

Category (ある種の代数系) 準同型

functor (関手)

$$\mathbb{C} \xrightarrow[\quad]{F} \mathbb{D}$$

natural transformation (自然変換)

$$\begin{array}{ccc} \mathbb{C} & \xrightarrow[\downarrow \beta]{F} \mathbb{D} & \xrightarrow[\downarrow \gamma]{F'} \mathbb{E} \\ & \downarrow \beta & \downarrow \gamma \\ & \mathbb{C} & \xrightarrow[\downarrow \beta]{G} \mathbb{D} & \xrightarrow[\downarrow \gamma]{G'} \mathbb{E} \end{array}$$

horizontal composition $\alpha' \circ \alpha$
 $(\beta' \circ \alpha') \circ (\beta \circ \alpha) = (\beta' \circ \beta) \circ (\alpha' \circ \alpha)$

$$\alpha: F \Rightarrow G \quad \beta \circ \alpha \quad \alpha': F' \Rightarrow G'$$

$$\beta: G \Rightarrow H$$

vertical composition

\mathbb{C}, \mathbb{D} : categories

$$\mathbb{C} \times \mathbb{D}$$

$$(A, B) \quad A', A \in \mathbb{C} \quad B', B \in \mathbb{D}$$

$$(f' \circ f, g' \circ g)$$

$$(A, B) \xrightarrow{(f, g)} (A', B') \xrightarrow{(f', g')} (A'', B'')$$

$$f: A \rightarrow A' \text{ in } \mathbb{C}$$

$$g: B \rightarrow B' \text{ in } \mathbb{D}$$

$\mathbb{C}, \mathbb{D}, \mathbb{E}$: categories

$$F: \mathbb{C} \times \mathbb{D} \rightarrow \mathbb{E} \text{ functor}$$

$B \in \mathbb{D}$ を固定

$$A \xrightarrow{f} A'$$

$$F^B: \mathbb{C} \rightarrow \mathbb{E}$$

$$F^B(A) = F(A, B)$$

$$F^B(f) = F(f, \text{id}_B)$$

$$B \xrightarrow{g} B' \text{ in } \mathbb{D}$$

$$(\alpha^g)_A = F(\text{id}_A, g)$$

$$F^B(A) = F(A, B) \xrightarrow{(\alpha^g)_A} F(A, B') = F^{B'}(A)$$

$\alpha^g = \text{natural transformation}$

$$\begin{array}{ccc} F^B(f) = F(f, \text{id}_B) & \downarrow & F(f, \text{id}_{B'}) \\ F^B(A') = F(A', B) & \xrightarrow{(\alpha^g)_{A'}} & F(A', B') = F^{B'}(A') \end{array}$$

$$F^{B'}(A') = F(A', B') = F^{B'}(A') \xrightarrow{F(\text{id}_{A'}, g)} F(f, \text{id}_{B'}) \circ F(\text{id}_A, g)$$

$$\forall B \in \mathbb{D} \quad F^B: \mathbb{C} \rightarrow \mathbb{E} \text{ functor}$$

$$= F(f, g)$$

$$\forall B \xrightarrow{g} B' \text{ in } \mathbb{D} \quad \alpha^g: F^B \Rightarrow F^{B'} \text{ natural transformation}$$

$$F(A, B) = F^B(A)$$

$$A \xrightarrow{f} A' \quad B \xrightarrow{g} B'$$

$$F(f, g) =$$

$$\begin{array}{ccc} F(A, B) = F^B(A) & \xrightarrow{F^B(f)} & F^B(A') \\ (\alpha^g)_A \downarrow & & \downarrow (\alpha^g)_{A'} \\ F(A, B') = F^{B'}(A) & \rightarrow & F^{B'}(A') \end{array}$$

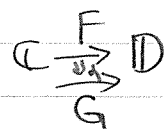
Sets

objects : categories. } category Cat
morphisms : functors

Cat(C, D) category

Sets(A, B)

objects : CからDへの functors

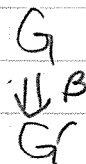
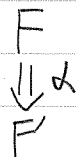


morphism : natural transformation.

II bifunctor になっていることを確認して下さい。

$$\text{Cat}(C, D) \times \text{Cat}(D, E) \xrightarrow{G \circ F} \text{Cat}(C, E)$$

horizontal composition



pod

Yoneda lemma

米田信夫

Category 1940年代

代数の位相幾何学

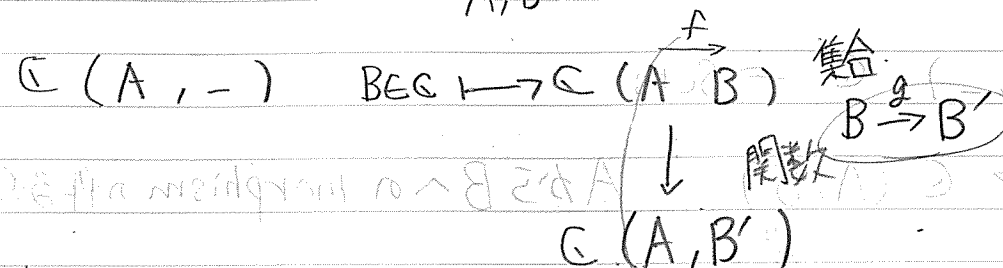
category

Morita duality

abstract nonsense

$A \in C$ 固定

$A, B \in C$



$C \rightarrow \text{Set}$

functor $C(A, B)$

$C(-, B)$ $C(A, B)$ $B \in C$

$C(A, B)$

$g \circ f$

$A \xrightarrow{f} A' \xrightarrow{g} B$

contra-variant functor

$C^{\text{op}} \rightarrow D$

$C \rightarrow D$

C^{op}