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HYBRID FINITE ELEMENT MODELING FOR SEISMIC STRUCTURAL RESPONSE ANALYSIS OF A REINFORCED CONCRETE STRUCTURE

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For rational seismic structural response analysis of a reinforced concrete structure, this paper presents a solid element in which a sophisticated concrete constitutive relation and cracking functionality are implemented. Hybrid finite element modeling that uses solid and beam elements for concrete and steel rebar is proposed, made tougher with a method of constructing the hybrid finite element. Well-balanced modeling is possible by first generating beam elements for the steel rebars and then generating solid elements for the concrete with nodes of the beam elements being shared by the solid element. A numerical experiment was carried out for a reinforced concrete column subjected to unilateral loading, in order to examine the potential applicability of the hybrid finite element modeling. The computed results are compared with the experimental data, and the non-linear relation between the displacement and reaction force is reproduced to some extent.

Keywords: seismic structural response analysis; reinforced concrete structure; hybrid element; concrete constitutive relation; crack propagation/generataion

1. Introduction

A structure has been designed that exhibits sufficient performance when it is subjected to ground motion. Numerical simulation is used to estimate the seismic performance of the structure. Higher accuracy is needed if it is to serve as a substitute for experimental evaluation of structural components. Numerical simulation of

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structural seismic response is the toughest, because it is dynamic and sometimes analyzes local or overall failure.

The most idealized problem setting for structural seismic response analysis is summarized as follows: 1) analysis of the interaction between structure and soil; 2) analysis of a possible occurrence of local damage or failure, and 3) analysis of the remaining capacity for to prepar for a large aftershock. The choice of input ground motion is necessary; a set of ground motions is given for the design purposes of a new structure, and various ground motions are used in damage evaluation of an existing structure.

The use of solid element finite element analysis is a unique solution to fulfill the above three requirements. However, a large analysis domain and the finest meshing are required in order to meet the first and second requirements. Suitable constitutive relations that cover linear and non-linear regimes of the material behavior have to be implemented for the second and third requirements. It is no wonder that the analysis model needed is of the largest scale; if the element size is of the order of 10^{-1} m to capture local damage and the dimension of the analysis domain that includes soil is of the order of 10^2 m, the number of elements will be 10^9.

A reinforced concrete (RC) structure needs special treatments. RC is a composite of concrete and steel of different material characteristics. An RC element could be developed that accounts for the overall material properties of RC. The properties depend on many parameters, such as the material properties of concrete and steel and the spatial arrangement of the reinforcement bars that includes hoop steel bars. It is not a simple task to develop a versatile element for RC. One solution is to use distinct solid elements for the concrete and steel. This treatment of RC has high applicability with various combination of concrete and steel. One drawback is the increase in the number of solid elements, since the radius of ordinary steel rebar is a few centimeters and around 10 elements are needed for accurate modeling of the bar.

One compromise is the use of a hybrid finite element, i.e., the combined use of a solid element and beam element for concrete and steel, respectively. This is logical since bending, one critical force induced by seismic motion, is carried out by the compressive forces of massive concrete and the tensile forces of the steel rebar, which are accurately computed by using solid elements and beam elements, respectively. Shearing, another critical force, is carried out by the concrete, and solid elements of concrete can be used to calculate this. A difficulty arises when combining the concrete solid elements and the steel rebar beam elements; for instance, nodes are shared by the different elements so that the displacement function becomes continuous.

In this paper, we propose hybrid finite element modeling of seismic structural response analysis for an RC structure. While various elements are developed for beams, a solid element of concrete is not available. We thus first developed a solid element in which suitable constitutive relations of concrete are implemented and which is able to analyze meso- or macro-cracking; the location and thickness of the cracking is an index of the damaged conditions of concrete used in an experiment.

The present paper is organized as follows: In Section 2, we present a solid element for concrete; the most sophisticated constitutive relations of concrete that were proposed by Maekawa and his colleagues are implemented, and functionality of cracking in an element is implemented. In Section 3, a method of combining solid elements and beam elements is discussed. We decided to first model steel reinforcement in terms of beam elements and then model concrete in terms of solid elements. A simple numerical experiment is carried out in Section 4. The numerical convergence is examined for a model that is constructed according to the proposed hybrid finite element modeling. A comparison with experimental data indicates the satisfactory performance of the modeling.

2. Elasto-Plastic Cracking Solid Element for Concrete

We developed a solid element for concrete for which the most sophisticated constitutive relations and the functionalities of cracking or the generation of displacement discontinuity are implemented. The element is linear in the sense that it has constant strain and stress within the element, and the configuration is tetrahedral; a hexagonal element is possible but the direction of the crack surface is fixed in this configuration.

2.1. *Maekawa's concrete constitutive relation*

Maekawa and his collaborators have developed non-linear concrete constitutive relations, which account for both elasto-plasticity and damage taking place in concrete. The relation is formulated in terms of a strain increment, $\mathrm{d}\boldsymbol{\epsilon}$, which is decomposed into an elastic part and plastic part, i.e., $\mathrm{d}\boldsymbol{\epsilon} = \mathrm{d}\boldsymbol{\epsilon}^E + \mathrm{d}\boldsymbol{\epsilon}^P$. Like an ordinary constitutive relation, the elastic strain, $\boldsymbol{\epsilon}^E$, gives the stress increment,

$$\boldsymbol{\sigma} = \mathbf{c} : \boldsymbol{\epsilon}^E, \tag{1}$$

where $\mathbf{c}$ is an isotropic elasticity tensor that is a function of $\boldsymbol{\epsilon}^E$, and : stands for the second-order contraction. A special characteristic of concrete is the following relation between $\mathrm{d}\boldsymbol{\epsilon}^E$ and $\mathrm{d}\boldsymbol{\epsilon}^P$:

$$\mathrm{d}\boldsymbol{\epsilon}^P = \mathbf{l} : \mathrm{d}\boldsymbol{\epsilon}^E, \tag{2}$$

where $\mathbf{l}$ is a fourth-order tensor that is also a function of $\boldsymbol{\epsilon}^E$.

It is thus straightforward to derive an elasto-plastic constitutive relation which gives $\mathrm{d}\boldsymbol{\sigma}$ in terms of $\mathrm{d}\boldsymbol{\epsilon}$,

$$\mathrm{d}\boldsymbol{\sigma} = \mathbf{c}^{EP} : \mathrm{d}\boldsymbol{\epsilon}, \tag{3}$$

where $\mathbf{c}^{EP}$ is the elasto-plasticity tensor, defined as

$$\mathbf{c}^{EP} = \mathbf{c} + \left(\boldsymbol{\nabla}\mathbf{c} : \mathrm{d}\boldsymbol{\epsilon}^E\right) : (\mathbf{I} + \mathbf{l})^{-1}, \tag{4}$$

Here, $\boldsymbol{\nabla}\mathbf{c}$ is the derivative of $\mathbf{c}$ with respect to $\boldsymbol{\epsilon}^E$, $\mathbf{I}$ is the fourth-order symmetric identity tensor, and $(.)^{-1}$ stands for an inverse tensor of a fourth-order tensor $(.)$.

The elasto-plasticity tensor, $\mathbf{c}^{EP}$, becomes non-symmetrical and non-positive definite, as $\boldsymbol{\epsilon}^E$ increases. In Maekawa's constitutive relation, this is modeled as the decrease in Young's modulus, which determines $\mathbf{c}$ as a function of $\boldsymbol{\epsilon}^E$. Also, $\mathbf{l}$, which gives $\mathrm{d}\boldsymbol{\epsilon}^P$ in terms of $\mathrm{d}\boldsymbol{\epsilon}^E$, changes as $\boldsymbol{\epsilon}^E$ changes, so that the principle directions of $\mathrm{d}\boldsymbol{\epsilon}^P$ are parallel to those of the deviatoric part of $\boldsymbol{\epsilon}^E$. The computation of Young's modulus and $\mathbf{l}$ is complicated, even though it is explicitly given. Computing the inverse of the fourth-order tensor, $(\mathbf{I}+\mathbf{l})^{-1}$, is time consuming; the tensor is converted to a six-by-six matrix, and the inverse of the matrix (which requires tedious computation) is used to compute the inverse of the tensor. A more suitable algorithm is developed to compute $\mathbf{c}^{EP}$.

2.2. *Cracking functionality*

It is essential to consider the multiple cracking that takes place in concrete due to tension, shearing, and compression. Tensile stress is a major factor that induces cracking, and the growth of cracks that lead to local and global failure is a key in analyzing the damage to and collapse of an RC structure. Cracking is displacement discontinuity or a facet across which displacement is no longer continuous. Numerical analysis of cracking is generally difficult since its first assumption is the continuity and smoothness for a target function.

We have developed a new discretization scheme, called the Particle Discretization Scheme (PDS), to implement cracking functionality into a finite element method. PDS is formulated in terms of two sets of basis functions for discretization; one set is for a function and the other set is for its derivative. That is, for an analysis domain, V, let $\{\phi^\alpha\}$ and $\{\psi^\beta\}$ be a set of characteristic functions for Voronoi and Delaunay tessellations, and discretize the displacement and strain functions as

$$\begin{aligned} \mathbf{u}(\mathbf{x}) &= \sum_\alpha \mathbf{u}^\alpha \phi^\alpha(\mathbf{x}), \\ \boldsymbol{\epsilon}(\mathbf{x}) &= \sum_\beta \boldsymbol{\epsilon}^\beta \psi^\beta(\mathbf{x}), \end{aligned} \tag{5}$$

where $\{\mathbf{u}^\alpha\}$ and $\{\boldsymbol{\epsilon}^\beta\}$ are coefficients to be determined. By definition, $\mathbf{u}$ of Eq. (5) becomes discontinuous across all boundaries of the Voronoi blocks, and $\boldsymbol{\epsilon}$ of Eq. (5) takes on a constant value on each Delaunay tetrahedron.

PDS is easily implemented in the finite element method. To show this, we consider the simplest case when V is linearly elastic, with $\mathbf{c}$ and ρ being the elasticity tensor and density, respectively. We consider a Lagrangian of

$$\mathcal{L}[\mathbf{u},\boldsymbol{\epsilon}] = \int_V \frac{1}{2}\rho\dot{\mathbf{u}}\cdot\dot{\mathbf{u}} - \frac{1}{2}\boldsymbol{\epsilon}:\mathbf{c}:\boldsymbol{\epsilon}\,\mathrm{d}v, \tag{6}$$

subjected to minimizing $\int_V |\mathrm{sym}\boldsymbol{\nabla}\mathbf{u}-\boldsymbol{\epsilon}|^2\,\mathrm{d}v$, where $\cdot$ stands for the inner product,

$\dot{(.)}$ is the time derivative of a function $(.)$, and $|(.)|^2$ is the norm of a second-order tensor $(.)$. A matrix equation for unknown $\{\mathbf{u}^\alpha\}$ is readily derived from $\partial \int \mathcal{L}\,\mathrm{d}t = 0$.

The derivative of $\mathbf{u}$ from Eq. (5) has delta functions across the Voronoi boundary, and the volume integration over V is computable. We thus construct $\boldsymbol{\epsilon}$ associated with $\mathbf{u}$. When a crack is initiated on the boundary, the displacement is discontinuous and its derivative loses the delta functions. It automatically changes $\boldsymbol{\epsilon}$ or a linear relation between $\mathbf{u}^\alpha$ and $\boldsymbol{\epsilon}^\beta$. Thus, the matrix equation for $\{\mathbf{u}^\alpha\}$ is altered, as multiple-cracking takes place. While possible cracking locations are restricted to the Voronoi boundaries, we can evaluate the initiation of cracking using a given failure condition.

2.3. *Elasto-plasticity and cracking*

The original constitutive relations proposed by Maekawa and his colleagues have a special part for a tensile condition, in which cracking is modeled as the decrease in the capacity of carrying tensile stress. This part includes a relation between shear strain and stress. This part is described in an element-wise manner, as multiple cracks are smeared within the element.

PDS provides a solid element the functionality of initiating and growing a crack in an element, by satisfying a failure criterion which is described in a point-wise manner. It is possible to impose boundary conditions on the crack surfaces; by adding a suitable surface integral to $\mathcal{L}$ of Eq. (6), a matrix equation that accounts for the boundary conditions derived from $\partial \int \mathcal{L}\,\mathrm{d}t = 0$.

In the solid element developed for concrete, we make use of Maekawa's constitutive relation for the compressive part that is described in a point-wise manner. The tensile part is omitted, and PDS is used, assuming that multiple-cracking of PDS models the tensile part. The failure criterion is based on strength, i.e., a critical value for stress. The boundary conditions on the crack surfaces are basically traction free; the functionality of closing an open crack under suitable compression and increasing the stiffness is implemented.

3. Hybrid Finite Element Modeling

In general, there are two methods for creating an analysis model based on hybrid finite element modeling for an RC structure or structural component, in which solid and beam elements are used for concrete and steel rebar, respectively. The procedures for the first method are as follows: 1) construct a set of beam elements for steel rebar; and then 2) wrap the beam elements with solid elements of concrete. the nodes of the beam elements are used to construct the solid elements, and the continuity of displacement among neighboring solid elements and beam elements is assured. The second method takes the reverse way, i.e., 1) solid elements of concrete are first made for the entire structure or structural components, and then 2) beam elements for steel rebar are embedded in the solid elements. The nodes of the solid and beam elements are not shared, and Multi-Point Constraint (MPC) must be implemented.

The two methods explained above have advantages and disadvantages. Regarding the displacement continuity of the different elements, the first method is better because it forces the continuity by sharing the nodes. The second method needs a suitable MPC design; the simplest way is to put beam element nodes on the surface of the solid elements and to connect the beam element displacement to the solid element displacement on the corner of the surface via MPC. Regarding the ease of constructing an analysis model, the second method has higher flexibility since it starts with ignoring the presence of reinforcements. For densely arranged steel bars, the first method might have ill-configured elements when wrapping the connection of adjacent steel bars, often vertical and horizontal, with one solid element.

Generating solid elements with some conditions, such as generate the nodes on the break line, is a classical problem in the field of mesh generation for the finite element method; for instance, connection of two solid element models consisting of different materials so that the nodes on the interface are shared has been studied. Various algorithms have been developed and software is available. Connecting solid elements to beam elements with sharing nodes is relatively easy because the nodes of the beam elements are distributed in one direction for a given segment of a steel rebar. Free software can be used to generate a set of solid elements of sufficiently high quality. The procedures explained for the first method are actually carried out using *Gmsh*, a free three-dimensional finite element mesh generator.

Based on the above considerations, we adopted the first method for constructing an analysis model based on hybrid finite element modeling. The length of the beam elements that are first generated controls the size of the solid elements, since the edges of the solid elements for concrete are configured as the beam elements for the steel rebars; see Fig. 1. Since the main rebar and hoop bars are both modeled by beam elements, their intersection point is set as a shared node of the beam element; this simplifies the present hybrid finite element modeling. It should be noted that, while the nodal displacement and the force of a beam element readily satisfy the continuity of those of a solid element, it is not straightforward to consider the continuity of nodal rotation and the moment of the beam elements to nodal displacement and the force of the solid elements. In this paper, we use a truss element rather than a beam element, in order to avoid this continuity problem.

4. Numerical Experiment

A numerical experiment was carried out in order to examine the potential applicability of the proposed hybrid finite element modeling to an RC structure. An RC column subjected to unilateral loading was studied. The constructed analysis model based on the hybrid modeling is shown in Fig. 2. The material properties are summarized in Table 1. As for the boundary conditions, the bottom surface of the column is fixed, and the displacement on the top surface is prescribed, with the four side surfaces being traction-free. The Euler beam element is used for a steel rebar, and Delaunay tessellation is used to generate tetrahedral elements; nodal displacement represents the rigid body motion of a Voronoi tessellation, according to PDS.

It should be emphasized that the edge of tetrahedral elements is forced to coincide with a beam element when the beam element is wrapped by the tetrahedral element. This guarantees that the nodes of all beam elements are shared by neighboring tetrahedral elements.

We prepared four resolutions of beam and solid meshes from 25 to 200 mm, in order to examine the convergence of the solution; the meshes of the four models are presented in Fig. 3. The results are summarized in Fig. 4; the relation of the posed displacement and computed reaction force is plotted for the four models. It can be seen that more or less similar relations are obtained for different meshes. Numerical computation ceased at an early loading stage when small elements were used. An element size of 200 mm would be the minimum to carry out numerical computation in a stable manner. This is comparable with the size of a cylindrical specimen of concrete with which the constitutive relations are measured.

Iso-surfaces of equivalent stress (or $|\mathbf{s}|$ with $\mathbf{s}$ being the deviatoric part of $\boldsymbol{\sigma}$) are plotted for the four models in Fig. 5. The distribution of cracked solid elements within the analysis models is shown in Fig. 6; the red elements are the cracked element. It can be seen that the iso-surfaces and the cracked element distribution are similar to those of the four elements, even though some models ceased. This confirms that an element size of 200 mm would be the minimum for hybrid finite element modeling.

In Fig. 4, we plot the relation that is observed in the experiment. The numerical computation of hybrid finite element modeling accurately reproduces the reaction force near its peak, which implies that the hybrid element modeling is applicable to an RC structure. The discrepancy of results between the numerical analysis and the experiment could be accepted for practical purposes. On the other hand, the discrepancy increases after the peak, in which the main steel rebars start to yield. This might be a less proper treatment of crack surfaces of the solid element for concrete, to which traction-free boundary conditions are posed.

We replace the boundary conditions on the crack surface from a traction-free condition to a shear traction transfer condition (no axial traction transfer) via a suitable spring when the cracks are closed or the separated parts of the cracked concrete element re-contact. The use of a spring to model the shear stress transfer mechanism is adapted from Maekawa's concrete constitutive relation. The shear traction transfer condition could contribute to an increase in the maximum shear force carried by the RC column model. The results of analyzing the model by replacing the crack boundary conditions are presented in Fig. 7, in comparison with Fig. 4, for the displacement-reaction force relation. As can be seen, the maximum shear force is overestimated by this model, while it is underestimated by the previous model with a traction-free boundary condition on the crack surface. The equivalent stress iso-surface and the cracked element distribution are shown in Figs. 8 and 9, which are not the same as but similar to Figs. 5 and 6. We may conclude that posing a more suitable boundary condition on the crack surface increases the accuracy of predicting the maximum shear force of the RC column.

5. Concluding Remarks

In this paper, we first presented a solid element for concrete, in which the non-linear elasto-plasticity of concrete and cracking functionality are implemented. With the use of this solid element, hybrid finite element modeling becomes possible for an RC structure; solid and beam elements are used for concrete and steel rebar, respectively. A numerical experiment with an RC column shows the potential applicability of the present hybrid finite element modeling, since it reproduces the experimentally observed relation of displacement and reaction force.

As explained in the preceding section, boundary conditions on the cracked surface of a solid element for concrete are not established. A traction free boundary condition results in underestimating the maximum shear force, and too strong shear traction transfer results in its overestimation. A proper boundary condition, possibly a more suitable (linear or non-linear) spring, must be found. Hybrid finite element modeling that uses such a solid element with an improved crack boundary condition could contribute to an improvement in the accuracy and efficiency of numerical analysis of seismic structural response analysis for an RC structure.

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Table 1. Material properties

Material	Young modulus	Poisson ratio	Yield stress	Compressive strength
Concrete	21.8GPa	0.2	–	26.0 MPa
Rebars (D22)	205.0GPa	0.3	443.3MPa	–
Rebars (D10)	205.0GPa	0.3	385.0MPa	–

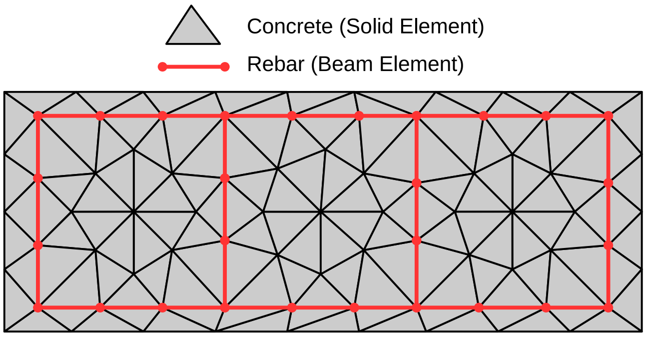

Fig. 1. Example of beam-solid hybrid mesh

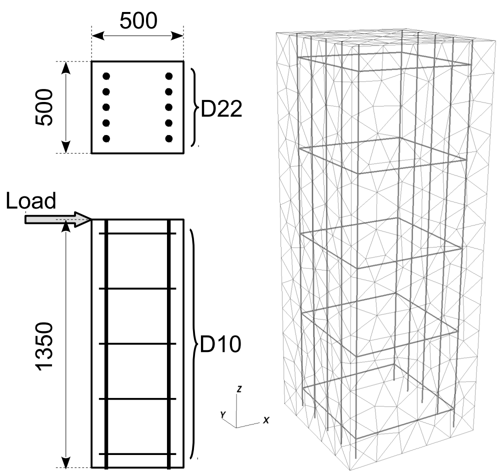

Fig. 2. Analysis model of RC column.

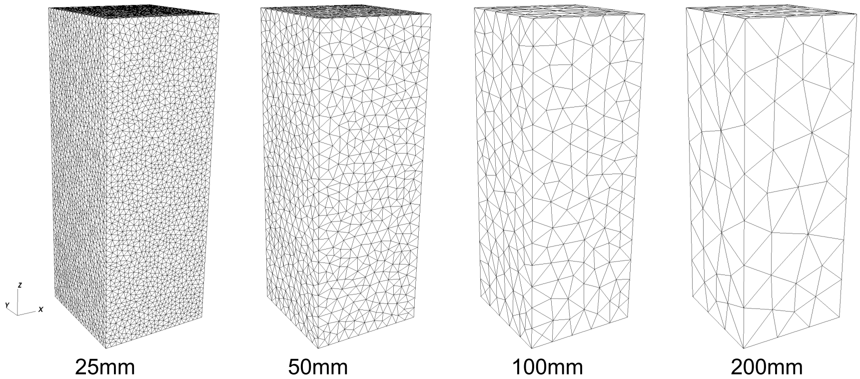

Fig. 3. Finite element meshes of four analysis models.

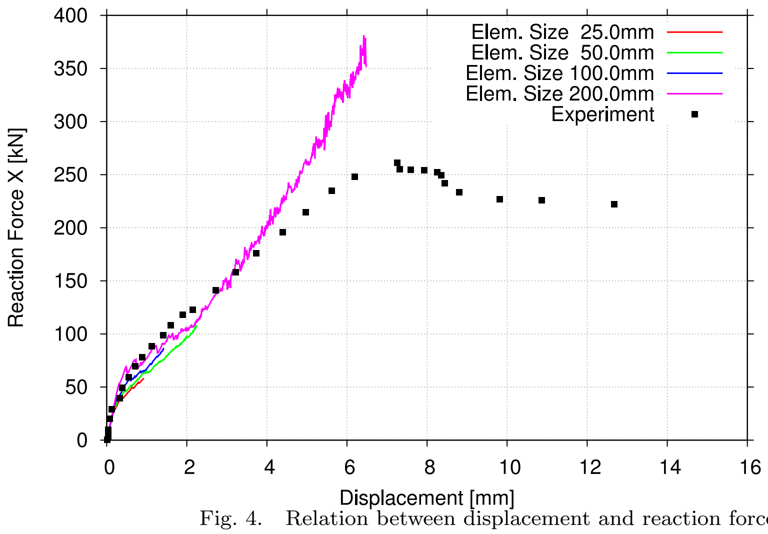

Fig. 4. Relation between displacement and reaction force.

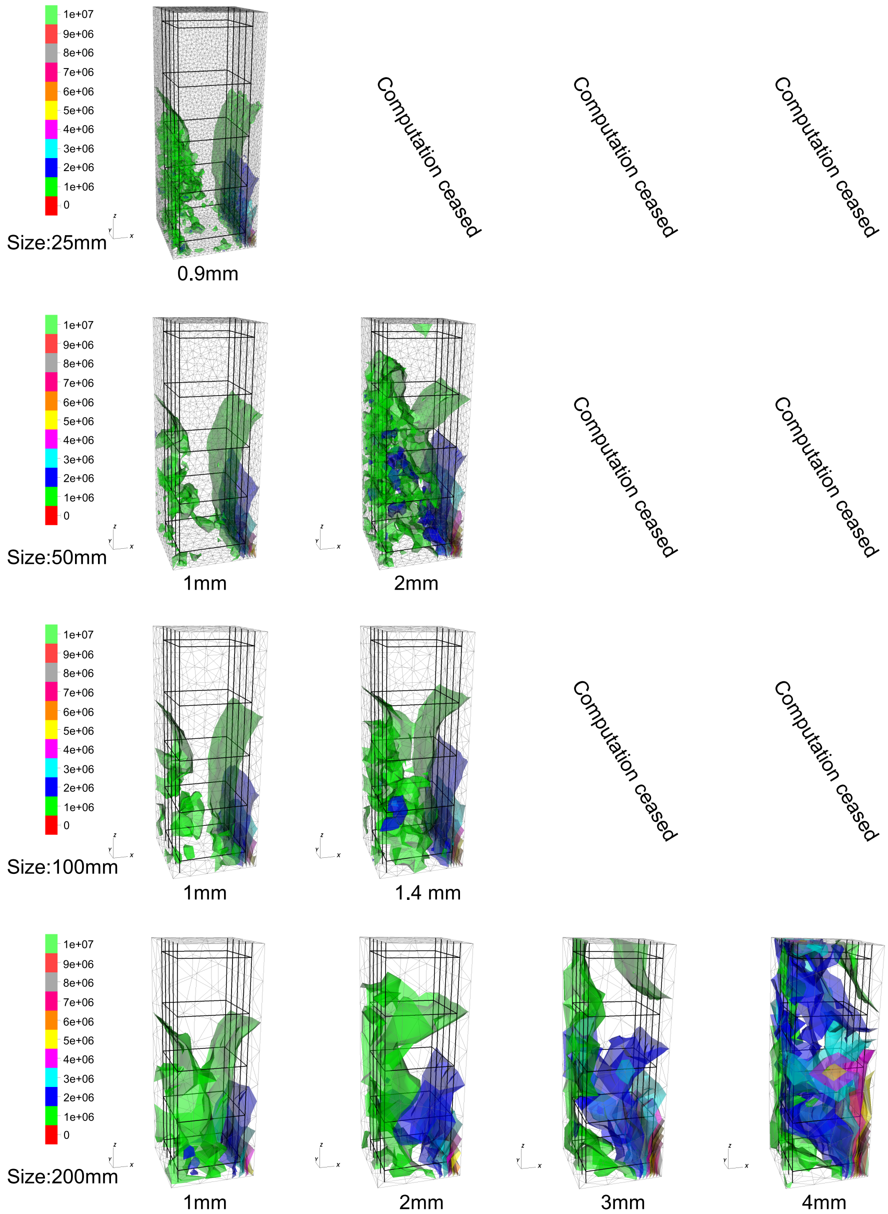

Fig. 5. Iso-surface of equivalent stress for indicated step of posing displacement boundary condition.

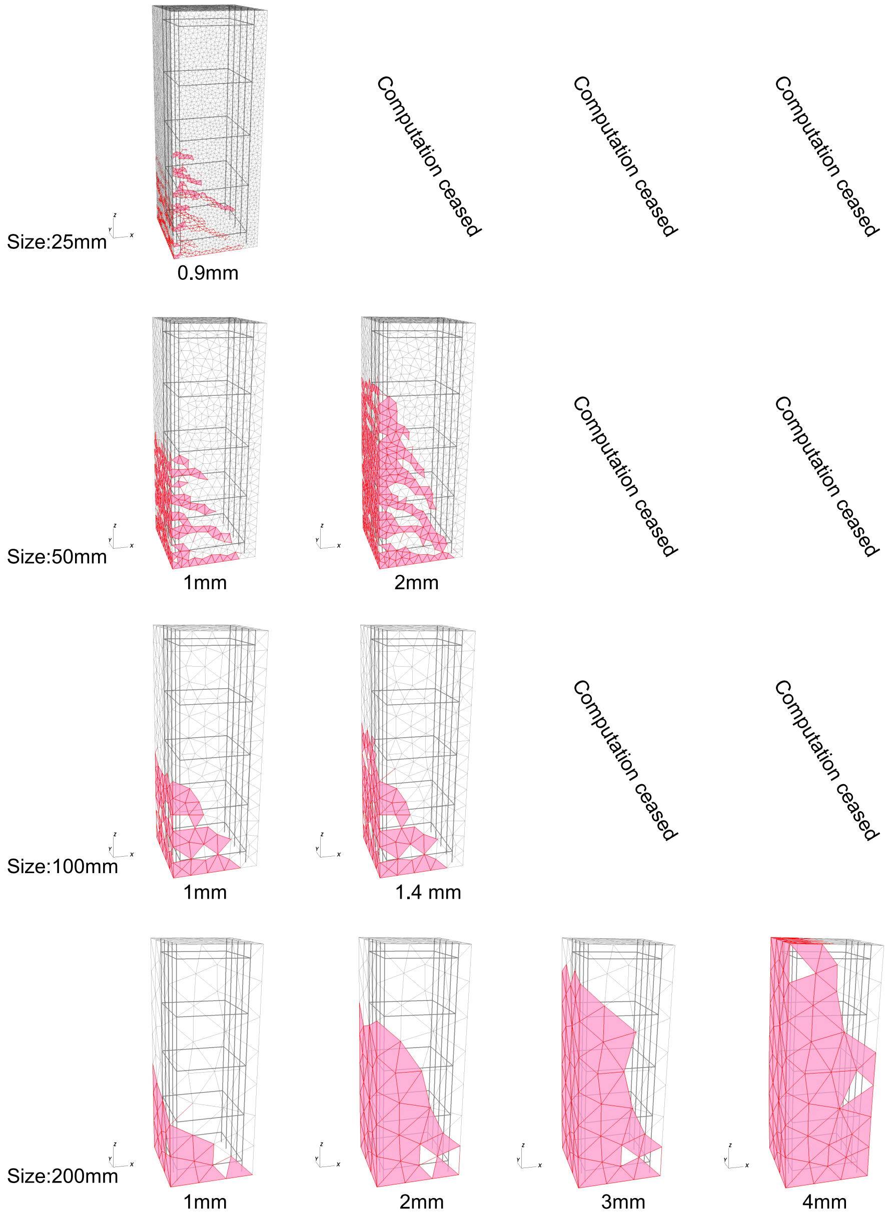

Fig. 6. Distribution of cracked solid elements for indicated step of posing displacement boundary condition.

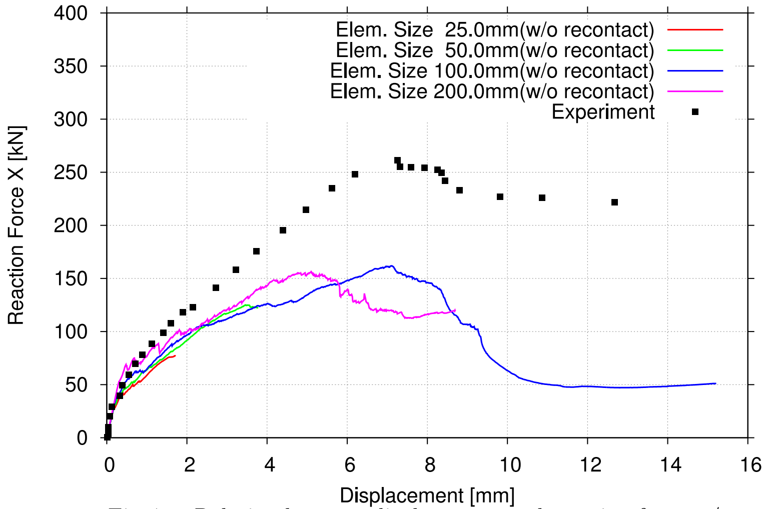

Fig. 7. Relation between displacement and reaction force w/o re-contacting.

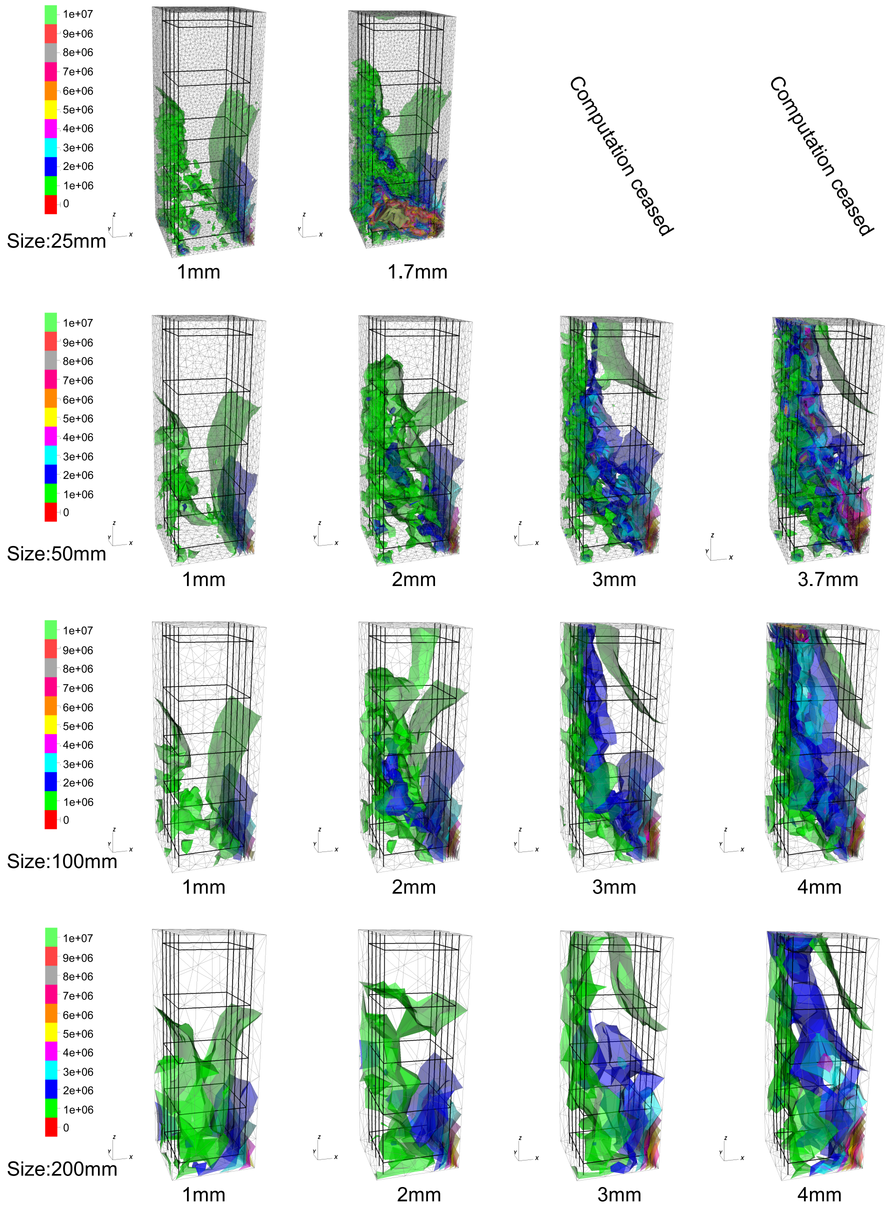

Fig. 8. Iso-surface of equivalent stress w/o re-contacting for indicated step of posing displacement boundary condition.

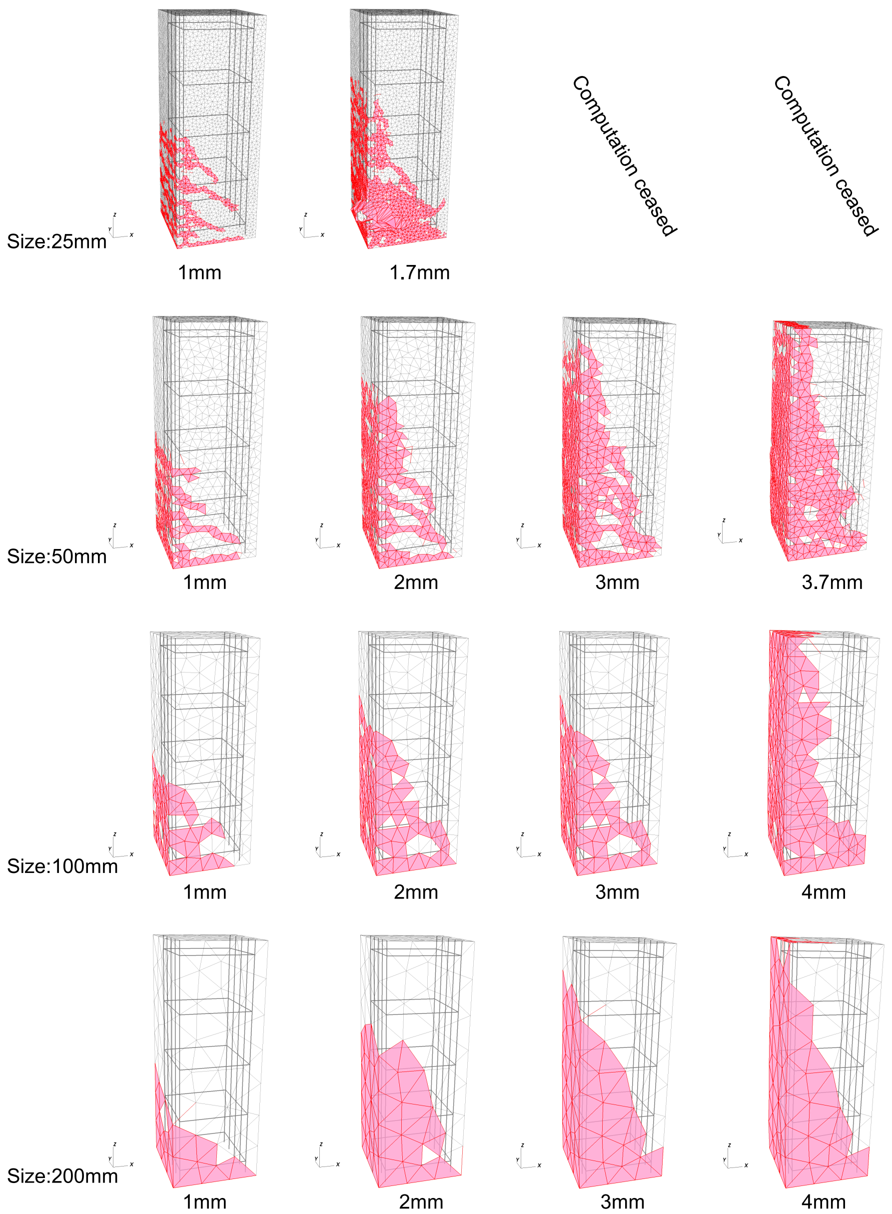

Fig. 9. Distribution of cracked solid elements w/o re-contacting for indicated step of posing displacement boundary condition.