07
Snoopy Street	0.11	0.10	0.07	0.19	0.13	0.15	0.10	0.14	0.12	0.10	0.08	0.08	0.09	0.08	0.08	0.08
Boiling OSSAN Eggs!		0.00	0.15	0.62	0.35	0.21	0.06	0.10	0.11	0.12	0.10	0.09	0.08	0.06	0.05	0.05
Ossan Train Stuffing!						0.01	0.32	0.31	0.24	0.14	0.08	0.15	0.17	0.14	0.09	0.06
Hamster Life	0.10	0.09	0.05	0.25	0.14	0.17	0.11	0.12	0.08	0.07	0.08	0.09	0.08	0.08	0.06	0.06
Naughty Little Devil			0.16	0.28	0.17	0.08		0.07	0.05	0.08	0.08	0.08	0.06	0.08	0.07	0.08
Constellation game		0.08	0.14	0.71	0.31	0.25	0.14	0.15	0.11	0.08	0.09	0.07	0.07	0.05	0.05	0.05
Jewels Star	0.07	0.07	0.07	0.22	0.14	0.14	0.08	0.10	0.06	0.07	0.06	0.07	0.06	0.05	0.03	0.04
Despicable Me	0.00	0.20														
Garden - escape game			0.00	1.00	0.39	0.27	0.07	0.07	0.07	0.08	0.09	0.09	0.09	0.07	0.07	0.08
Pocket Dairy		0.00	0.28	0.62	0.38	0.19	0.07	0.09	0.06	0.07	0.06	0.07	0.06	0.05	0.05	0.07
Ah! Monster		0.00		0.77	0.39	0.26	0.08	0.08	0.05	0.06	0.05	0.06	0.05	0.07	0.06	0.10
Candy Crush Saga	0.28															
Pet Rescue Saga	0.00	0.12	0.21	0.67	0.33	0.25	0.13	0.15	0.12	0.11	0.10	0.09	0.10	0.10	0.10	0.10
Burger	0.05	0.06	0.05	0.14	0.09	0.11	0.08	0.10	0.09	0.10	0.10	0.08	0.08	0.07	0.05	0.04
My principal	0.28	0.20	0.08	0.07	0.04	0.07	0.06	0.10	0.08	0.05	0.07	0.06	0.04	0.04	0.05	0.07
Scary Stories		0.00	0.08	0.84	0.46	0.36	0.08	0.09	0.06	0.06	0.06	0.07	0.08	0.10	0.10	0.14
Kumatomo							0.01	0.17	0.33	0.30	0.20	0.13	0.09	0.07	0.06	0.06
Kamen rider Raidabout!	0.11	0.12	0.09	0.23	0.14	0.15	0.11	0.12	0.10	0.08	0.11	0.09	0.09	0.07	0.08	0.09
Dots			0.00	1.00	0.51	0.40	0.11	0.11	0.08	0.10	0.12	0.10	0.06	0.10	0.09	0.10
Kanji ability diagnosis	0.10	0.10	0.08	0.17	0.11	0.13	0.10	0.13	0.10	0.09	0.09	0.10	0.08	0.06	0.05	0.06
My Talking Tom						0.00	0.25	0.31	0.30	0.20	0.14	0.12	0.10	0.12	0.12	0.09
Escape from the subway	0.03	0.45	0.20	0.32	0.09	0.07	0.05	0.08	0.06	0.06	0.05	0.05	0.05	0.04	0.03	0.03
Sudden Kiss of Vow						0.43	0.34	0.26	0.15	0.11	0.09	0.10	0.12	0.11	0.11	0.09
Brain power + payment	0.24	0.12	0.06	0.11	0.08	0.10	0.09	0.09	0.12	0.12	0.14	0.08	0.08	0.06	0.08	0.10
Poop Rearing Simulator	0.06	0.07	0.05	0.13	0.07	0.08	0.06	0.07	0.06	0.05	0.04	0.06	0.08	0.07	0.09	0.04
Fire a match	0.09	0.08	0.04		0.10	0.11	0.05	0.10	0.07	0.09	0.07	0.09	0.07	0.08	0.06	0.05
Match DE Rhythm	0.29	0.23	0.09	0.23	0.14	0.13	0.06	0.09	0.06	0.07	0.05	0.05	0.07	0.08	0.06	0.05
Please reply			0.00	1.00	0.40	0.26	0.05	0.07	0.04	0.06	0.06	0.08	0.07	0.08	0.06	0.05
Homo-tampering	0.09	0.09	0.05		0.07	0.10	0.08	0.11	0.08	0.07	0.07	0.07	0.06	0.07	0.06	0.06
Mushroom Garden	0.08	0.09	0.06	0.12	0.08	0.09	0.06	0.08	0.06	0.06	0.06	0.09	0.09	0.07	0.06	0.05
Mushroom Garden Seaso	0.09				0.09											
Touche Investigator Gard		0.07	0.14	0.64	0.32	0.25	0.09	0.12	0.07	0.05	0.05	0.06	0.06	0.08	0.09	0.09
Ultra! Gourmet life form	0.00	0.43	0.35	0.55	0.25	0.15	0.08	0.11	0.08	0.07	0.07	0.19	0.17	0.14	0.07	0.06
Kotatsu cat	0.08	0.10	0.06	0.14	0.09	0.11	0.08	0.11	0.07	0.07	0.07	0.07	0.04	0.06	0.07	0.06
Nyanko Dai Senso	0.16	0.08	0.06	0.15	0.10	0.13			0.11	0.09	0.09	0.11	0.15	0.14	0.12	0.10
COLOPL	0.11	0.12	0.09	0.29	0.20	0.20	0.06	0.08	0.06	0.06	0.06	0.06	0.06	0.06	0.05	0.05
Sparkle Drop!	0.15	0.10	0.11	0.31	0.18	0.21	0.11	0.13	0.09	0.07	0.06	0.08	0.09	0.06	0.05	0.05
Rhythm coin!	0.09	0.19	0.07	0.17	0.11	0.14	0.08	0.10	0.07	0.08	0.07	0.08	0.07	0.07	0.05	0.06
Rhythm coin 2	0.07	0.10	0.06	0.17	0.10	0.12	0.07	0.12	0.10	0.09	0.09	0.10	0.09	0.07	0.05	0.06
Disney Magician	0.08	0.09	0.05	0.20	0.11	0.14	0.09	0.11	0.07	0.07	0.05	0.07	0.06	0.06	0.08	0.08
Go home!	0.29	0.11	0.18	0.22	0.09	0.12	0.06	0.07	0.05	0.06	0.06	0.07	0.06	0.05	0.06	0.06
Game Gift		0.34		1.00	0.29	0.31	0.27	0.28	0.28	0.24	0.24	0.23	0.20	0.15	0.15	0.17
Crazy Tower	0.11		0.06	0.13	0.09	0.09	0.07	0.09	0.07	0.06	0.06	0.07	0.07	0.07	0.07	0.07
Abepyon	0.00	0.10		0.42	0.23	0.08	0.04	0.07	0.07	0.06	0.07	0.05	0.07	0.07	0.06	0.04
Don't step the white tile	0.05	0.13	0.03	0.09	0.06	0.07	0.04	0.07	0.07	0.04	0.11	0.35	0.28	0.19	0.06	0.05
Tokimeki Restaurant	0.42	0.05	0.13	0.26	0.14	0.17	0.12	0.16	0.13	0.14	0.14	0.15	0.13	0.10	0.09	0.10
Big Cat in Advance		0.30	0.00	1.00	0.41	0.31	0.06	0.06	0.03	0.06	0.06	0.06	0.05	0.11	0.11	0.12
Angry Yoshida & Hungr						0.00	0.41	0.30	0.25	0.14	0.08	0.08	0.09	0.06	0.06	0.05
Nyanko dai bōsō							0.33	0.32	0.28	0.17	0.09	0.08	0.09	0.09	0.08	0.06

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Application	2013/06	2013/07	2013/08	2013/09	2013/10	2013/11	2013/12	2014/01	2014/02	2014/03	2014/04	2014/05	2014/06	2014/07	2014/08	2014/09
LINE Theater Town	0.30			0.18	0.11	0.15	0.10	0.13	0.09	0.09	0.08	0.08	0.08	0.07	0.06	0.05
LINE quiz		0.19	0.18	0.55	0.29		0.21	0.22	0.16	0.09	0.07	0.07	0.07	0.07	0.06	0.05
LINE ranch life	0.28	0.00	0.10	0.26	0.15	0.19	0.11	0.15	0.10	0.10	0.09	0.10	0.10	0.08	0.07	0.05
Runaway Let's tiger's	0.12	0.28	0.09	0.19	0.11	0.14	0.10	0.11	0.10	0.08	0.07	0.06	0.07	0.08	0.09	0.08
SkySwings		0.13			0.00	0.09	0.41	0.30	0.18	0.07	0.03	0.06	0.08	0.06	0.06	0.09
Wall Jump	0.01		0.23	0.41	0.16	0.11	0.09	0.14	0.11	0.10	0.07	0.11	0.11	0.09	0.07	0.11
Touch the Numbers for A	0.12	0.40	0.08	0.13	0.08	0.08	0.07	0.09	0.07	0.06	0.07	0.09	0.09	0.08	0.07	0.06
Nyanko Hazard		0.12	0.23	0.39	0.22	0.09	0.05	0.07	0.04	0.03	0.04	0.03	0.03	0.13	0.16	0.13
Pou	0.08	0.01	0.06	0.36	0.17	0.17	0.11	0.14	0.11	0.10	0.10	0.09	0.09	0.09	0.09	0.09
iStair+		0.09	0.00	1.00	0.40	0.32	0.07	0.06	0.04	0.04	0.05	0.06	0.07	0.08	0.09	0.06
ForbiddenEgg	0.00		0.16	0.24	0.10	0.05	0.04	0.07	0.07	0.05	0.05	0.06	0.08	0.05	0.06	0.05
MindStep	0.28	0.28	0.15	0.26	0.11	0.10	0.05	0.07	0.06	0.07	0.06	0.05	0.04	0.07	0.05	0.04
PressZombie		0.30	0.00	1.00	0.34	0.26	0.07		0.04	0.05	0.05	0.06	0.08	0.06	0.07	0.04
ReacheeE	0.18		0.06	0.22	0.14	0.14	0.06	0.08	0.07	0.08	0.07	0.07	0.08	0.09	0.09	0.08
ShotZombie	0.18	0.10		0.57	0.27	0.19	0.07	0.07	0.04	0.05	0.05	0.05	0.06	0.07	0.07	0.08
ShotZombieHalloween		0.11			0.00	0.34	0.33	0.24	0.11	0.05	0.05	0.06	0.06	0.07	0.08	0.06
YureteruBose			0.10	0.85	0.45	0.31	0.11	0.10	0.08	0.07	0.06	0.06	0.06	0.07	0.08	0.09
Your dream story	0.09	0.00	0.07	0.19	0.12	0.12	0.09	0.13	0.11	0.09	0.09	0.10	0.09	0.09	0.09	0.10
Creative pastry chef		0.10			0.00	0.32	0.40	0.31	0.19	0.11	0.09	0.10	0.10	0.09	0.10	0.08
Pancake Tower	0.00		0.09	0.80	0.28	0.27	0.14	0.09	0.17	0.16	0.15	0.10	0.07	0.05	0.09	0.12
Spoon mania		0.25	0.00	1.00	0.30	0.34	0.15	0.10	0.17	0.15	0.13	0.08	0.05	0.06	0.10	0.10
More than 700 stories!			0.00		0.02	0.40	0.35	0.28	0.11	0.06	0.06	0.05	0.06	0.05	0.06	0.06
Busu diagnosis			0.13		0.05	0.03	0.03	0.06	0.05	0.06	0.05	0.07	0.06	0.08	0.10	0.08
Pesoguin Battery 3D Per	0.26	0.29	0.11	0.59	0.30	0.30	0.12	0.14	0.08	0.07	0.06	0.05	0.05	0.03	0.05	0.07
Hardest Game Ever 2	0.27	0.21	0.15	0.41	0.20	0.15	0.06	0.09	0.06	0.06	0.04	0.06	0.05	0.09	0.10	0.09
One million eggs	0.27	0.20	0.12	0.09	0.05	0.06	0.05	0.07	0.05	0.04	0.04	0.04	0.04	0.05	0.05	0.05

Table A. 5 - App-Dec Values (Sub-Category: Casual)

Application	2013/06	2013/07	2013/08	2013/09	2013/10	2013/11	2013/12	2014/01	2014/02	2014/03	2014/04	2014/05	2014/06	2014/07	2014/08	2014/09
Dumb Ways to Die			0.00				0.10	0.32	0.50	0.58	0.53	0.46	0.45	0.48	0.54	0.59
Come Here... Cat!	0.71		0.70	0.62	0.60		0.55	0.45	0.51	0.51	0.50	0.52	0.53	0.55	0.52	0.48
Matchless Cat Sew	0.55	0.57	0.61	0.54	0.57	0.51	0.54	0.48	0.61	0.64	0.64	0.62	0.62	0.61	0.60	0.59
STICK NINJA	0.79	0.74	0.74	0.65	0.70	0.64			0.60	0.60	0.65	0.62	0.63	0.61	0.56	0.55
Star Vanisher	0.00	0.00	0.02	0.41	0.79	0.82	0.78	0.64	0.71	0.69	0.66	0.63	0.58	0.55	0.52	0.47
Star Vanisher -S		0.00	0.00	0.00	0.56	0.63	0.74	0.64	0.73	0.69	0.64	0.57	0.52	0.52	0.52	0.54
Kamehame Beam!				0.00	0.00	0.16	0.56	0.70	0.78	0.74	0.67	0.63	0.60	0.56	0.57	0.56
Alpaca Evolution	0.39	0.62	0.78	0.75	0.77	0.71	0.69	0.55	0.65	0.66	0.65	0.64	0.62	0.61	0.61	0.58
Alpaca Evolution Begins	0.00	0.00	0.00	0.08	0.61	0.67	0.72	0.58	0.67	0.67	0.62	0.64	0.62	0.63	0.64	0.61
Star Chef			0.00	0.00	0.00	0.16	0.52		0.86	0.84	0.80	0.71	0.70	0.61	0.64	0.60
Bound Monsters	0.00	0.32	0.62	0.71	0.73	0.66	0.67	0.59	0.69	0.70	0.67	0.65	0.64	0.63	0.64	0.60
Snoopy Street	0.61	0.58	0.60	0.52	0.52	0.46	0.46	0.44	0.53	0.54	0.52	0.53	0.51	0.50	0.49	0.46
Boiling OSSAN Eggs!		0.00	0.00	0.00	0.38	0.45	0.63	0.51	0.57	0.53	0.57	0.60	0.65	0.65	0.68	0.66
Ossan Train Stuffing!						0.00	0.00	0.00	0.30	0.54	0.58	0.51	0.47	0.53	0.65	0.68
Hamster Life	0.60	0.61	0.59	0.54	0.51	0.42	0.50	0.50	0.62	0.61	0.59	0.56	0.58	0.59	0.57	0.55
Naughty Little Devil			0.00	0.03	0.75	0.80		0.66	0.74	0.71	0.64	0.56	0.53	0.54	0.54	0.58
Constellation game		0.14	0.06	0.05	0.22	0.28	0.42	0.49	0.61	0.64	0.63	0.62	0.63	0.64	0.64	0.63
Jewels Star	0.69	0.65	0.55	0.46	0.51	0.46	0.53	0.48	0.58	0.59	0.57	0.55	0.52	0.53	0.55	0.53
Despicable Me	0.00	0.00														
Garden - escape game			0.00	0.00	0.00	0.14	0.55	0.60	0.67	0.62	0.59	0.60	0.59	0.60	0.57	0.57
Pocket Dairy		0.00	0.00	0.00	0.42	0.50	0.64	0.59	0.66	0.63	0.63	0.63	0.61	0.59	0.60	0.58
Ah! Monster		0.00		0.00	0.23	0.33	0.62	0.63	0.74	0.74	0.72	0.72	0.71	0.68	0.66	0.58
Candy Crush Saga	0.31															
Pet Rescue Saga	0.00	0.00	0.00	0.03	0.29	0.34	0.47	0.47	0.58	0.59	0.58	0.57	0.54	0.55	0.54	0.46
Burger	0.73	0.73	0.70	0.59	0.59	0.49	0.51	0.44	0.53	0.56	0.57	0.55	0.54	0.51	0.51	0.51

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Application	2013/06	2013/07	2013/08	2013/09	2013/10	2013/11	2013/12	2014/01	2014/02	2014/03	2014/04	2014/05	2014/06	2014/07	2014/08	2014/09
My principal	0.00	0.16	0.72	0.83	0.85	0.70	0.65	0.56	0.67	0.72	0.67	0.67	0.59	0.65	0.57	0.55
Scary Stories		0.00	0.00	0.00	0.17	0.27	0.55	0.59	0.69	0.67	0.65	0.62	0.60	0.56	0.55	0.49
Kumatomo							0.00	0.00	0.00	0.14	0.40	0.59	0.67	0.69	0.71	0.68
Kamen rider Raidabout!	0.57	0.55	0.55	0.49	0.50	0.43	0.47	0.42	0.53	0.52	0.53	0.54	0.54	0.54	0.55	0.51
Dots			0.00	0.00	0.00	0.15	0.48	0.57	0.64	0.61	0.57	0.60	0.57	0.56	0.53	0.55
Kanji ability diagnosis	0.61	0.60	0.59	0.50	0.48	0.43	0.46	0.42	0.49	0.48	0.48	0.48	0.48	0.51	0.52	0.51
My Talking Tom						0.00	0.00	0.00	0.15	0.39	0.53	0.58	0.52	0.49	0.46	0.49
Escape from the subway	0.00	0.00	0.03	0.37	0.69	0.72	0.70	0.60	0.70	0.68	0.71	0.68	0.64	0.61	0.65	0.65
Sudden Kiss of Vow						0.00	0.00	0.32	0.54	0.63	0.62	0.57	0.55	0.53	0.59	0.58
Brain power + payment	0.46	0.63	0.73	0.65	0.63	0.54	0.56	0.44	0.51	0.48	0.49	0.52	0.49	0.48	0.45	0.42
Poop Rearing Simulator	0.63	0.62	0.63	0.56	0.60	0.54	0.56	0.49	0.58	0.59	0.58	0.59	0.58	0.58	0.50	0.53
Fire a match	0.63	0.65	0.71		0.64	0.54	0.61	0.51	0.58	0.54	0.59	0.58	0.61	0.54	0.54	0.48
Match DE Rhythm	0.00	0.29	0.55	0.63	0.62	0.54	0.63	0.58	0.65	0.64	0.63	0.61	0.60	0.60	0.57	0.53
Please reply			0.00	0.00	0.00	0.17	0.62	0.69	0.73	0.71	0.65	0.64	0.58	0.59	0.63	0.62
Homo-tampering	0.65	0.69	0.73		0.67	0.58	0.60	0.50	0.61	0.62	0.64	0.63	0.63	0.62	0.62	0.59
Mushroom Garden	0.61	0.59	0.62	0.54	0.56	0.51	0.53	0.46	0.55	0.55	0.51	0.50	0.51	0.53	0.56	0.51
Mushroom Garden Seaso	0.58				0.53											
Touche Investigator Gar		0.16	0.10	0.08	0.27	0.32	0.48	0.54	0.65	0.66	0.68	0.68	0.65	0.60	0.55	0.53
Ultra! Gourmet life form	0.00	0.00	0.00	0.14	0.46	0.50	0.53	0.44	0.55	0.56	0.46	0.44	0.42	0.52	0.58	0.58
Kotatsu cat	0.70	0.56	0.66	0.59	0.59	0.52	0.54	0.46	0.55	0.53	0.54	0.56	0.57	0.52	0.53	0.53
Nyanko Dai Senso	0.55	0.64	0.68	0.63	0.62	0.52			0.53	0.54	0.53	0.49	0.48	0.47	0.51	0.50
COLOPL	0.53	0.62	0.43	0.35	0.37	0.36	0.46	0.44	0.51	0.50	0.51	0.52	0.50	0.50	0.50	0.48
Sparkle Drop!	0.47	0.50	0.43	0.41	0.44	0.43	0.49	0.49	0.58	0.58	0.54	0.53	0.52	0.55	0.51	0.48
Rhythm coin!	0.59	0.42	0.58	0.51	0.52	0.47	0.50	0.48	0.56	0.59	0.56	0.54	0.51	0.51	0.51	0.49
Rhythm coin 2	0.67	0.56	0.61	0.55	0.56	0.50	0.53	0.43	0.51	0.53	0.53	0.53	0.54	0.56	0.57	0.53
Disney Magician	0.64	0.62	0.58	0.54	0.55	0.49	0.55	0.49	0.58	0.57	0.57	0.53	0.51	0.47	0.47	0.49
Go home!	0.00	0.59	0.39	0.57	0.71	0.64	0.62	0.57	0.68	0.67	0.62	0.59	0.52	0.51	0.55	0.54
Game Gift		0.00		0.00	0.00	0.05	0.18	0.23	0.32	0.34	0.36	0.38	0.41	0.45	0.43	0.39
Crazy Tower	0.62		0.67	0.60	0.65	0.59	0.61	0.55	0.63	0.61	0.60	0.59	0.58	0.58	0.59	0.56
Abepyon	0.00	0.63		0.09	0.62	0.68	0.75	0.60	0.68	0.67	0.65	0.63	0.63	0.62	0.58	0.59
Don't step the white tile	0.75	0.00	0.74	0.66	0.67	0.58	0.60	0.49	0.59	0.38	0.17	0.13	0.21	0.51	0.69	0.71
Tokimeki Restaurant	0.18	0.72	0.46	0.45	0.48	0.44	0.48	0.43	0.48	0.45	0.44	0.47	0.49	0.54	0.54	0.54
Big Cat in Advance		0.33	0.00	0.00	0.00	0.15	0.67	0.76	0.83	0.78	0.74	0.68	0.58	0.50	0.45	0.46
Angry Yoshida & Hungr						0.00	0.00	0.00	0.35	0.61	0.72	0.71	0.69	0.68	0.63	0.60
Nyanko dai bōsō							0.00	0.00	0.28	0.54	0.68	0.68	0.65	0.62	0.62	0.61
LINE Theater Town	0.33			0.58	0.60	0.52	0.54	0.48	0.57	0.58	0.58	0.57	0.59	0.60	0.62	0.60
LINE quiz		0.49	0.00	0.00	0.47		0.41	0.28	0.52	0.64	0.70	0.69	0.67	0.66	0.66	0.63
LINE ranch life	0.03	0.00	0.50	0.55	0.55	0.47	0.52	0.47	0.57	0.58	0.58	0.58	0.59	0.61	0.60	0.61
Runaway Let's tiger's	0.55	0.33	0.54	0.49	0.51	0.44	0.46	0.40	0.51	0.54	0.55	0.58	0.57	0.54	0.50	0.45
SkySwings		0.53			0.00	0.00	0.00	0.09	0.47	0.73	0.79	0.75	0.67	0.63	0.57	0.50
Wall Jump	0.00		0.01	0.21	0.59	0.61	0.61	0.47	0.62	0.63	0.64	0.60	0.58	0.56	0.59	0.54
Touch the Numbers for A	0.56	0.00	0.60	0.55	0.57	0.51	0.52	0.46	0.53	0.53	0.50	0.48	0.47	0.48	0.49	0.49
Nyanko Hazard		0.55	0.00	0.01	0.63	0.68	0.77	0.65	0.76	0.74	0.70	0.67	0.53	0.49	0.40	0.44
Pou	0.68	0.00	0.56	0.41	0.45	0.38	0.46	0.44	0.55	0.58	0.59	0.58	0.56	0.54	0.53	0.54
iStair+		0.69	0.00	0.00	0.00	0.15	0.58	0.70	0.79	0.76	0.75	0.73	0.73	0.71	0.63	0.60
ForbiddenEgg	0.00		0.00	0.37	0.79	0.82	0.78	0.62	0.71	0.71	0.70	0.70	0.70	0.66	0.64	0.59
MindStep	0.00	0.00	0.35	0.51	0.68	0.63	0.68	0.62	0.72	0.69	0.67	0.66	0.70	0.66	0.61	0.53
PressZombie		0.00	0.00	0.00	0.00	0.14	0.61		0.83	0.79	0.76	0.70	0.68	0.65	0.70	0.67
ReacheeE	0.51		0.64	0.56	0.58	0.52	0.61	0.55	0.65	0.62	0.60	0.56	0.56	0.53	0.54	0.51
ShotZombie	0.00	0.62		0.08	0.38	0.45	0.64	0.65	0.77	0.75	0.73	0.70	0.67	0.67	0.65	0.63
ShotZombieHalloween		0.01			0.00	0.00	0.00	0.29	0.62	0.76	0.79	0.74	0.70	0.67	0.66	0.62
YureteruBose			0.00	0.00	0.16	0.27	0.54	0.57	0.69	0.69	0.68	0.65	0.65	0.61	0.62	0.63
Your dream story	0.62	0.00	0.59	0.54	0.56	0.48	0.50	0.43	0.54	0.54	0.55	0.54	0.53	0.52	0.53	0.49
Creative pastry chef		0.58			0.00	0.00	0.00	0.19	0.48	0.61	0.63	0.61	0.61	0.59	0.59	0.54
Pancake Tower	0.00		0.00	0.07	0.15	0.24	0.44	0.46	0.53	0.49	0.57	0.66	0.69	0.65	0.60	0.51
Spoon mania		0.00	0.00	0.00	0.00	0.11	0.39	0.45	0.53	0.51	0.58	0.71	0.74	0.68	0.59	0.54
More than 700 stories!			0.00		0.00	0.01	0.02	0.37	0.64	0.74	0.73	0.68	0.65	0.64	0.63	0.64
Busu diagnosis			0.13		0.84	0.85	0.78	0.63	0.69	0.72	0.68	0.68	0.62	0.62	0.63	0.57
Pesoguin Battery 3D Pe	0.36	0.00	0.22	0.19	0.26	0.28	0.44	0.51	0.62	0.63	0.63	0.62	0.59	0.58	0.56	0.51
Hardest Game Ever 2	0.00	0.28	0.18	0.24	0.53	0.54	0.69	0.63	0.73	0.71	0.68	0.65	0.62	0.60	0.59	0.63
One million eggs	0.24	0.07	0.63	0.71	0.78	0.69	0.67	0.53	0.65	0.65	0.69	0.67	0.65	0.62	0.61	0.58

Table A. 6 - App-Dec Values (Sub-Category: Puzzle)

Application	2013/06	2013/07	2013/08	2013/09	2013/10	2013/11	2013/12	2014/01	2014/02	2014/03	2014/04	2014/05	2014/06	2014/07	2014/08	2014/09
Pota-cats	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Block Puzzle Original	0.00	0.00	0.00	0.01	0.01	0.01	0.02	0.03	0.03	0.03	0.03	0.04	0.03	0.03	0.03	0.03
Flow Free	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Puzzles with Matches	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Learning Japan Map Puz	0.02	0.01	0.02	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Learning World Map Pu	0.01	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
One touch Drawing	0.05	0.02	0.04	0.03	0.03	0.03	0.03	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
Puzzle & Dragons Comb	0.03	0.02	0.04	0.04	0.04	0.04	0.04	0.03	0.03	0.02	0.02	0.02	0.01	0.01	0.01	0.01
Search for mistakes in ka	0.01	0.00	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Unblock Me FREE	0.03	0.01	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.01	0.01
DOOORS	0.03	0.01	0.03	0.02	0.02	0.02	0.02	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01
DOOORS2	0.01	0.01	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Jinma no tō	0.00	0.00	0.02	0.02	0.03	0.03	0.04	0.05	0.04	0.04	0.03	0.03	0.03	0.03	0.02	0.02
Bad Piggies	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00
Puyo Puyo !! Quest	0.01	0.01	0.03	0.03	0.04	0.04	0.05	0.05	0.04	0.04	0.05	0.05	0.05	0.05	0.05	0.05
Shoot Bubble Deluxe	0.02	0.01	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Pop Star for Android	0.01	0.00	0.01	0.01	0.01	0.01		0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00
Cut the Rope	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Cut the Rope: Time Trav	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Chokokushi	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Tracing a cat!	0.01	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
PIYOMORI	0.07	0.03	0.05	0.04	0.04	0.03	0.03	0.02	0.02	0.02	0.02	0.01	0.01	0.01	0.01	0.01
MORE!PIYOMORI	0.01	0.01	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Shogi app everyday	0.02	0.01	0.02	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
ZOOKEEPER BATTLE	0.04	0.02	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.02	0.02	0.02
LINE Tours	0.00	0.01	0.03	0.04	0.04	0.05	0.05	0.04	0.03	0.03	0.02	0.02	0.01	0.01	0.01	0.01
Flame at LINE	0.00	0.00	0.02	0.02	0.04	0.04	0.04	0.05	0.04	0.03	0.02	0.02	0.01	0.01	0.01	0.01
LINE HIDDEN CATCH	0.19	0.09	0.18	0.16	0.13	0.11	0.10	0.08	0.07	0.06	0.06	0.05	0.04	0.04	0.04	0.03
LINE Pocopan	0.06	0.07														
LINE JELLY	0.16	0.09	0.20	0.23	0.22	0.19	0.17	0.14	0.12	0.11	0.09	0.08	0.07	0.06	0.05	0.05
LINE ZOOKEEPER	0.05	0.02	0.03	0.03	0.03	0.02	0.02	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01
ChickPusher	0.05	0.02	0.03	0.03	0.03	0.02	0.02	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Romanization		0.00	0.01	0.01	0.01	0.01	0.02	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00
Yokai Taiso Pazuru da N		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.02	0.02	0.03	0.04	0.05
LINE Bubble		0.22														
LINE POP		0.25														
Buttons and Scissors			0.00	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00
Jigty Jigsaw Puzzle			0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
123 Guess Guess (Hong			0.00	0.00	0.00	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
123 Guess Guess (Taiwa			0.01	0.01	0.01	0.01	0.01	0.02	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00
DivineGateJP			0.00	0.00	0.01	0.01	0.02	0.02	0.02	0.02	0.02	0.02	0.01	0.01	0.01	0.01
LINE PON PON PON			0.01	0.02	0.02	0.05	0.06	0.07	0.06	0.06	0.06	0.05	0.05	0.04	0.03	0.03
Dwarf observation puzzl			0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00
Three Kingdoms Puzzle					0.00	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.01	0.01	0.01
Puzzle Combo						0.00	0.01	0.01	0.01	0.02	0.02	0.02	0.02	0.02	0.02	0.02
Hello Kitty's puzzle chai						0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
DOOORS3 - room esca							0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
100 million puzzles - refr							0.00	0.01	0.03	0.04	0.05	0.05	0.06	0.06	0.05	0.05
LINE puzzle de Inazuma							0.00	0.01	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01
LINE Disney Zum Zum								0.01	0.10	0.15	0.19	0.23	0.27	0.32	0.35	0.36

Table A. 7 - App-Stab Values (Sub-Category: Puzzle)

Application	2013/06	2013/07	2013/08	2013/09	2013/10	2013/11	2013/12	2014/01	2014/02	2014/03	2014/04	2014/05	2014/06	2014/07	2014/08	2014/09
Pota-cats	0.14	0.18	0.17	0.25	0.26	0.29	0.33	0.36	0.32	0.33	0.32	0.33	0.33	0.35	0.37	0.36
Block Puzzle Original	0.00	0.02	0.04	0.05	0.06	0.04	0.06	0.04	0.04	0.09	0.14	0.16	0.19	0.20	0.22	0.23
Flow Free	0.06	0.06	0.08	0.13	0.12	0.12	0.15	0.14	0.13	0.13	0.15	0.15	0.17	0.19	0.18	0.20
Puzzles with Matches	0.12	0.15	0.15	0.23	0.25	0.27	0.29	0.26	0.21	0.20	0.20	0.20	0.19	0.19	0.17	0.16
Learning Japan Map Puz	0.18	0.21	0.20	0.27	0.29	0.32	0.34	0.33	0.28	0.27	0.26	0.28	0.29	0.30	0.31	0.31
Learning World Map Pu	0.16	0.18	0.17	0.25	0.25	0.29	0.33	0.32	0.26	0.25	0.24	0.28	0.30	0.31	0.30	0.29
One touch Drawing	0.13	0.15	0.13	0.20	0.20	0.20	0.23	0.22	0.19	0.20	0.19	0.20	0.20	0.20	0.21	0.19
Puzzle & Dragons Comb	0.00	0.03	0.06	0.11	0.15	0.17	0.23	0.25	0.25	0.25	0.26	0.29	0.31	0.34	0.35	0.36
Search for mistakes in ka	0.00	0.00	0.01	0.13	0.13	0.18	0.25	0.32	0.27	0.28	0.30	0.31	0.32	0.32	0.31	0.30
Unblock Me FREE	0.14	0.15	0.14	0.22	0.21	0.21	0.23	0.23	0.20	0.20	0.21	0.23	0.24	0.26	0.27	0.27
DOOORS	0.11	0.12	0.11	0.17	0.18	0.21	0.24	0.24	0.20	0.21	0.20	0.22	0.24	0.26	0.28	0.27
DOOORS2	0.00	0.07	0.08	0.12	0.15	0.18	0.22	0.19	0.17	0.17	0.17	0.18	0.20	0.22	0.24	0.23
Jinma no tō	0.00	0.00	0.00	0.01	0.00	0.03	0.12	0.09	0.11	0.12	0.13	0.14	0.16	0.16	0.16	0.16
Bad Piggies	0.07	0.08	0.10	0.18	0.18	0.19	0.21	0.23	0.21	0.22	0.21	0.22	0.23	0.22	0.21	0.22
Puyo Puyo !! Quest	0.00	0.00	0.00	0.00	0.07	0.10	0.17	0.15	0.14	0.16	0.16	0.17	0.17	0.17	0.20	0.22
Shoot Bubble Deluxe	0.10	0.12	0.12	0.19	0.17	0.18	0.21	0.20	0.19	0.20	0.22	0.22	0.23	0.22	0.22	0.24
Pop Star for Android	0.16	0.18	0.14	0.20	0.19	0.21		0.28	0.24	0.24	0.27	0.28	0.29	0.33	0.33	0.34
Cut the Rope	0.05	0.06	0.06	0.10	0.12	0.12	0.16	0.16	0.14	0.14	0.14	0.15	0.16	0.18	0.20	0.23
Cut the Rope: Time Trav	0.00	0.00	0.04	0.08	0.08	0.09	0.15	0.14	0.12	0.13	0.13	0.15	0.18	0.18	0.21	0.21
Chokokushi	0.25	0.28	0.27	0.35	0.34	0.36	0.39	0.41	0.38	0.39	0.39	0.41	0.41	0.39	0.39	0.38
Tracing a cat!	0.00	0.00	0.02	0.11	0.11	0.14	0.23	0.23	0.22	0.25	0.27	0.27	0.30	0.30	0.30	0.25
PIYOMORI	0.17	0.19	0.17	0.23	0.25	0.28	0.30	0.35	0.31	0.32	0.32	0.34	0.35	0.36	0.37	0.36
MORE!PIYOMORI	0.00	0.00	0.00	0.10	0.13	0.16	0.24	0.25	0.24	0.27	0.26	0.28	0.29	0.30	0.29	0.28
Shogi app everyday	0.15	0.17	0.16	0.24	0.23	0.25	0.28	0.28	0.26	0.26	0.26	0.28	0.31	0.31	0.32	0.32
ZOOKEEPER BATTLE	0.15	0.19	0.19	0.26	0.27	0.28	0.30	0.29	0.27	0.28	0.28	0.30	0.32	0.33	0.34	0.34
LINE Tours	0.00	0.00	0.00	0.00	0.02	0.07	0.12	0.12	0.11	0.12	0.14	0.16	0.19	0.21	0.22	0.24
Flame at LINE	0.00	0.00	0.00	0.00	0.01	0.01	0.09	0.07	0.09	0.09	0.11	0.15	0.16	0.19	0.21	0.21
LINE HIDDEN CATCH	0.11	0.13	0.12	0.18	0.19	0.21	0.23	0.25	0.22	0.23	0.23	0.26	0.28	0.29	0.30	0.30
LINE Pocopan	0.00	0.00														
LINE JELLY	0.00	0.00	0.08	0.15	0.15	0.17	0.20	0.21	0.18	0.18	0.19	0.20	0.22	0.24	0.25	0.26
LINE ZOOKEEPER	0.09	0.11	0.11	0.15	0.16	0.18	0.20	0.22	0.19	0.20	0.20	0.21	0.22	0.23	0.26	0.27
ChickPusher	0.12	0.14	0.14	0.20	0.21	0.25	0.28	0.32	0.28	0.29	0.27	0.28	0.29	0.29	0.30	0.30
Romanization		0.00	0.00	0.00	0.00	0.04	0.09	0.10	0.08	0.10	0.13	0.14	0.18	0.22	0.20	0.21
Yokai Taiso Pazuru da N		0.00	0.00	0.00	0.00	0.03	0.13	0.07	0.04	0.03	0.02	0.04	0.06	0.08	0.09	0.11
LINE Bubble		0.17														
LINE POP		0.19														
Buttons and Scissors			0.00	0.00	0.00	0.00	0.14	0.10	0.12	0.14	0.17	0.17	0.17	0.17	0.17	0.20
Jigty Jigsaw Puzzle			0.00	0.00	0.00	0.00	0.04	0.02	0.07	0.07	0.09	0.11	0.13	0.14	0.16	0.16
123 Guess Guess (Hong			0.00	0.00	0.00	0.00	0.13	0.08	0.09	0.09	0.12	0.16	0.20	0.18	0.21	0.21
123 Guess Guess (Taiwa			0.00	0.00	0.00	0.00	0.07	0.04	0.05	0.04	0.05	0.06	0.08	0.09	0.14	0.17
DivineGateJP			0.00	0.00	0.00	0.00	0.01	0.01	0.06	0.07	0.10	0.13	0.15	0.16	0.17	0.18
LINE PON PON PON			0.00	0.00	0.00	0.00	0.09	0.06	0.06	0.13	0.14	0.16	0.18	0.20	0.21	0.21
Dwarf observation puzzle			0.00	0.00	0.00	0.00	0.11	0.08	0.08	0.09	0.13	0.16	0.19	0.20	0.20	0.19
Three Kingdoms Puzzle					0.00	0.00	0.00	0.00	0.01	0.12	0.11	0.16	0.18	0.20	0.24	0.25
Puzzle Combo						0.00	0.01	0.01	0.01	0.09	0.14	0.19	0.21	0.23	0.25	0.26
Hello Kitty's puzzle chair						0.00	0.00	0.00	0.00	0.08	0.12	0.12	0.14	0.17	0.20	0.21
DOOORS3 - room escap							0.00	0.00	0.00	0.00	0.09	0.14	0.18	0.20	0.22	0.21
100 million puzzles - refre							0.00	0.00	0.00	0.00	0.02	0.04	0.15	0.15	0.20	0.21
LINE puzzle de Inazuma							0.00	0.00	0.00	0.00	0.04	0.05	0.06	0.13	0.14	0.14
LINE: Disney Zum Tum								0.00	0.00	0.00	0.00	0.02	0.21	0.24	0.27	0.28

Table A. 8 - App-Pop Values (Sub-Category: Puzzle)

Application	2013/06	2013/07	2013/08	2013/09	2013/10	2013/11	2013/12	2014/01	2014/02	2014/03	2014/04	2014/05	2014/06	2014/07	2014/08	2014/09
Pota-cats	0.42	0.44	0.44	0.51	0.52	0.55	0.56	0.65	0.58	0.59	0.58	0.58	0.58	0.58	0.59	0.58
Block Puzzle Original	0.27	0.29	0.27	0.37	0.36	0.29	0.29	0.33	0.35	0.39	0.41	0.43	0.46	0.48	0.50	0.50
Flow Free	0.27	0.28	0.34	0.41	0.40	0.41	0.40	0.45	0.40	0.41	0.42	0.43	0.44	0.45	0.45	0.45
Puzzles with Matches	0.39	0.41	0.42	0.49	0.50	0.53	0.53	0.54	0.47	0.46	0.46	0.45	0.45	0.45	0.43	0.43
Learning Japan Map Puz	0.47	0.48	0.47	0.53	0.54	0.58	0.57	0.61	0.53	0.52	0.52	0.53	0.54	0.55	0.56	0.57
Learning World Map Puz	0.45	0.45	0.44	0.51	0.51	0.55	0.56	0.61	0.53	0.54	0.51	0.53	0.55	0.55	0.56	0.56
One touch Drawing	0.41	0.42	0.41	0.47	0.47	0.48	0.48	0.52	0.46	0.46	0.46	0.46	0.46	0.47	0.46	0.46
Puzzle & Dragons Comb	0.30	0.34	0.36	0.42	0.43	0.46	0.48	0.60	0.53	0.53	0.55	0.55	0.58	0.60	0.60	0.63
Search for mistakes in ka	0.23	0.29	0.31	0.36	0.39	0.46	0.50	0.60	0.52	0.54	0.56	0.57	0.57	0.57	0.58	0.57
Unblock Me FREE	0.40	0.40	0.42	0.48	0.47	0.49	0.48	0.53	0.47	0.47	0.47	0.48	0.50	0.51	0.52	0.53
DOOORS	0.38	0.38	0.37	0.43	0.44	0.47	0.47	0.53	0.47	0.48	0.48	0.48	0.49	0.51	0.51	0.52
DOOORS2	0.29	0.32	0.33	0.39	0.42	0.46	0.46	0.50	0.44	0.45	0.44	0.45	0.46	0.48	0.48	0.48
Jinma no tō	0.25	0.19	0.19	0.32	0.31	0.35	0.38	0.41	0.38	0.39	0.41	0.42	0.44	0.44	0.44	0.44
Bad Piggies	0.31	0.32	0.36	0.44	0.44	0.46	0.46	0.53	0.48	0.49	0.50	0.51	0.51	0.50	0.48	0.49
Puyo Puyo !! Quest	0.17	0.23	0.24	0.34	0.37	0.40	0.41	0.46	0.41	0.44	0.42	0.42	0.44	0.46	0.47	0.48
Shoot Bubble Deluxe	0.37	0.38	0.40	0.46	0.45	0.46	0.46	0.52	0.46	0.47	0.48	0.48	0.50	0.50	0.50	0.50
Pop Star for Android	0.43	0.44	0.42	0.47	0.46	0.49		0.59	0.51	0.52	0.54	0.54	0.55	0.59	0.59	0.58
Cut the Rope	0.28	0.29	0.33	0.39	0.39	0.42	0.41	0.47	0.41	0.42	0.42	0.42	0.42	0.43	0.45	0.48
Cut the Rope: Time Trav	0.27	0.29	0.29	0.35	0.36	0.39	0.40	0.46	0.39	0.40	0.41	0.43	0.43	0.44	0.46	0.47
Chokokushi	0.52	0.54	0.53	0.59	0.58	0.61	0.61	0.68	0.61	0.63	0.63	0.64	0.65	0.64	0.64	0.63
Tracing a cat!	0.25	0.30	0.30	0.37	0.40	0.45	0.47	0.56	0.48	0.51	0.53	0.54	0.56	0.57	0.57	0.51
PIYOMORI	0.47	0.47	0.46	0.50	0.50	0.54	0.54	0.65	0.57	0.58	0.59	0.59	0.61	0.61	0.62	0.62
MORE!PIYOMORI	0.23	0.28	0.30	0.37	0.40	0.45	0.47	0.57	0.50	0.53	0.53	0.54	0.55	0.56	0.55	0.55
Shogi app everyday	0.43	0.43	0.42	0.48	0.48	0.52	0.52	0.58	0.52	0.53	0.53	0.54	0.55	0.55	0.56	0.58
ZOOKEEPER BATTLE	0.43	0.45	0.46	0.52	0.52	0.55	0.53	0.58	0.53	0.54	0.54	0.55	0.57	0.58	0.59	0.58
LINE Tours	0.17	0.18	0.21	0.31	0.34	0.37	0.40	0.48	0.39	0.41	0.42	0.44	0.47	0.48	0.50	0.52
Flame at LINE	0.17	0.23	0.19	0.32	0.31	0.36	0.39	0.42	0.35	0.37	0.39	0.41	0.44	0.46	0.49	0.49
LINE HIDDEN CATCH	0.39	0.42	0.41	0.46	0.46	0.50	0.50	0.59	0.50	0.50	0.51	0.52	0.54	0.55	0.56	0.57
LINE Pocopan	0.17	0.22														
LINE JELLY	0.30	0.35	0.37	0.42	0.43	0.46	0.46	0.55	0.47	0.47	0.48	0.48	0.50	0.51	0.52	0.54
LINE ZOOKEEPER	0.36	0.38	0.38	0.43	0.43	0.46	0.46	0.54	0.46	0.46	0.47	0.47	0.48	0.49	0.51	0.52
ChickPusher	0.41	0.42	0.42	0.47	0.48	0.52	0.52	0.62	0.54	0.55	0.54	0.54	0.54	0.55	0.55	0.56
Romanization		0.17	0.19	0.27	0.30	0.33	0.35	0.46	0.36	0.39	0.42	0.44	0.46	0.50	0.49	0.51
Yokai Taiso Pazuru da N		0.17	0.19	0.29	0.34	0.38	0.39	0.34	0.28	0.28	0.30	0.32	0.35	0.37	0.38	0.40
LINE Bubble		0.46														
LINE POP		0.47														
Buttons and Scissors			0.17	0.33	0.35	0.38	0.42	0.45	0.39	0.41	0.43	0.44	0.45	0.45	0.45	0.46
Jigty Jigsaw Puzzle			0.17	0.33	0.25	0.30	0.32	0.33	0.30	0.34	0.36	0.37	0.39	0.40	0.41	0.42
123 Guess Guess (Hong			0.17	0.33	0.34	0.39	0.43	0.45	0.36	0.37	0.41	0.44	0.47	0.49	0.53	0.53
123 Guess Guess (Taiwa			0.17	0.33	0.30	0.34	0.37	0.38	0.32	0.31	0.33	0.35	0.36	0.38	0.42	0.43
DivineGateJP			0.17	0.33	0.18	0.23	0.27	0.30	0.32	0.35	0.38	0.40	0.42	0.43	0.44	0.46
LINE PON PON PON			0.17	0.33	0.37	0.31	0.35	0.39	0.38	0.41	0.43	0.44	0.46	0.48	0.49	0.50
Dwarf observation puzzle			0.17	0.33	0.35	0.38	0.41	0.43	0.33	0.36	0.41	0.43	0.46	0.47	0.49	0.48
Three Kingdoms Puzzle					0.17	0.18	0.24	0.28	0.32	0.35	0.38	0.42	0.45	0.48	0.51	0.52
Puzzle Combo						0.19	0.24	0.29	0.35	0.40	0.44	0.47	0.50	0.52	0.54	0.55
Hello Kitty's puzzle chair						0.17	0.23	0.29	0.33	0.36	0.38	0.41	0.44	0.45	0.46	0.47
DOOORS3 - room escap							0.17	0.22	0.27	0.32	0.35	0.37	0.42	0.45	0.46	0.47
100 million puzzles - refre							0.17	0.20	0.20	0.27	0.31	0.36	0.41	0.44	0.47	0.49
LINE puzzle de Inazuma							0.17	0.23	0.27	0.27	0.30	0.33	0.34	0.37	0.40	0.42
LINE: Disney Zum Tum								0.17	0.18	0.27	0.34	0.41	0.48	0.51	0.53	0.55

Table A. 9 - App-Adv Values (Sub-Category: Puzzle)

Application	2013/06	2013/07	2013/08	2013/09	2013/10	2013/11	2013/12	2014/01	2014/02	2014/03	2014/04	2014/05	2014/06	2014/07	2014/08	2014/09
Pota-cats	0.14	0.12	0.08	0.17	0.11	0.12	0.07	0.10	0.08	0.08	0.07	0.07	0.07	0.07	0.07	0.06
Block Puzzle Original	0.28	0.27	0.15	0.51	0.25	0.16	0.26	0.35	0.31	0.23	0.14	0.13	0.13	0.12	0.11	0.09
Flow Free	0.04	0.06	0.08	0.32	0.16	0.17	0.11	0.15	0.10	0.11	0.09	0.08	0.08	0.07	0.11	0.09
Puzzles with Matches	0.06	0.06	0.04	0.15	0.10	0.10	0.07	0.10	0.13	0.13	0.12	0.10	0.09	0.11	0.11	0.15
Learning Japan Map Puz	0.10	0.11	0.08	0.18	0.12	0.13	0.08	0.11	0.10	0.14	0.15	0.14	0.11	0.11	0.12	0.11
Learning World Map Puz	0.13	0.12	0.10	0.23	0.15	0.17	0.10	0.11	0.11	0.14	0.12	0.13	0.10	0.10	0.11	0.11
One touch Drawing	0.07	0.07	0.05	0.18	0.12	0.12	0.10	0.12	0.11	0.09	0.09	0.10	0.09	0.10	0.09	0.09
Puzzle & Dragons Comb	0.38	0.35	0.18	0.29	0.17	0.17	0.10	0.11	0.05	0.04	0.04	0.04	0.04	0.04	0.03	0.04
Search for mistakes in ka	0.39	0.34	0.20	0.14	0.06	0.09	0.06	0.07	0.08	0.10	0.08	0.07	0.07	0.07	0.07	0.05
Unblock Me FREE	0.10	0.11	0.08	0.22	0.13	0.15	0.11	0.15	0.11	0.11	0.10	0.10	0.09	0.07	0.08	0.09
DOOORS	0.12	0.11	0.06	0.18	0.12	0.11	0.07	0.11	0.12	0.10	0.08	0.07	0.06	0.07	0.07	0.08
DOOORS2	0.26	0.16	0.08	0.22	0.16	0.15	0.08	0.13	0.11	0.08	0.07	0.08	0.07	0.08	0.07	0.09
Jinma no tō	0.21	0.05	0.09	0.77	0.30	0.33	0.16	0.17	0.12	0.10	0.10	0.08	0.07	0.06	0.07	0.07
Bad Piggies	0.05	0.06	0.07	0.21	0.13	0.13	0.11	0.15	0.10	0.08	0.09	0.09	0.07	0.08	0.08	0.09
Puyo Puyo !! Quest	0.00	0.36	0.22	0.58	0.26	0.19	0.10	0.18	0.17	0.16	0.11	0.15	0.17	0.16	0.13	0.10
Shoot Bubble Deluxe	0.08	0.07	0.07	0.21	0.14	0.15	0.12	0.14	0.11	0.08	0.07	0.07	0.08	0.07	0.08	0.09
Pop Star for Android	0.06	0.07	0.06	0.25	0.15	0.16		0.08	0.04	0.05	0.07	0.08	0.06	0.07	0.04	0.03
Cut the Rope	0.07	0.09	0.09	0.29	0.16	0.17	0.11	0.15	0.13	0.09	0.08	0.09	0.08	0.08	0.07	0.08
Cut the Rope: Time Trav	0.32	0.27	0.11	0.29	0.16	0.15	0.08	0.11	0.09	0.10	0.10	0.10	0.09	0.09	0.08	0.10
Chokokushi	0.10	0.10	0.06	0.14	0.09	0.10	0.06	0.09	0.07	0.07	0.07	0.06	0.06	0.07	0.07	0.08
Tracing a cat!	0.41	0.33	0.17	0.31	0.17	0.16	0.08	0.11	0.07	0.07	0.08	0.11	0.09	0.10	0.09	0.09
PIYOMORI	0.07	0.07	0.04	0.10	0.07	0.08	0.06	0.07	0.05	0.04	0.05	0.06	0.06	0.05	0.04	0.04
MORE!PIYOMORI	0.40	0.38	0.24	0.33	0.16	0.17	0.09	0.13	0.09	0.08	0.08	0.09	0.07	0.06	0.07	0.07
Shogi app everyday	0.14	0.15	0.10	0.19	0.12	0.14	0.10	0.14	0.11	0.11	0.11	0.11	0.10	0.09	0.10	0.11
ZOOKEEPER BATTLE	0.12	0.13	0.09	0.20	0.12	0.15	0.10	0.16	0.14	0.13	0.11	0.10	0.10	0.09	0.08	0.06
LINE Tours	0.00	0.11	0.22	0.56	0.28	0.16	0.07	0.09	0.06	0.05	0.04	0.04	0.05	0.04	0.05	0.03
Flame at LINE	0.05	0.33	0.04	0.83	0.27	0.32	0.14	0.11	0.04	0.03	0.04	0.05	0.05	0.06	0.06	0.05
LINE HIDDEN CATCH	0.16	0.10	0.05	0.14	0.10	0.11	0.07	0.09	0.07	0.05	0.05	0.06	0.06	0.05	0.05	0.06
LINE Pocopan	0.00	0.35														
LINE JELLY	0.43	0.38	0.15	0.19	0.12	0.12	0.07	0.10	0.06	0.06	0.05	0.05	0.05	0.05	0.05	0.04
LINE ZOOKEEPER	0.08	0.09	0.05	0.15	0.10	0.11	0.08	0.09	0.07	0.06	0.07	0.08	0.07	0.07	0.07	0.06
ChickPusher	0.10	0.10	0.07	0.15	0.10	0.11	0.07	0.11	0.08	0.07	0.07	0.09	0.09	0.08	0.08	0.07
Romanization		0.00	0.11	0.53	0.25	0.15	0.04	0.06	0.04	0.05	0.05	0.04	0.04	0.03	0.03	0.04
Yokai Taiso Pazuru da N		0.00	0.12	0.61	0.39	0.25	0.06	0.07	0.22	0.29	0.36	0.32	0.33	0.31	0.29	0.28
LINE Bubble		0.15														
LINE POP		0.15														
Buttons and Scissors			0.00	1.00	0.39	0.37	0.16	0.15	0.10	0.09	0.10	0.11	0.12	0.10	0.09	0.07
Jigty Jigsaw Puzzle			0.00	1.00	0.12	0.34	0.22	0.17	0.12	0.15	0.14	0.11	0.09	0.08	0.08	0.08
123 Guess Guess (Hong			0.00	1.00	0.38	0.36	0.11	0.08	0.02	0.02	0.03	0.04	0.03	0.01	0.04	0.06
123 Guess Guess (Taiwa			0.00	1.00	0.29	0.31	0.11	0.07	0.02	0.02	0.02	0.02	0.01	0.03	0.04	0.03
DivineGateJP			0.00	1.00	0.04	0.33	0.28	0.25	0.18	0.12	0.09	0.07	0.06	0.10	0.13	0.14
LINE PON PON PON			0.00	1.00	0.45	0.23	0.31	0.28	0.21	0.11	0.07	0.07	0.07	0.07	0.07	0.05
Dwarf observation puzzle			0.00	1.00	0.34	0.28	0.09	0.08	0.04	0.05	0.07	0.09	0.09	0.08	0.05	0.09
Three Kingdoms Puzzle					0.00	0.05	0.41	0.29	0.26	0.09	0.07	0.08	0.07	0.06	0.04	0.03
Puzzle Combo						0.04	0.39	0.44	0.48	0.37	0.28	0.23	0.18	0.17	0.15	0.13
Hello Kitty's puzzle chair						0.00	0.40	0.39	0.33	0.20	0.15	0.17	0.16	0.12	0.12	0.11
DOOORS3 - room escap							0.00	0.31	0.36	0.30	0.14	0.08	0.08	0.09	0.07	0.08
100 million puzzles - refre							0.00	0.20	0.14	0.48	0.33	0.32	0.14	0.10	0.07	0.06
LINE puzzle de Inazuma							0.00	0.37	0.29	0.16	0.21	0.15	0.11	0.09	0.06	0.05
LINE: Disney Zum Tum								0.00	0.08	0.56	0.55	0.50	0.26	0.18	0.19	0.17

Table A. 10 - App-Dec Values (Sub-Category: Puzzle)

Application	2013/06	2013/07	2013/08	2013/09	2013/10	2013/11	2013/12	2014/01	2014/02	2014/03	2014/04	2014/05	2014/06	2014/07	2014/08	2014/09
Pota-cats	0.54	0.56	0.58	0.50	0.53	0.48	0.51	0.43	0.52	0.52	0.51	0.49	0.50	0.47	0.46	0.44
Block Puzzle Original	0.06	0.17	0.21	0.27	0.33	0.18	0.14	0.12	0.25	0.43	0.53	0.56	0.55	0.54	0.55	0.52
Flow Free	0.81	0.77	0.52	0.40	0.46	0.40	0.47	0.44	0.56	0.57	0.61	0.62	0.62	0.57	0.52	0.49
Puzzles with Matches	0.72	0.69	0.66	0.55	0.54	0.48	0.50	0.37	0.42	0.39	0.45	0.51	0.54	0.48	0.45	0.43
Learning Japan Map Puz	0.58	0.57	0.57	0.50	0.48	0.42	0.46	0.39	0.44	0.41	0.41	0.43	0.47	0.49	0.47	0.44
Learning World Map Puz	0.56	0.53	0.53	0.45	0.47	0.39	0.43	0.40	0.49	0.48	0.47	0.47	0.48	0.47	0.48	0.44
One touch Drawing	0.68	0.67	0.65	0.56	0.54	0.45	0.48	0.42	0.53	0.53	0.53	0.53	0.52	0.52	0.51	0.50
Puzzle & Dragons Comb	0.09	0.21	0.38	0.46	0.54	0.53	0.56	0.53	0.65	0.66	0.65	0.63	0.60	0.58	0.58	0.54
Search for mistakes in ka	0.00	0.04	0.49	0.65	0.75	0.66	0.62	0.47	0.54	0.50	0.47	0.53	0.54	0.54	0.53	0.53
Unblock Me FREE	0.62	0.60	0.54	0.47	0.50	0.42	0.46	0.40	0.50	0.52	0.53	0.52	0.53	0.53	0.53	0.51
DOOORS	0.53	0.57	0.60	0.55	0.58	0.49	0.52	0.41	0.50	0.51	0.57	0.60	0.58	0.54	0.50	0.48
DOOORS2	0.27	0.50	0.60	0.57	0.54	0.46	0.49	0.43	0.51	0.55	0.60	0.63	0.61	0.56	0.50	0.48
Jinma no tō	0.19	0.05	0.02	0.02	0.15	0.21	0.35	0.44	0.56	0.60	0.61	0.63	0.64	0.63	0.63	0.63
Bad Piggies	0.77	0.74	0.63	0.49	0.54	0.46	0.49	0.42	0.54	0.57	0.59	0.59	0.54	0.55	0.54	0.54
Puyo Puyo !! Quest	0.00	0.00	0.00	0.14	0.41	0.45	0.47	0.39	0.46	0.48	0.46	0.45	0.44	0.46	0.52	0.53
Shoot Bubble Deluxe	0.71	0.68	0.60	0.51	0.51	0.43	0.47	0.44	0.54	0.58	0.57	0.59	0.57	0.57	0.57	0.52
Pop Star for Android	0.70	0.61	0.53	0.44	0.45	0.43		0.52	0.64	0.63	0.58	0.57	0.58	0.53	0.56	0.54
Cut the Rope	0.70	0.68	0.53	0.46	0.51	0.43	0.49	0.43	0.53	0.56	0.57	0.59	0.61	0.60	0.60	0.55
Cut the Rope: Time Trav	0.00	0.16	0.38	0.45	0.57	0.49	0.56	0.49	0.59	0.58	0.60	0.61	0.57	0.58	0.56	0.52
Chokokushi	0.55	0.52	0.54	0.46	0.48	0.43	0.46	0.40	0.48	0.47	0.45	0.45	0.46	0.49	0.48	0.48
Tracing a cat!	0.00	0.10	0.38	0.49	0.56	0.48	0.56	0.56	0.62	0.59	0.53	0.53	0.50	0.50	0.48	0.43
PIYOMORI	0.65	0.64	0.68	0.61	0.61	0.54	0.57	0.49	0.57	0.58	0.57	0.55	0.54	0.53	0.53	0.50
MORE!PIYOMORI	0.00	0.00	0.28	0.40	0.56	0.50	0.54	0.46	0.56	0.55	0.54	0.53	0.54	0.53	0.55	0.52
Shogi app everyday	0.49	0.46	0.53	0.49	0.51	0.43	0.46	0.41	0.50	0.51	0.49	0.49	0.47	0.48	0.45	0.44
ZOOKEEPER BATTLE	0.59	0.54	0.53	0.47	0.48	0.42	0.43	0.36	0.44	0.45	0.47	0.48	0.48	0.48	0.49	0.45
LINE Tours	0.00	0.00	0.00	0.06	0.45	0.53	0.67	0.64	0.75	0.75	0.73	0.72	0.69	0.68	0.66	0.64
Flame at LINE	0.00	0.00	0.01	0.09	0.11	0.22	0.45	0.65	0.80	0.81	0.77	0.71	0.70	0.65	0.63	0.60
LINE HIDDEN CATCH	0.53	0.64	0.70	0.63	0.62	0.56	0.59	0.54	0.63	0.64	0.62	0.61	0.60	0.59	0.57	0.54
LINE Pocopan	0.00	0.00														
LINE JELLY	0.00	0.19	0.49	0.55	0.62	0.56	0.60	0.54	0.65	0.66	0.67	0.66	0.66	0.65	0.64	0.60
LINE ZOOKEEPER	0.68	0.65	0.66	0.60	0.62	0.55	0.58	0.51	0.60	0.59	0.56	0.57	0.58	0.58	0.58	0.57
ChickPusher	0.64	0.60	0.63	0.56	0.60	0.53	0.55	0.46	0.54	0.53	0.52	0.51	0.49	0.51	0.52	0.51
Romanization		0.00	0.00	0.00	0.46	0.54	0.76	0.73	0.81	0.77	0.76	0.78	0.75	0.70	0.72	0.72
Yokai Taiso Pazuru da N		0.00	0.00	0.00	0.38	0.48	0.63	0.31	0.19	0.11	0.12	0.17	0.22	0.24	0.25	0.27
LINE Bubble		0.52														
LINE POP		0.51														
Buttons and Scissors			0.00	0.00	0.00	0.12	0.41	0.55	0.67	0.63	0.57	0.52	0.51	0.55	0.59	0.61
Jigty Jigsaw Puzzle			0.00	0.00	0.00	0.04	0.25	0.48	0.53	0.52	0.50	0.57	0.59	0.58	0.56	0.57
123 Guess Guess (Hong			0.00	0.00	0.00	0.17	0.50	0.72	0.86	0.85	0.83	0.79	0.74	0.74	0.70	0.68
123 Guess Guess (Taiwa			0.00	0.00	0.00	0.13	0.39	0.70	0.87	0.91	0.89	0.88	0.86	0.83	0.79	0.74
DivineGateJP			0.00	0.00	0.00	0.01	0.03	0.29	0.49	0.63	0.69	0.70	0.65	0.59	0.52	0.50
LINE PON PON PON			0.00	0.00	0.00	0.08	0.22	0.27	0.48	0.61	0.68	0.68	0.66	0.64	0.64	0.65
Dwarf observation puzzle			0.00	0.00	0.00	0.15	0.56	0.72	0.79	0.76	0.66	0.62	0.62	0.62	0.61	0.57
Three Kingdoms Puzzle					0.00	0.00	0.00	0.13	0.46	0.63	0.68	0.66	0.65	0.67	0.68	0.66
Puzzle Combo						0.00	0.01	0.01	0.09	0.21	0.33	0.39	0.44	0.46	0.49	0.50
Hello Kitty's puzzle chain						0.00	0.00	0.00	0.24	0.45	0.51	0.52	0.53	0.51	0.52	0.51
DOOORS3 - room escap							0.00	0.00	0.00	0.31	0.57	0.66	0.61	0.52	0.49	0.47
100 million puzzles - refre							0.00	0.00	0.00	0.05	0.12	0.37	0.53	0.63	0.64	0.62
LINE puzzle de Inazuma							0.00	0.00	0.00	0.27	0.46	0.53	0.58	0.62	0.64	0.67
LINE: Disney Zum Tum								0.00	0.00	0.00	0.01	0.17	0.34	0.40	0.41	0.39