第10回 数理科学IIIA

Category (ある種の代数系)　　準同型

functor (関手)

$$\begin{tikzcd} \mathbb{C} \arrow[r, bend left, "F"] \arrow[r, bend right, "G"'] \arrow[r, phantom, "\Downarrow\alpha"] & \mathbb{D} \end{tikzcd}$$

natural transformation (自然変換)

$$\begin{tikzcd} \mathbb{C} \arrow[r, bend left=50, "F"] \arrow[r, "G" description] \arrow[r, bend right=50, "H"'] \arrow[r, phantom, shift left=3, "\Downarrow\alpha"] \arrow[r, phantom, shift right=3, "\Downarrow\beta"] & \mathbb{D} \arrow[r, bend left=50, "F'"] \arrow[r, "G'" description] \arrow[r, bend right=50, "H'"'] \arrow[r, phantom, shift left=3, "\Downarrow\alpha'"] \arrow[r, phantom, shift right=3, "\Downarrow\beta'"] & \mathbb{E} \end{tikzcd}$$

horizontal composition　$\alpha'\circ\alpha$

$(\beta'\circ\alpha')\circ(\beta\circ\alpha) = (\beta'\circ\beta)\circ(\alpha'\circ\alpha)$

$\alpha : F \Rightarrow G$　　$\beta\circ\alpha$　　$\alpha' : F' \Rightarrow G'$

$\beta : G \Rightarrow H$

vertical composition

$\mathbb{C}, \mathbb{D}$: categories

$\mathbb{C}\times\mathbb{D}$　　(A, B)　　$A', A \in \mathbb{C}$, $B', B \in \mathbb{D}$　　$(f'\circ f, g'\circ g)$

$$(A, B) \xrightarrow{(f,g)} (A', B') \xrightarrow{(f',g')} (A'', B'')$$

$f : A \to A'$ in $\mathbb{C}$

$g : B \to B'$ in $\mathbb{D}$

$\mathbb{C}, \mathbb{D}, \mathbb{E}$: categories

$F : \mathbb{C}\times\mathbb{D} \to \mathbb{E}$ functor

$B \in \mathbb{D}$ を固定　　$A \xrightarrow{f} A'$

$F^B : \mathbb{C} \to \mathbb{E}$　　$F^B(A) = F(A, B)$

$F^B(f) = F(f, id_B)$

$B \xrightarrow{g} B'$ in $\mathbb{D}$　　$(\alpha^g)_A = F(id_A, g)$

α^g : natural transformation

$$\begin{tikzcd} F^B(A) = F(A,B) \arrow[r, "(\alpha^g)_A"] \arrow[d, "F^B(f) = F(f, id_B)"'] \arrow[dr, phantom, "\circlearrowleft"] & F(A, B') = F^{B'}(A) \arrow[d, "F(f, id_{B'}) = F^{B'}(f)"] \\ F^B(A') = F(A', B) \arrow[r, "(\alpha^g)_{A'}"'] & F(A', B') = F^{B'}(A') \end{tikzcd}$$

$A \xrightarrow{f} A'$

$F(id_{A'}, g)$

$F(f, id_{B'})\circ F(id_A, g)$

bi functor

$= F(f, g)$

(I) $\mathbb{C}\to\mathbb{E}$ への bifunctorになることのproof)

逆に

$\forall B \in \mathbb{D}$　$F^B : \mathbb{C} \to \mathbb{E}$ functor

$\forall B \xrightarrow{g} B'$ in $\mathbb{D}$　$\alpha^g : F^B \Rightarrow F^{B'}$ natural transformation

$F(A, B) = F^B(A)$　　$A \xrightarrow{f} A'$　　$B \xrightarrow{g} B'$

$F(f, g) =$

$$\begin{tikzcd} F(A,B) = F^B(A) \arrow[r, "F^B(f)"] \arrow[d, "(\alpha^g)_A"'] & F^B(A') \arrow[d, "(\alpha^g)_{A'}"] \\ F(A, B') = F^{B'}(A) \arrow[r] & F^{B'}(A') \end{tikzcd}$$

No.

Date

Sets

objects : categories. } category $\mathbb{C}at$

morphisms : functors

category

$\underline{\mathbb{C}at(\mathbb{C},\mathbb{D})}$ Sets(A, B)

└ objects : $\mathbb{C}$から$\mathbb{D}$への Functors $\mathbb{C} \overset{F}{\underset{G}{\Downarrow\alpha}} \mathbb{D}$

morphism : natural transformation.

☆ Ⅱ bifunctor になっていることを確認して下さい。

$$\underset{\alpha}{\underline{\mathbb{C}at(\mathbb{C},\mathbb{D})}}^{F} \times \mathbb{C}at(\mathbb{D},\mathbb{E})^{G} \longrightarrow \mathbb{C}at(\mathbb{C},\mathbb{E})^{G\circ F}$$

horizontal composition

$F \overset{\Downarrow\alpha}{} F'$ $\quad$ $G \overset{\Downarrow\beta}{} G'$ $\quad$ $\beta\circ\alpha$

No. (水)

Date

米田信夫 $\quad$ Category 1940年代

$\underline{\text{Yoneda lemma}}$ $\quad$ 森田紀一 → 森田 $\quad$ 代数的位相幾何学

$\mathbb{C}$: category $\quad$ Morita duality

abstract non sense

$A \in \mathbb{C}$ 固定

$A, B \in \mathbb{C}$

$\mathbb{C}(A, -)$ $\quad B \in \mathbb{C} \longmapsto \mathbb{C}(A \xrightarrow{f} B)$ 集合

$\downarrow$ 関数 $\quad B \xrightarrow{g} B'$

$\mathbb{C}(A, B')$

$\mathbb{C} \longrightarrow Set$ $\quad g\circ f$

functor

$\mathbb{C}(\_, B)$ $\quad \mathbb{C}(A,B)$ $\quad B \in \mathbb{C}$

$\mathbb{C}(f, id) \uparrow$

$\mathbb{C}(A', B)$ $\quad g\circ f$

$A \xrightarrow{f} A' \xrightarrow{g} B$

Contravariant functor.

$\mathbb{C}^{op} \to \mathbb{D}$ $\quad \mathbb{C} \to \mathbb{D}$ $\quad \mathbb{C}^{op}$