7月12日

$f: \mathbb{R}^n \to \mathbb{R}^m$ $x_0 \in \mathbb{R}^n$で微分

$f'(x_0): \mathbb{R}^n \to \mathbb{R}^m$への線形写像

$a \in \mathbb{R}^n$

$f'(x_0)(a)$ $\exists! \in \mathbb{R}^m$

$f(x_0 + a d) = f(x_0) + ?\, d$

$f'(x_0)(a)$としましょう

(1) $f'(x_0)(a_1 + a_2) = f'(x_0)(a_1) + f'(x_0)(a_2)$

(2) $f'(x_0)(\alpha a) = \alpha f'(x_0)(a)$

$f'(x_0): \mathbb{R}^n \to \mathbb{R}^m$ 線型

$a \in \mathbb{R}^n$ 線型空間

$e_1 = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$ $e_2 = \begin{pmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{pmatrix}$ $\cdots$ $e_n \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix}$

$a = a_1 e_1 + \cdots + a_n e_n$ $\exists! (a_1, \cdots, a_n)$ n個の実数の組

$n=2$ $\mathbb{R}^2$ 平面のベクトル 基底

$e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $a_1 e_1 + a_2 e_2$ 線型和

$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \alpha_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} + \alpha_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$$\begin{array}{rl} & a_1 = \alpha_1 + \alpha_2 \\ +) & a_2 = -\alpha_1 + \alpha_2 \\ \hline & a_1 + a_2 = 2\alpha_2 \end{array} \qquad \begin{array}{rl} & \cdots \\ -) & \cdots \\ \hline & a_1 - a_2 = 2\alpha_1 \end{array}$$

$\varphi : \mathbb{R}^n \to \mathbb{R}$ への線型写像の全体 $L(\mathbb{R}^n, \mathbb{R})$

$$a = \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix} = a_1 e_1 + \cdots + a_n e_n$$

n次元

$$\varphi(a) = \varphi(a_1 e_1 + \cdots + a_n e_n) = a_1 \varphi(e_1) + \cdots + a_n \varphi(e_n)$$

$\dfrac{dx}{dy}$

$$\begin{array}{lll} \mathbb{R}^n & \mathbb{R} & \\ \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix} \longmapsto a_1 & & dx_1 \\ \longmapsto a_2 & & dx_2 \\ \vdots & & \vdots \\ \longmapsto a_n & & dx_n \end{array}$$

$$\varphi = \varphi(e_1)\, dx_1 + \cdots + \varphi(e_n)\, dx_n$$

$f : \mathbb{R}^n \to \mathbb{R}$ 一般の関数

$x_0 \in \mathbb{R}^n$ で微分 $\quad x_0 = \begin{pmatrix} x_0^1 \\ x_0^2 \\ \vdots \\ x_0^n \end{pmatrix}$

$f'(x_0) : \mathbb{R}^n \to \mathbb{R}$ 線型

$$= \alpha_1 dx_1 + \cdots + \alpha_n dx_n$$

$(\alpha_1, \cdots, \alpha_n)$ n個の実数の組

$$\alpha_1 = f'(x_0)(e_1) \quad \cdots \quad \alpha_n = f'(x_0)(e_n)$$

$$f(x_0 + d e_1) = f(x_0) + f'(x_0)(e_1)\, d$$

$$f\left(\begin{pmatrix} x_0^1 \\ \vdots \\ x_0^n \end{pmatrix} + d \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \right) = f\begin{pmatrix} x_0^1 \\ \vdots \\ x_0^n \end{pmatrix} + ?d$$

$$f\begin{pmatrix} x_0^1 + d \\ x_0^2 \\ \vdots \\ x_0^n \end{pmatrix} = f\begin{pmatrix} x_0^1 \\ \vdots \\ x_0^n \end{pmatrix} + ?d \qquad \frac{\partial f}{\partial x_1}(x_0)$$

$$f'(x_0) = \frac{\partial f}{\partial x_1}(x_0)\, dx_1 + \cdots + \frac{\partial f}{\partial x_n}(x_0)\, dx_n$$

No.

Date

report 次の関数を微分せよ

(1) $f(x,y) = 3x^2 + 4xy + 5xy^3$

(2) $f(x,y) = \sin xy$

(3) $f(x,y,z) = 3x^2y + 5xz + 7yz^4$

極値 $f: \mathbb{R} \to \mathbb{R}$

増減表 必要条件 $f'(x_0) = 0$

$y = f(x)$

$z = f(x,y)$ $f'(x_0) = 0$

$f''(x_0) > 0$

< 0

$f: \mathbb{R}^n \to \mathbb{R}^m$

$f': \mathbb{R}^n \to \underbrace{\mathcal{L}(\mathbb{R}^n, \mathbb{R}^m)}_{nm次元}$