7月19日

$f: \mathbb{R}^n \to \mathbb{R}^m$

点 $x \in \mathbb{R}^n$ で微分

$f'(x)$ : $\mathbb{R}^n$ から $\mathbb{R}^m$ への線型写像

$\mathcal{L}(\mathbb{R}^n ; \mathbb{R}^m)$ 　$\mathbb{R}^n$ から $\mathbb{R}^m$ への線型写像の全体

線型空間
足し算とスカラー倍が
定義されている空間

$\varphi, \psi \in \mathcal{L}(\mathbb{R}^n, \mathbb{R}^m)$ mn次元

$\alpha \in \mathbb{R}$

$(\varphi + \psi)(x) = \varphi(x) + \psi(x)$

$(\alpha \varphi)(x) = \alpha \varphi(x)$

成分

$\mathcal{L}(\mathbb{R}^2, \mathbb{R}^2)$ 　4次元

m×n の行列

$m\left(\overset{n}{\quad\quad}\right)$ 　A, B

m×n 個の実数

$\varphi \longleftrightarrow A$

$\psi \longleftrightarrow B$

$\varphi + \psi \longleftrightarrow A + B$

$\alpha \varphi \longleftrightarrow \alpha A$

高階の微分 → 極値

$f: \mathbb{R} \to \mathbb{R}$

$f': \mathbb{R} \to \mathbb{R}$

$f: \mathbb{R}^n \to \mathbb{R}^m$

$f': \mathbb{R}^n \to \mathcal{L}(\mathbb{R}^n ; \mathbb{R}^m)$

nm次元

$f'': \mathbb{R}^n \to \mathcal{L}(\mathbb{R}^n, \underline{\mathcal{L}(\mathbb{R}^n, \mathbb{R}^m)})$

# 内積

$\varphi : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R} \qquad \alpha, \beta \in \mathbb{R}$

$$\left.\begin{aligned}
\varphi(a_1 + a_2, b) &= \varphi(a_1, b) + \varphi(a_2, b) \\
\varphi(\alpha a, b) &= \alpha \varphi(a, b) \\
\varphi(a, b_1 + b_2) &= \varphi(a, b_1) + \varphi(a, b_2) \\
\varphi(a, \beta b) &= \beta \varphi(a, b)
\end{aligned}\right\} \text{二重線型性}$$

$$a \in \mathbb{R}^2 \longmapsto \overset{\text{線型}}{\varphi(a, \_)} : \mathbb{R}^2 \to \mathbb{R}$$

$\mathbb{R}^2 \longrightarrow \mathcal{L}(\mathbb{R}^2, \mathbb{R})$ 線型

$\varphi(a_1 + a_2, \_) = \varphi(a_1, \_) + \varphi(a_2, \_)$

$\varphi(\alpha a, \_) = \alpha \varphi(a, \_)$

$\varphi \in \mathcal{L}(\mathbb{R}^2 ; \mathcal{L}(\mathbb{R}^2 ; \mathbb{R}))$

$\mathcal{L}(\mathbb{R}^2, \mathbb{R}^2, \mathbb{R})$ $\quad \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$ 二重線型写像

$f : \mathbb{R}^2 \to \mathbb{R}$

$x \in \mathbb{R}^2$

$f'(x) \in \mathcal{L}(\mathbb{R}^2 ; \mathbb{R})$

$f' : \mathbb{R}^2 \to \mathcal{L}(\mathbb{R}^2 ; \mathbb{R})$

$x \in \mathbb{R}^2$ で微分

$$f''(x) = \mathcal{L}(\mathbb{R}^2 ; \mathcal{L}(\mathbb{R}^2 ; \mathbb{R}))$$

$$\|$$

$$\mathcal{L}(\mathbb{R}^2, \mathbb{R}^2, \mathbb{R})$$

$\mathcal{L}(\mathbb{R}^2;\mathbb{R})$  1×2の行列  $1(\underbrace{a_1, a_2}_{2})$

$$(a_1, a_2)\begin{pmatrix}x_1\\x_2\end{pmatrix} = a_1x_1 + a_2x_2$$

$\varphi: \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$  二重線型  $a = \begin{pmatrix}a_1\\a_2\end{pmatrix} = a_1e_1 + a_2e_2$

$b = \begin{pmatrix}b_1\\b_2\end{pmatrix} = b_1e_1 + b_2e_2$

$$\begin{aligned}
&\varphi(a, b)\\
&= \varphi(a_1e_1 + a_2e_2,\ b_1e_1 + b_2e_2)\\
&= a_1b_1\varphi(e_1, e_1) + a_1b_2\varphi(e_1, e_2)\\
&\quad + a_2b_1\varphi(e_2, e_1) + a_2b_2\varphi(e_2, e_2)
\end{aligned}$$

$$\overset{1\times 2}{(a_1, a_2)}\ \overset{2\times 2}{\begin{pmatrix}\varphi(e_1, e_1) & \varphi(e_1, e_2)\\ \varphi(e_2, e_1) & \varphi(e_2, e_2)\end{pmatrix}}\ \overset{2\times 1}{\begin{pmatrix}b_1\\b_2\end{pmatrix}}$$

$$= \big(a_1\varphi(e_1, e_1) + a_2\varphi(e_2, e_1),\ a_1\varphi(e_1, e_2) + a_2\varphi(e_2, e_2)\big)\begin{pmatrix}b_1\\b_2\end{pmatrix}$$

$\mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$

$\begin{pmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{pmatrix}$ 対称 $\overset{\Rightarrow}{\underset{\Leftarrow}{\Longleftrightarrow}}$ $a_{21} = a_{12}$

レポ $a_{21} = a_{12}$ のとき 対称であることを示せ

$f: \mathbb{R}^2 \to \mathbb{R}$

$$f(x) = \left(\frac{\partial f}{\partial x}(x),\ \frac{\partial f}{\partial y}(x)\right)$$

$$f''(x) = \begin{pmatrix}\dfrac{\partial^2 f}{\partial x^2}(x) & \dfrac{\partial^2 f}{\partial x \partial y}(x)\\[2ex] \dfrac{\partial^2 f}{\partial y \partial x}(x) & \dfrac{\partial^2 f}{\partial y^2}(x)\end{pmatrix}$$

$$\frac{\partial}{\partial y}\frac{\partial f}{\partial x} = \frac{\partial^2 f}{\partial x \partial y}$$

常に同じ ⇒ 対称