# BEHIND SRAFFA'S SILENCE The Transformation Problem in Retrospect

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This is a synopsis of my book *Behind Sraffa's Silence* which was written in Japanese and published in December 2001. The book contains a number of mathematical formulae, which were necessary for me to formulate the controversy over the transformation problem and to support my arguments as well. I suspect, however, that some of the readers not familiar with the mathematical gadgets such as vectors, matrices, etc., might not only be perplexed by them, but also find it difficult to see the points I wanted to make. My prospect was that a brief summary written in English with no mathematical formulae should be included in the book. But when I proposed my intention to the publisher, it was too late to be included therein. The synopsis I will publish here as an independent article.

### Chapter 1 The Traps Laid by Bortkiewicz

The controversy over the transformation problem has now gone on over a hundred years. The crucial question at the heart of the transformation problem is, as was formulated by Engels in 1885, how an equal average ratio of profit can and must come out on the basis of the law of value. Since Bortkiewicz claimed that the equal average rate of profit and the relative

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prices of commodities could, without any magnitude of value and surplus value, be obtained mathematically, the primacy of mathematical method over Marx's has appealed even to supporters of Marxian economics.

As was claimed by Bortkiewicz, Marx's alleged postulate of the equality of total profit with total surplus value is incompatible with his other postulate requiring the equality of total value with total price of production. To show that the profit consists in the exploitation of labor, another way of explanation was needed —an explanation which demonstrates the existence of surplus value as the only origin of profit.

As an answer to this, Okishio offered in 1963 the now so called Fundamental Marxian Theorem, which in its turn presupposes the labor theory of value. Okishio and other Marxist economists take it for granted that the labor theory of value can be formulated in the value-determining equation. Actually the Fundamental Marxian Theorem follows merely from the value-determining equation, provided the Hawkins-Simon condition is satisfied.

The reason why the Fundamental Marxian Theorem has not been accepted is, I suggest, that the value-determining equation is itself begging a question. In the next chapter we will focus on the value-determining equation.

# Chapter 2 Sraffa's Way Out

The second stage of the controversy over the transformation problem started with the problem of a multi-product system which could produce a commodity with a negative value. The problem was foreshadowed by Sraffa's *Production of Commodities by Means of Commodities* (1960), and culminated in Steedman's rebuttal against the Fundamental Marxian Theorem. Steedman, in his *Marx After Sraffa* (1977), exemplified a case in

which some commodity has a negative value with a positive price. Sraffa, however, passed over this controversy in silence. I asked myself what was the reason behind his silence.

The points I propose to make in this chapter are the following three.

- (1) Sraffa's standard commodity as "an invariable measure of value" dispenses with Marxian value—the value as the *Schwerpunkt* of, or the first approximation to, the price of production.
- (2) The so-called value-determining equation which calculates a negative value is mathematically not discriminable from a special case of Sraffa's production equation—the case in which the rate of profit is zero.
- (3) In the sense in which one of the joint products has a negative value, the "value" is not the value which Marx required to be always positive.

The thee points taken together will reveal the reason why Sraffa was reluctant to take part in the controversy. The reason, I conclude, is that on the one hand "there is no transformation problem." And on the other, such an attempt as that of Steedman who tried to refute the Marxian concept of value on the basis of Sraffian economics turns out to be impossible—logically impossible, because the value-determining equation which calculates "a negative value with a positive price" proves to be nothing but a Sraffa's production equation in which the rate of profit is zero.

### Chapter 3 The Problem Solved

The assumptions recurring in the controversy over the transformation problem are the following:

- (a) Divergence of production-prices from values.
- (b) Existence of an equal rate of profit.
- (c) Validity of the value-determining equation.
- (d) Inequality of the organic composition of capital.

The premises of the dispute are (b) and (d), which added by (c) entail (a). But (a), in general, does not permit to hold the Marxian proposition that the total profit equals to the total surplus value. The proposition is guaranteed to hold if only the product of each industry is proportional, in an economic system as a whole, to the quantity of it used up as a means of production. This condition is satisfied by transforming an actual system with the property (d) into a standard system which would produce only one standard commodity. That is to say, if we transform an actual system in which the organic composition of capitals is unequal to one another into the standard system, both of the two Marxian propositions at issue obtain.

Any solution which suffices to warrant both of the Marxian propositions has to make use of a procedure of standardization. Thus Morishima and Catefores, in their treatment of this problem, proved the Marxian propositions by adjusting the quantities of the production of commodities, and U. Krause, in his system of "standard reduction of labor," re-evaluated the concrete labors in order to get the standardized abstract labor.

The mathematical tools of linear algebra, unknown to the age of Marx, made "the average commodity" and "the average industry," by which Marx intended to represent an economic system as a whole, obsolete. The "value" was indispensable to Marx, not only for the purpose of disclosing the craziness and antagonism in the capitalist society, but also for the determination of the prices of individual commodities. This was the original sin of the transformation problem. With Sraffa's device of standard system comes the transformation problem to a close.

## Chapter 4 Okishio's Theory Reconsidered

In this chapter Okishio's theory as to the problem of whether the general rate of profit in the long term has the tendency to fall down is discussed in relation to his solution of the transformation problem. According to Okishio's answer to the transformation problem, Marx's postulate of the equality of total profits with total surplus values does not hold. This result makes Okishio's discussion of the problem of the falling rate of profit extremely complicated, because it makes some of the calculi in terms of value irrelevant, and therefore unavailable.

My argument in this chapter is that the reverse of each of Okishio's mathematical theorems proves to be valid as well, and that each of his theorems as well as its reverse can be embedded in Sraffa's system of economic theory. The merit of Sraffa's system consists in the fact that it enables one to observe an actual economic system as a whole "as in a vacuum." My argument not only removes a delusive aspect of Okishio's theory on the falling rate of profit, but contributes to its simplification.

By making use of the device of Sraffa's standard system as a method of scientific idealization, we can restore Marx's postulate of the equality of total profits to total surplus value. The latter guarantees to identify Marx's concept of value with Sraffa's concept of price, as far as an economic system as a whole is concerned. Looked at in this perspective, the problem of the falling rate of profits has no definite answer, that is, there is no simple relation to be found between the general rate of profit and the successive processes of reswitching to more profitable methods of production.

# Appendix Wittgenstein and Sraffa

In the preface of his *Philosophical Investigations* Wittgenstein avowed his indebtedness to the criticism which Sraffa practiced for many years on his thought. This has provoked the conjecture that there must be some similarities between the philosophy of the later Wittgenstein and the economic theory of Sraffa; but no agreement has ever been reached on what

the similarities between them are. Far from that, Sraffa's *Production of Commodities by Means of Commodities* has some likeness, not to the philosophy of the later Wittgenstein, but rather to that of the *Tractatus Logico-Philosophicus*.

Wittgenstein left us only two sayings about Sraffa's influence upon him. One was that what he gained from talking to Sraffa was an "anthropological" way of looking at philosophical problems. The other was that his discussions with Sraffa "made him feel like a tree from which all branches had been cut." I believe that both of them are true, but they are too general to help us to specify the alleged similarities. Still less can we draw the conclusion that Wittgenstein's later philosophy has some tinge of anthropology as a discipline. His obsession with ethical and religious matters did not permit him to indulge in any kind of social studies, I suggest.

The point I want to make in this appendix is this. The anthropological point of view he gained from Sraffa was a naturalistic way of looking at our language. He transformed it into his analytical tool which he called "language games." This standpoint makes any searching for the similarities between them basically unsuited to his philosophy, because it presupposes the "craving for generality" which, according to him, is "the real source of metaphysics." It was this presupposition which Wittgenstein has time and again tried to dissuade us to make. What Wittgenstein and Sraffa have in common is, I definitely believe, the attitude against any swollen thought whatsoever. Boasting was the abhorrence to them both.