7月5日

微分

$f: R \to R$

$D = \{d \in R \mid d^2 = 0\}$

Kock-Lawvere（コック ラウィアー）の公理

$\varphi: D \to R$

$(\exists! a \in R)(\forall d \in D)$

$(\varphi(d) = \varphi(0) + ad$

fをxで微分

$d \in D \mapsto f(x+d) \in R$

$\varphi(0) = f(x)$

$\varphi(d) = f(x+d)$

$f(x+d) = f(x) + ad$

$a = f'(x)$

$\varphi: D \to R^m$ (mは自然数)

$\varphi = \begin{pmatrix} \varphi_1 \\ \varphi_m \end{pmatrix}$

$\varphi_i: D \to R$ $\exists!$

$\varphi_i(d) = \varphi_i(0) + a_i d$

$$\varphi(d) = \begin{pmatrix} \varphi_1(d) \\ \vdots \\ \varphi_m(d) \end{pmatrix} = \begin{pmatrix} \varphi_1(0) + a_1 d \\ \vdots \\ \varphi_m(0) + a_m d \end{pmatrix} = \begin{pmatrix} \varphi(0) \\ \vdots \\ \varphi_m(0) \end{pmatrix} + \begin{pmatrix} a_1 \\ \vdots \\ a_m \end{pmatrix} d$$

$(\exists! a \in R^m)(\forall d \in D)$

$(\varphi(d) = \varphi(0) + ad)$ 一般化された

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$f: \mathbb{R} \to \mathbb{R}^m$

時間

$m=2$ 平面での運動

$m=3$ 空間での運動

xで微分

$\varphi: d \in D \mapsto f(x+d)$

$\varphi(0) = f(x)$

$\varphi(d) = f(x+d)$

$f(x+d) = f(x) + ad$

$\exists! \, a \in \mathbb{R}^m$

$f'(x)$ 微分係数

$f: \mathbb{R}^n \to \mathbb{R}^m$ を $x \in \mathbb{R}^n$ で微分

$f'(x)$ $\mathbb{R}^n \to \mathbb{R}^m$ への線形写像

$a \in \mathbb{R}^n$

$f'(x)(a) \in \mathbb{R}^m$

これをどう決めましょうか?

① $t \in \mathbb{R} \mapsto x + at \in \mathbb{R}^n$

② $f: y \in \mathbb{R}^n \mapsto f(y) \in \mathbb{R}^m$

合成

$t \mapsto f(x+at) \in \mathbb{R}^m$ $t=0$で微分

$f(x+ad) = f(x) + bd$

$\exists! \, b \in \mathbb{R}^m$

合成関数の微分

$f'(x)(a)$

$s \mapsto as$

(1) $f'(x)(a_1 + a_2) = f'(x)(a_1) + f'(x)(a_2)$

(ホ) (2) $f'(x)(\alpha a) = \alpha f'(x)(a)$

$\alpha \in \mathbb{R}$ $a, a_1, a_2 \in \mathbb{R}^n$

$x \in \mathbb{R}^n$

(1) の証

$f($

$f($

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(1) の証明

$$f(x+(a_1+a_2)d) = f(x) + f'(x)(a_1+a_2)d$$

$$f(x+a_1d+a_2d) = f(x+a_1d) + f'(x+a_1d)(a_2)d$$

$$= f(x) + f(x)(a_1)d + \{f'(x) + f'(x)(a_1)d\}(a_2)d$$

$$= f(x) + \{f(x)(a_1) + f'(x)(a_2)\}d$$