7月26日

極値

2回続けて偏微分した折に順序によらない

$$\boxed{\begin{gathered} f: \mathbb{R}^n \to \mathbb{R}^m \\ \frac{\partial}{\partial x_j}\frac{\partial f}{\partial x_i}(x) = \frac{\partial}{\partial x_i}\frac{\partial f}{\partial x_j}(x) \end{gathered}}$$

$$\frac{\partial f}{\partial x_i}(x)$$

$$f(x + d_i e_i) - f(x) = \frac{\partial f}{\partial x_i}(x) d_i \qquad d_j, d_i \in D$$

$$\frac{\partial f}{\partial x_i} \quad \mathbb{R}^n \to \mathbb{R}^m$$

$$\boxed{\frac{\partial f}{\partial x_i}(x + d_j e_j) - \frac{\partial f}{\partial x_i}(x) = \frac{\partial}{\partial x_j}\frac{\partial f}{\partial x_i}(x) d_j}$$

両辺に d_i を掛ける

$$\frac{\partial f}{\partial x_i}(x + d_j e_j) d_i - \frac{\partial f}{\partial x_i}(x) d_i = \frac{\partial}{\partial x_j}\frac{\partial f}{\partial x_i}(x\, d_i d_j)$$

$$\{f(x + d_j e_j + d_i e_i) - f(x + d_j e_j)\} - \{f(x + d_i e_i) - f(x)\}$$

$$f(x + d_j e_j + d_i e_i) - f(x + d_j e_j) - f(x + d_i e_i) + f(x)$$

$$\frac{\partial}{\partial x_j}\frac{\partial f}{\partial x_i}(x) d_i d_j = \frac{\partial}{\partial x_i}\frac{\partial f}{\partial x_j}(x) d_i d_j$$

等しい

極値　$f: \mathbb{R} \to \mathbb{R}$

必要条件　$f'(x) = 0$　十分条件　$f''(x) > 0$ 極小

$f''(x) < 0$ 極大

Ⅰ (1) $f(x,y) = 2 - 3(x+y) + x^3 + y^3$

(2) $f(x,y) = x^3 + y^3 - 9xy + 27$

(3) $f(x,y) = x^4 + y^4 - 3(x-y)^2$　の極大．極小

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = 0$$

$f: \mathbb{R}^2 \to \mathbb{R}$

fがxで極大 $\Leftrightarrow$ 任意の$a \in \mathbb{R}^2$に対して

$g_a : t \in \mathbb{R} \to f(x + ta)$が$t=0$で極大

$g''_a(0) < 0$

$f' : \mathbb{R}^2 \to L(\mathbb{R}^2 ; \mathbb{R})$

$$f''(x) \in \underset{\substack{\| \\ L(\mathbb{R}^2, \mathbb{R}^2, \mathbb{R})}}{\underline{L(\mathbb{R}^2, L(\mathbb{R}^2, \mathbb{R}))}}$$

$L(\mathbb{R}^2, \mathbb{R}^2, \mathbb{R}) \leftarrow \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$ への2重線型写像の全体

Ⅱ　$g''_a(0) = f''(x)(a, a)$ を示せ

$$(a_1, a_2) \underbrace{\begin{bmatrix} \frac{\partial}{\partial x}\frac{\partial f}{\partial x}(x), & \frac{\partial}{\partial y}\frac{\partial f}{\partial x}(x) \\ \frac{\partial}{\partial x}\frac{\partial f}{\partial y}(x), & \frac{\partial}{\partial y}\frac{\partial f}{\partial x}(x) \end{bmatrix}}_{f''(x)} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

$$= a_1^2 \frac{\partial^2 f}{\partial x^2}(x) + 2a_1 a_2 \frac{\partial f}{\partial x \partial y}(x) + a_2^2 \frac{\partial^2 f}{\partial y^2}(x) < 0$$　$a_2 \neq 0$　a_2^2で割る

$$\left(\frac{a_1}{a_2}\right)^2 \frac{\partial^2 f}{\partial x^2}(x) + 2\frac{a_1}{a_2}\frac{\partial f}{\partial x \partial y}(x) + \frac{\partial^2 f}{\partial y^2}(x) < 0$$　$\frac{a_1}{a_2} = t$

判別式 $D < 0 \to$ 極大