No. 5/30(火)
Date 西村

第7回数理科学IIIA

Seifert − van Kampen の定理

knot theory (結び目理論) (筑波大学 石井 丹下)

$I=[0,1]$ a simple closed curve in Euclidean 3-space

$t : I \to \mathbb{R}^3 \quad t(0)=t(1)$ 分類

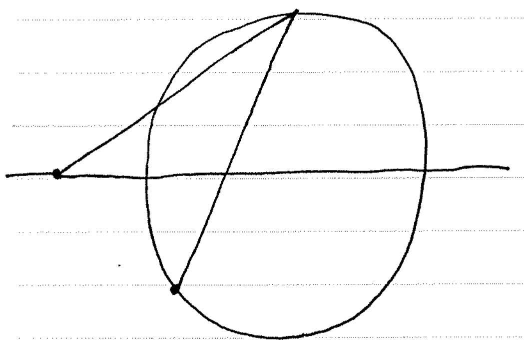
ex) 線形代数 $P^{-1}AP=B$ Jordon標準形

有限生成のAbel群 ～同型

k_1, k_2 contained in $\mathbb{R}^3$ are equivalent
if $\exists$ orientation-preserving homeomorphism $h : \mathbb{R}^3 \to \mathbb{R}^3$
with $h(k_1)=k_2$

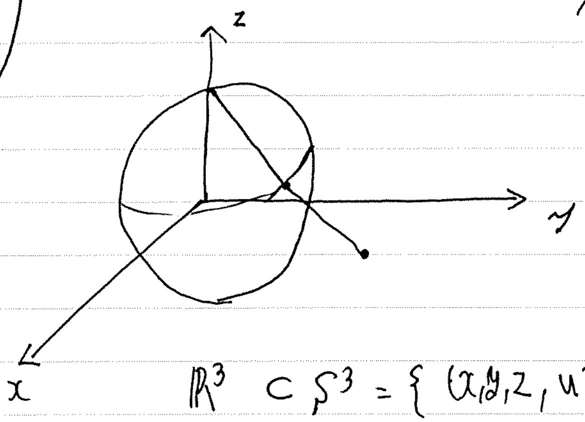
$\mathbb{R}^3 - k_1 \qquad \mathbb{R}^3 - k_2$ homeomorphic 同相

$\pi(\mathbb{R}^3 - k_1) = \pi(\mathbb{R}^3 - k_2)$



$\mathbb{R} \subset S^1$ 1点 compactification

$\mathbb{R}^2 \subset S^2 = \{(x,y,z)\in\mathbb{R}^3 \mid x^2+y^2+z^2=1\}$



$\mathbb{R}^3 \subset S^3 = \{(x,y,z,u)\in\mathbb{R}^4 \mid x^2+y^2+z^2+u^2=1\}$

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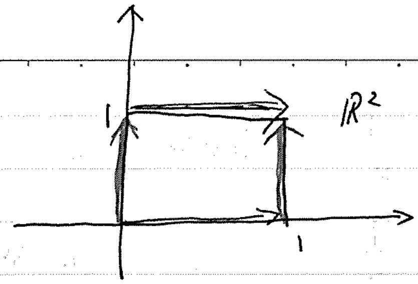
torus knots

torus

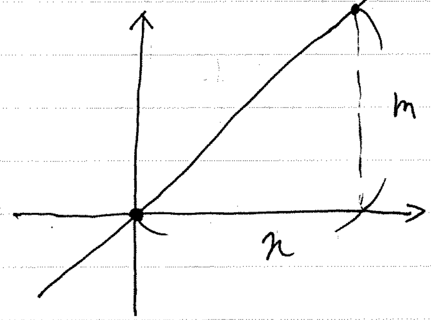


$(x,y) \sim (x',y')$
$x-x' \in \mathbb{Z}$
$y-y' \in \mathbb{Z}$

$\mathbb{R}^2$

L line through the origin with slope m/n $(1<m<n)$

m と n は relatively prime (互いに素)



torus knot of type (m,n)

目標 → $(2,5)$ $(7,9)$ は同じではない。

torus knot of type (m,n) $\quad S^3$

$A=\{(x_1,x_2,x_3,x_4)\in S^3 \mid x_1^2+x_2^2 \le x_3^2+x_4^2\}$

$B=\{(x_1,x_2,x_3,x_4)\in S^3 \mid x_1^2+x_2^2 \ge x_3^2+x_4^2\}$

A, B : closed subsets of S^3 with $A\cup B = S^3$

$A\cap B = \{(x_1,x_2,x_3,x_4)\in S^3 \mid x_1^2+x_2^2 = x_3^2+x_4^2 = \frac{1}{2}\}$

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$x_1^2+x_2^2=\frac{1}{2}$ circle in the (x_1,x_2) plane

$x_3^2+x_4^2=\frac{1}{2}$ circle in the (x_3,x_4) plane

AもBも solid torus (homeomorphic ~~to the~~ product of a disc and a circle)

$D=\{(x_1,x_2)\in\mathbb{R}^2 : x_1^2+x_2^2\leqq\frac{1}{2}\}$ closed disc

$S^1=\{(x_3,x_4)\in\mathbb{R}^2 : x_3^2+x_4^2=\frac{1}{2}\}$ circle

radius $\frac{\sqrt{2}}{2}$
(半径)

$f: D\times S^1\to A$ ☆ report I 全単射であることを示せ。

$f(x_1,x_2,x_3,x_4)=(x_1,x_2,\sqrt{2}x_3(1-(x_1^2+x_2^2))^{\frac{1}{2}},\sqrt{2}x_4(1-(x_1^2+x_2^2))^{\frac{1}{2}})$
continuous

$A\cap B$

k torus knot of type (m,n)

$S^3-K=(A-K)\cup(B-K)$

$A-K, B-K, (A-K)\cap(B-K)$: arcwise connected

Seifert - van kampen の定理.

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小さな $\varepsilon>0$ AとBは deformation retracts of U and V

N: tubular neighborhood of K of radius ε
チューブ状の近傍

S^3-N deformation retract of S^3-K

U, V $\frac{1}{2}\varepsilon$ neighborhoods of A and B respectively.

U, V homeomorphic to the product of an open disc and circle

$U\cap V$ thickened torus
the product of $A\cap B$ and the open interval $(-\frac{1}{2}\varepsilon,\frac{1}{2}\varepsilon)$

$S^3-N=(U-N)\cup(V-N)$
$\pi(S^3-N)=\pi(S^3-K)$

$\left.\begin{array}{l}U-N\\V-N\end{array}\right\}$ homotopy type of a circle 示せ II

無限巡回群

$\left.\begin{array}{l}(U-N)\cap(V-N)=(U\cap V)-N\\(A-K)\cap(B-K)=(A\cap B)-K\end{array}\right\}$ same homotopy type

$(A-N)\cap(B-N)=(A\cap B)-N$ deformation retract of each of these spaces.

$(A\cap B)-K$ subset of $\underbrace{A\cap B}_{\text{torus}}$ homeomorphic to the product of a circle and an open interval

$\left.\begin{array}{l}\varphi_1:\pi(U\cap V-N)\to\pi(U-N)\\\varphi_2:\pi(U\cap V-N)\to\pi(V-N)\end{array}\right\}$ これを決定せよ III

$\boxed{\text{命題 torus knot of type (m,n) } \{\alpha,\beta\} \text{ で生成される } \alpha^m\beta^n \text{ 証明}}$