11/17(木) 第7回 数理科学ⅢB

tangent vector (接ベクトル)

M: microlinear space $x \in M$

$t: D \to M$ $t(0) = x$ 無限小の流れ

時間

vector field (ベクトル場)

$\pi: M^D \xrightarrow{\text{projection}} M$ $\pi(t) = t(0)$

$\leftarrow X$ 切断 (section) $\pi \circ X = id_M$

無限小の流れ

$X \in (M^D)^M = M^{D \times M} = (M^M)^D$

集合論における指数法則

無限小変換

3つの観点からとらえることができる.

命題 $X_{d_1} \circ X_{d_2} = X_{d_1 + d_2}$ $(d_1, d_2) \in D(2) = \{(d_1, d_2) \in D^2 \mid d_1 d_2 = 0\}$ を好む

quasi-colimit diagram

$$\begin{tikzcd} 1 \arrow[r] \arrow[d] & D \arrow[d] \\ D \arrow[r] & D(2) \end{tikzcd}$$

証明 $(d_1, d_2) \in D(2) \longmapsto$

(1) $d_1 = d$ $d_2 = 0$ $X_d \circ X_0 = X_{d+0}$ (X_0: id_M)

$d_1 = 0$ $d_2 = d$ $X_0 + X_d = X_{0+d}$

Corallary $\forall (d, -d) \in D(2)$

$(-d, d)$

$X_d \circ X_{-d} = X_0$ id_M

$X_{-d} \circ X_d = id_M$

命題 X, Y: vector fields on M

$(X+Y)_d = X_d \circ Y_d$ $l_{(X,Y)}: D(2) \longrightarrow M^M$

$= Y_d \circ X_d$

$(d_1, d_2) \in D(2) \longmapsto X_{d_1} \circ Y_{d_2} \in M^M$

$d_1 = d$ $d_2 = 0$ X_d たし算の場合ともに d とおく

$d_1 = 0$ $d_2 = d$ Y_d

$\mathfrak{X}(M)$ M 上のベクトル場の全体 $\mathbb{R}$ 加群

Lie bracket (Lie かっこ積)

$n \times n$ の行列の全体 $A + B$ αA

$[A, B] = AB - BA$ 二重線型

$[A, B] = -[B, A]$ antisymmetric

$[A, A] = 0$

A, B, C Le 代数 Jacobi の恒等式

$[[A,B],C] + [[B,C],A] + [[C,A],B]$

$= (AB - BA)C - C(AB - BA) + (BC - CB)A - A(BC - CB) + (CA - AC)B - B(CA - AC)$

$= 0$

$X_{-d} \circlearrowleft X_d \quad X_{-d}\circ X_d = id_M$

$[X,Y] \quad (d_1,d_2)\in D\times D \to M^M$

$Y_{-d_2}\circ X_{-d_1}\circ Y_{d_2}\circ X_{d_1}$

$$\begin{tikzcd} & \bullet \arrow[dl, bend right, "X_{-d_1}"'] \\ \bullet \arrow[d, bend right, "Y_{-d_2}"'] & \bullet \arrow[u, "Y_{d_2}"'] \\ \bullet \arrow[ur, bend left, "X_{d_1}"'] & \end{tikzcd}$$

$[X,Y]$

Z_{d_1,d_2}

$d_1=d,\ d_2=0$

$\underbrace{Y_0}_{id_M}\circ X_{-d}\circ \underbrace{Y_0}_{id_M}\circ X_d = id_M$

quasi-colimit

$$\begin{tikzcd} D \arrow[r, shift left=2, "i_1"] \arrow[r, "i_2"] \arrow[r, shift right=2, "0"'] & D\times D \arrow[r, "m"] & D \end{tikzcd}$$

$d_1=0 \quad d_2=d$

id_M

anti symmetric $\qquad (X_d)^{-1}=X_{-d}$

$[XY] = -[Y,X]$

$[X,Y]_{d_1d_2} = [X,Y]_{-((-d_1)d_2)}$

$= ([X,Y]_{(-d_1)d_2})^{-1}$

$= (Y_{-d_2}\circ X_{d_1}\circ Y_{d_2}\circ X_{-d_1})^{-1}$

$= (X_{-d_1})^{-1}\circ (Y_{d_2})^{-1}\circ (X_{d_1})^{-1}\circ (Y_{-d_2})^{-1}$

$= X_{d_1}\circ Y_{-d_2}\circ X_{-d_1}\circ Y_{d_2}$

$= [Y,X]_{d_2(-d_1)}$

$= (-[Y,X])_{d_1d_2}$

No.

Date

[,] 二重線型

$\varphi: D\to \mathbb{R}$

$\varphi(d)=\varphi(0)+ad$

$\mathbb{R}^n$

$E \quad \mathbb{R}$-module

Euclidean

$\varphi: D \to E \quad \exists! a\in E$

$\varphi(d)=\varphi(0)+a\,d$

V : $\mathbb{R}$-module

E : Euclidean $\mathbb{R}$-module

$x\in V$

$f: V\to E$

$a \in V$

$d\in D \mapsto f(x+ad)\in E$

$f(x+ad) = f(x) + \underset{\in E}{?}\, d$

$f'(x)(a) \qquad f'(x): V\to E$ <u>線型</u>

$f'(x)(a_1+a_2) = f'(x)(a_1)+f'(x)(a_2) \qquad \cancel{f}: V\to ?$

$f(x+(a_1+a_2)d) = f(x) + f'(x)(a_1+a_2)d$

$f(x+a_1d+a_2d) = f(x+a_1d) + f'(x+a_1d)(a_2)d$

$f(x)+f'(x)(a_1)d + \{f'(x)+f''(x)(a_1)d\}(a_2)d$

$= f(x)+f'(x)(a_1)d + f'(x)(a_2)d$

宿 $f(x)(\alpha a) = \alpha f'(x)(a)$

補題 $f: V\to E$ homogeneous $f(\alpha x)=\alpha f(x)$

$f(x)$