Portfolio Selection and Insurance Period

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1. Introduction

This paper examines market risks, portfolio selection and period of insurance in a life insurance market. Date from many countries indicate that over ninety percent of bankruptcies of life insurers are due to failures in financial risk management. Financial risk management is therefore central. The contract period of life insurance is long, whereas that of loans is generally less than 10 years in Japan. As interest rates reduce, the assumed interest rates of life insurance increase above loan interest rates and the spread becomes negative. This negative spread is liable to bankrupt life insurers that do not have bad loans. In Japan a policy of very low interest rates has caused negative spread, and some Japanese life insurers have gone bankrupt.

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Of life insurers' employed assets, the largest are loans to corporations, followed by securities. Life insurers sell securities to make up for defaults or for negative spread arising from the gap between the loan period and the insurance period. Portfolio selections of life insurers are therefore concerned with the life insurance period. This paper studies how reductions in the insurance period affect the portfolio selections of life insurers.

2. Life insurers' portfolio selection

Macroeconomic changes affect investment income. In employing their assets, life insurers face the risk of default and negative spread. When negative spread occurs, life insurers sell securities. If the sales profit of securities is smaller than the negative spread, life insurers face bankruptcy.

Of life insurers' employed assets, the largest are loans to corporations, followed by securities. This study therefore concentrates loans to corporations and securities.

In loans to corporations, there is risk of default and of negative spread. Securities investment therefore carries the risk of price fluctuation. In explaining the effects of negative spread we assume for simplicity that there is no risk of default. We also suppose that life insurers choose loans to corporations and securities that maximize expected profit.

The model proposed here consists of two consecutive periods (specified by three points in time). A life insurance market includes life insurers and consumers. At the start of the first period a consumer purchases a contract and pays a premium. The insurers employ the entire premium in loans to companies and purchase of securities in each period. For simplicity it is supposed that an accident occurs at the end of the second period. We assume all interest rates of loans to companies are the same and given. A period of loans is one period. Since there is taken to be no risk of default, the loans which life insurers make at the beginning of the first period are returned at the end of this period. The interest rate on loans in the second period is determined at the beginning of that period, and life insurers will offer loans to companies at this rate. At the beginning of the first period, the interest rate on loans in the second period is uncertain. If business is good, the interest rate on loans in the second period is not less than the rate in the first period. If business is bad, the interest rate on loans in the second period is lower than in the first period, and negative spread occurs. We denote the probability of good business as g, and that of bad business as $1-g$. Consumers, insurers, and firms know these values. At the beginning of the first period, profits from loans in the second period are uncertain, and we treat them probability variable.

Generally, life insurers purchase securities for bad debt reserve or negative spread reserve. We therefore suppose that life insurers purchase securities at the beginning of the first period and sell them at the end of the second period. Profits from securities at the beginning of the first period are also treated probabilistically as the stock price changes.

When there is no risk of default, insurers that are not able to make up for the negative spread face bankruptcy. Insurers use securities as compensation for negative spread. A fall in the price of securities therefore increases the probability of bankruptcy of life insurers.

3. Longperiod contracts

At the beginning of the first period, consumers purchase contracts with premium y and benefit z, insurers employ all the premium in loans to companies and purchase of securities. Therefore if we denote loans to corporations as D and securities as M, then $y=D+M$. For simplicity we assume that benefit z is given for life insurers and consumers and is the same across insurers.

Successful investment increases a firm's profit and in turn increases its dividend ; we suppose that profit from securities consists of dividend and capital gains. We also assume for simplicity that the price earnings rate in the 'success' state is greater than one, and in the 'failure' state is smaller than one. When negative spread occurs, life insurers sell securities to make up for it. A decline of stock price therefore reduces the value of the negative spread reserve. It follows that if insurers purchase securities that have high probability of success, their probability of bankruptcy decreases.

We denote by m_S the price earnings rate in the 'success' state of insurers' holding securities, and denote by m_F in the 'failure' state, and we suppose that the price earnings rate is greater than one with probability q, and smaller than one with probability $1-q$. Here m_S and m_F are concave decreasing functions of the risk q. ($m_j=m_j(q)<0$, $m_j''(q)<0$, $j=S, F$). The expected insurance with benefit z and death rate π is πz. For simplicity, we assume that life insurers are identical in all respects. Life insurers discount their premium so as to give consumers part of the investment income, so that the expected insurance with premium y, benefit z, and assumed rate of interest p is $\pi z/(1+p)$.

Suppose that consumers are risk-neutral and the market is competitive. Then the expected insurance equals the premium :

$$\pi z=(1+p)y \qquad (1)$$

Let the interest rate of loans in the first period be r_1, and the interest rate of loans in the second period with good business be r_2^G. If business is bad the interest rate of loans in the second period is $r_2^B(0<r_2^B<r_1<r_2^G<1)$. If business is bad and the price earnings rate is smaller than one, life insurers may go bankrupt. Let D denote insurers' loans to companies; then an insurers' expected profit from loans is $[1+r_1+gr_2^G+(1-g)r_2^B]D$.

Next, Let M denote securities insurers purchase, so that an insurers' expected profit from securities is $[qm_S(q)+(1-q)m_F(q)]M$.

Life insurers must pay sales expenses, expenses of monitoring for assurance and loans, the expenses of computers and buildings etc., and investment costs. We refer to the expenses of sales and monitoring for assurance as insurance costs, denoted by $C(>0)$. Suppose that C is an increasing concave function of premium with respect to marginal costs, so that, $C'>0$, $C''<0$, $C(0)=0$.

Suppose now that the marginal investment costs of loans and securities increase. Let C_D denote the loan costs and C_M the investment costs of securities, where C_D an is increasing convex function of loans and C_M is increasing convex function of securities so that, $C_D'>0$, $C_D''>0$, $C_D(0)=0$, $C_M'>0$, $C_M''>0$, $C_M(0)=0$. The insurers' expected profits $E\Pi$ are how specified described as follows:

$$E\Pi=[1+r_1+gr_2^G+(1-g)r_2^B]D+[qm_S(q)+(1-q)m_F(q)]M$$
$$-(1+p)(D+M)-C(y)-C_D(D)-C_M(M) \qquad (2)$$

For simplicity, defind $g(1+r_1+r_2^G)\equiv g(1+r_G)$,$(1-g)(1+r_1)+(1-g)r_2^B \equiv(1-g)(1+r_B)$, (2) is rewritten as follows:

$$E\Pi=[g(1+r_G)+(1-g)(1+r_B)]D+[qm_S(q)+(1-q)m_F(q)]M$$
$$-(1+p)(D+M)-C(y)-C_D(D)-C_M(M) \qquad (2)'$$

Insurers choose loans and securities to maximize their expected profits (2). Let D^*,M^* denote equilibrium loans and equilibrium securities. Then, by differentiating (2), first order conditions for maximization are:

$$g(1+r_G)+(1-g)(1+r_B)=C_D'(D^*)+C'(D^*+M^*) \qquad (3)$$
$$qm_S(q)+(1-q)m_F(q)=C_M'(M^*)+C'(D^*+M^*) \qquad (4)$$

With quadratic dependencies $C(y)=cy^2$, $C_D(D)=c_dD^2$, $C_M(M)=c_MM^2$, (3) and (4) reduce as follows:

$$g(1+r_G)+(1-g)(1+r_B)=2(c+c_d)D^*+2cM^* \qquad (5)$$

$$qm_S(q)+(1-q)m_F(q)=2(c+c_m)M^*+2cD^* \qquad (6)$$

These can be solved for the equilibrium loans and equilibrium securities :

$$D^*=\frac{(c+c_m)\lfloor g(1+r_G)+(1-g)(1+r_B)\rfloor -c[qm_S(q)+(1-q)m_F(q)]}{2[c(c_d+c_m)+c_dc_m]}$$

$$M^*=\frac{[qm_S(q)+(1-q)m_F(q)](c+c_d)-c\lfloor g(1+r_G)+(1-g)(1+r_B)\rfloor}{2[c(c_d+c_m)+c_dc_m]}$$

Comparative statics

By differentiating (5) and (6) we obtain the Hessian matrix of the second derivatives and its determinant :

$$|H|=\begin{vmatrix}2(c+c_d) & 2c\\ 2c & 2(c+c_m)\end{vmatrix}=4\,c_dc_m)+4c(c_dc_m)>0$$

From Cramer's rule for matrix inversion we have,

$$\frac{\partial D^*}{\partial r_G}=\frac{2g(c+c_m)}{|H|}>0,\ \frac{\partial M^*}{\partial r_G}=\frac{-2gc}{|H|}<0$$

$$\frac{\partial D^*}{\partial r_B}=\frac{2(1-g)(c+c_m)}{|H|}>0,\ \frac{\partial M^*}{\partial r_B}=\frac{-2c(1-g)}{|H|}<0$$

$$\frac{\partial D^*}{\partial g}=\frac{2(c+c_m)(r_G-r_B)}{|H|}>0,\ \frac{\partial M^*}{\partial g}=\frac{-2c(r_G-r_B)}{|H|}<0$$

Proposition 1

No matter what the state of the business, a rise of the interest rate on loans increases loans and decreases securities. As the probability increases that the interest rate on loans will go up, loans to corporations increase and securities decrease.

A rise of the interest rate on loans means an increase in profit from loans and a decrease in the risk of negative spread. Hence, a rise of the interest rate increases loans and decreases securities. Since an increase in the probability of good business means a decrease in the probability of negative spread, an increase in its probability increases loans to corporations and decreases securities.

Next, we examine how changes in the risk of price fluctuation affect the equilibrium loans and securities.

$$\frac{\partial D^*}{\partial q}=\frac{-2c[m_S(q)+qm_S'(q)-m_F+(1-q)m_F'(q)]}{|H|} \qquad (7)$$

$$\frac{\partial M^*}{\partial q}=\frac{2(c+c_d)[m_S(q)+qm_S'(q)-m_F(q)+(1-q)m_F'(q)]}{|H|} \qquad (8)$$

From (7) , (8) and $m_F<1<m_S$, we obtain the following results :

(1) $q<-\frac{m_S-m_F-m_F'(q)}{m_F'(q)-m_S'(q)}\Rightarrow\frac{\partial D^*}{\partial q}<0,\ \frac{\partial M^*}{\partial q}>0$

(2) $\frac{m_S-m_F-m_F'}{m_F'(q)-m_S'(q)}\leq q\Rightarrow\frac{\partial D^*}{\partial q}\geq 0,\ \frac{\partial M^*}{\partial q}\leq 0$

Proposition 2

When the risk of price fluctuation is small (large q), an increase in risk (decrease in q) decreases loans and increases securities. By contrast, when the risk is large (small q), an increase in risk (decrease in q) imcreases loans and decreases securities.

Since the marginal profit from securities is larger than the marginal profit from loans in the case of low risk of price fluctuation, an increase in risk decreases loans and increases securities. By contrast, since the marginal profit from securities is less than the marginal profit from loans in the case of high risk, an increase in risk increases loans and decreases securities.

Next, we examine how changes in risk affect the expected profit. Let $E\Pi^*$ denote the equilibrium expected profit. By differentiating (2) we may derive the following equation :

$$\frac{\partial E\Pi^*}{\partial q} = [m_S(q) + q m_S'(q) - m_F(q) + (1-q) m_F'(q)] M^*$$

Proposition 3

When the risk of price fluctuation is small (large q), an increase in risk (decrease in q), increases the expected profit. By contrast, when the risk is large (small q), an increase in risk (decrease in q) decreases the expected profits.

When the risk of price fluctuation is small, the price earnings rate in the 'success' state is large but the probability of success is low and consequently an increase in risk of price fluctuatiom increases the expected profit. By contrast, when the risk is large, it is small but the probability of success is high and consequently an increase in risk decreases the expected profit.

4. Shortperiod contracts

Negative spread arises from the discrepancy between the loans period and the insurance period. Hence, by reducing the insurance period, negative spread will not arise. This section examines the influence of the insurance period on portfolio selection. As in the previous section, we assume there is no risk of default. In general, an increase in the interest rate on loans increases investment profit, and a decrease reduces it. Therefore, if the insurance is the same, insurers can reduce the premium when business is in a good state. Consumers do not know the assumed rate of interest at the beginning of the first period, which is uncertain.

Let y_G, p_G denote the premium and the assumed rate of interest when business is in a good state, and y_B, p_B denote them in the bad state. We can rewrite (1) as follows :

$$\pi z = g(1+p_G)y_G + (1-g)(1+p_B)y_B \qquad (1)'$$

Here, D_G, M_G denote loans to corporations and securities when business is in a good state, and D_B, M_B denote these when business is bad. We can now rewrite the expected profits (2) as follows :

$$\begin{aligned} E\Pi = g\lfloor\ &(1+r_G)D_G + \{qm_S(q)+(1-q)m_F(q)\}M_G - C(y_G) \\ &- C_D(D_G) - C_M(M_G)\rfloor\ + (1-g)[(1+r_B)D_B \\ &+ \{qm_S(q)+(1-q)m_F(q)\}M_B - C(y_B) - C_D(D_B) - C_M(M_B)] \\ &- [g(1+p_G)(D_G+M_G) + (1-g)(1+p_B)(D_B+M_B)] \end{aligned} \qquad (2)''$$

Life insurers choose loans D_G, D_B and securities M_G, M_B to maximize the expected profits (2)″. Then first order conditions for maximization are :

$$(D_G)(1+r_G)=C'(D_G+M_G)+C_D'(D_G) \qquad (9)$$

$$(D_B)(1+r_B)=C'(D_B+M_B)+C_D'(D_B) \qquad (10)$$

$$(M_G)qm_S(q)+(1-q)m_F(q)=C_M'(M_G)+C'(D_G+M_G) \qquad (11)$$

$$(M_B)qm_S(q)+(1-q)m_F(q)=C_M'(M_B)+C'(D_B+M_B) \qquad (12)$$

Again we choose the quadratic dependencies $C(y)=cy^2$, $C_D(D)=c_dD^2$, $C_M(M)=C_mM^2$, following which (9), (10) reduce to.

$$(1+r_G)=2(c+c_d)D_G+2cM_G \qquad (13)$$

$$(1+r_B)=2(c+c_d)D_B+2cM_B \qquad (14)$$

$$qm_S(q)+(1-q)m_F(q)=2(c+c_m)M_G+2cD_G \qquad (15)$$

$$qm_S(q)+(1-q)m_F(q)=2(c+c_m)M_B+2cD_B \qquad (16)$$

We denote D_G^*, M_G^* by the equilibrium loans and securities in the good business state and D_B^*,M_B^* in the bad business strate. Then equilibrium loans and securities become :

$$D_G^*=\frac{(1+r_G)(c+c_m)-c[qm_S(q)+(1-q)m_F(q)]}{2[c(c_d+c_m)+c_dc_m]}$$

$$D_B^*=\frac{(1+r^B)(c+c_m)-c[qm_S(q)+(1-q)m_F(q)]}{2[c(c_d+c_m)+c_dc_m]}$$

$$M_G^* = \frac{(c+c_d)[qm_S(q)+(1-q)m_F(q)]-c(1+r_G)}{2[c(c_d+c_m)+c_dc_m]},$$

$$M_B^* = \frac{(c+c_d)[qm_S(q)+(1-q)m_F(q)]-c(1+r_B)}{2[c(c_d+c_m)+c_dc_m]}$$

Comparative Statics

Upon defining $2[c(c_d+c_m)+c_dc_m] \equiv K$ and differentiating the equilibrium loans and securities, it eventually follows that

$$\frac{\partial D^{G*}}{\partial r^G} = \frac{\partial D^{B*}}{\partial r^B} = \frac{c+c_m}{K} > 0, \quad \frac{\partial M^{G*}}{\partial r^G} = \frac{\partial M^{B*}}{\partial r^B} = \frac{-c}{K} < 0$$

Proposition 4

Whether business is good or bad, a rise in the interest rate on loans increases loans to corporations and decreases securities.

Multiperiod contracts have no negative spread, and this has consecquences for price fluctuations. By assumption, a high risk of price fluctuation is associated with a high price earnings rate. Hence, for low risk the marginal expected profit from securities decreases, while for high risk it increases. The equilibrium loans and securities change with risk of price fluctuation as follows :

$$\frac{\partial D^{G*}}{\partial q} = \frac{\partial D^{B*}}{\partial q} = \frac{-c[m_S(q)+qm_S'(q)-m_F(q)+(1-q)m_F'(q)]}{K}$$

$$\frac{\partial M^{G*}}{\partial q} = \frac{\partial M^{B*}}{\partial q} = \frac{[m_S(q)+qm_S'(q)-m_F(q)+(1-q)m_F'(q)](c+c_d)}{K}$$

$$q < -\frac{m_S(q) - m_F(q) + m_F'(q)}{m_F'(q) - m_S'(q)} \Rightarrow \frac{\partial D^{G*}}{\partial q} = \frac{\partial D^{B*}}{\partial q} < 0,\ \frac{\partial M^{G*}}{\partial q} = \frac{\partial M^{B*}}{\partial q} > 0$$

$$q \geq -\frac{m_S(q) - m_F(q) + m_F'(q)}{m_F'(q) - m_S'(q)} \Rightarrow \frac{\partial D^{G*}}{\partial q} = \frac{\partial D^{B*}}{\partial q} \geq 0,\ \frac{\partial M^{G*}}{\partial q} = \frac{\partial M^{B*}}{\partial q} \leq 0$$

Proposition 5

When the risk of price fluctuation is small (large q), an increase in risk (decrease in q) decreases loans and increases securities. By contrast, when the risk is large (small q), an increase in risk (decrease in q) increases loans and decreases securities.

Since the marginal profit from securities is large than the marginal profit from loans in the case of low risk of price fluctuation, an increase in risk decreases loans and increases securities. By contrast, since the marginal profit from securities is less than the marginal profit from loans in the case of high risk, an increase in risk increases loans and decreases securities.

Next we examine the behavior of the equilibrium expected profit as the risk of price fluctuation changes. We denote $E\Pi^{**}$ by the equilibrium expected profit of short period insurance. By differentiating (2)′ we eventually derive the relation :

$$\frac{\partial E\Pi^{**}}{\partial q} = \left[m_S(q) + q m_S'(q) - m_F(q) + (1-q) m_F'(q) \right] \left[g M^{G*} + (1-g) M^{B*} \right]$$

$$q < -\frac{m_S(q) - m_F(q) + m_F'(q)}{m_F'(q) - m_S'(q)} \Rightarrow \frac{\partial E\Pi^{**}}{\partial q} > 0$$

$$q \geq -\frac{m_S(q) - m_F(q) + m_F'(q)}{m_F'(q) - m_S'(q)} \Rightarrow \frac{\partial E\Pi^{**}}{\partial q} \leq 0$$

Proposition 6

At high risk of risk fluctuation, an increase in the risk reduces the equilibrium expected profit. At low risk, by contrast, an increase in the risk increases the equilibrium expected profit.

5. Comparison between equilibria

Here, we compare equilibrium loans, securities and expected profit for long and short insurance periods. With $2[c(c_d + c_m) + c_d c_m] \equiv K$ again, we obtain :

$$D_G^* - D^* = \frac{(1-g)(c+c_m)(r_G - r_B)}{K} \geq 0$$

$$D_B^* - D^* = \frac{g(c+c_m)(r_B - r_G)}{K} \leq 0$$

$$M_G^* - M^* = \frac{-c(1-g)(r_G - r_B)}{K} \leq 0$$

$$M_B^* - M^* = \frac{-cg(r_B - r_G)}{K} \geq 0$$

$$E\Pi^* - E\Pi^{**} = -\frac{3g(1-g)(r_G - r_B)^2(c+c_m)}{K} \leq 0$$

Proposition 7

$D_B^* \leq D^* \leq D_G^*$

$M_G^* \leq M^* \leq M_B^*$

$E\Pi^* \leq E\Pi^{**}$

Since short period insurance has no negative spread, the equilibrium loans in the good business of short period insurance (D_G^*) is larger than the equilibrium loans of long period insurance (D^*) and the equilibrium loans in the bad business (D_B^*) is smaller than the equilibrium loans of long period insurance (D^*). By assumption, short period insurance has only the risk of price fluctuation. Since the interest rate of loans in the second period is smaller than the interest rate in the first period, the equilibrium securities in the bad business of short period insurance (M_B^*) is larger than the equilibrium securities of long period insurance (M^*) and the equilibrium securities in the good business (M_G^*) is smaller than the equilibrium securitties of long period insurance (M^*).

Since life insurers transfer risk of megative spread to consumers by reducing the insurance period, the equilibrium expected profit for short period insurance is larger than for long period insurance.

6. Conclusion

Since negative spread arises from the discrepancy between the loan period and the insurance period, life insurers can avoid negative spread by reducing the insurance period. This paper has examined how reduction of the life insurance period affects portfolio selection by the insurer.

The consequences of comparative statics for long and short insurance period are the same.

A rise of the interest rate on loans means an increase in profit from loans and a decrease in the risk of negative spread. Hence, a rise of the interest rate increases loans and decreases securities. Since an increase in the probability of good business means a decrease in the probability of negative spread, an increase in its probability increases loans to corporations and decreases securities.

Since the marginal profit from securities is larger than the marginal profit from loans in the case of low risk of price fluctuation, an increase in risk decreases loans and increases securities. By contrast, since the marginal profit from securities is less than the marginal profit from loans in the case of high risk, an increase in risk increases loans and decreases securities.

When the risk of price fluctuation is small, the price earnings rate in the 'success' state is large but the probability of success is low and consequently an increase in risk of price fluctuation increase the expected profit. By contrast, when the risk is large, it is small but the probability of success is high and consequently an increase in risk decreases the expected profit.

When we compare equilibrium loans, securities and expected profit for long and short insurance periods, we can derive the following consequence. Since short insurance has no negative spread, the equilibrium loans in the good business of short period insurance (D_G^*) is larger than the equilibrium loans of long period insurance (D^*) and the equilibrium loans in the bad business (D_B^*) is smaller than the equilibrium loans of long period insurance (D^*). By assumption, short period insurance has only the risk of price fluctuation. Since the interest rate of loans in the second period is smaller than the interest rate in the first period, the equilibrium securities in the bad business of short period insurance (M_B*) is larger than the equilibrium securities of long period insurance (M*) and the equilibrium securities in the good business (M_G*) is smaller than the equilibrium securities of long period insurance (M*).

Since life insurers transfer risk of negative spread to consumers by reducing the insurance period, the equilibrium expected profit for short period insurance is larger than for long period insurance.

In this paper, we assum that all life insurers face the same risk of price fluctuation and negative spread. But they practically face different risk and consumers can not describe their risk. We need to analyze the relation between the portfolio selection and life insurance period under asymmetric information.

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