第4回 数理科学III B　10/25(火)

Weil 代数

$\mathbb{R}$代数

$\mathbb{R}$代数

$\mathbb{R}[x_1, \cdots, x_n] / (x_1^{m_1}, \ldots, x_n^{m_n}, f_1(x_1, \cdots, x_n), \cdots, f_k(x_1, \cdots, x_n))$

finitely presented $\mathbb{R}$-algebra　$\mathbb{R}[x]/(x^4) \to \mathbb{R}$

$\mathrm{Spec}_{\mathbb{R}} W$　infinitesimal space 無限小空間　W: Weil代数

$=W$から$\mathbb{R}$への$\mathbb{R}$代数としての準同型の全体

$$\mathrm{Spec}_{\mathbb{R}} \mathbb{R}[x]/(x^2) = D$$

$$\mathrm{Spec}_{\mathbb{R}} \mathbb{R}[x,y]/(x^2,y^2) = D^2$$

$$\mathrm{Spec}_{\mathbb{R}} \mathbb{R}[x,y]/(x^2,y^2,xy) = D(2) := \{(d_1,d_2) \in D^2 \mid d_1 d_2 = 0\}$$

W　$\mathrm{Spec}_{\mathbb{R}} W$　$\mathbb{R}^{\mathrm{Spec}_{\mathbb{R}} W}$　$\mathbb{R}$代数

$W \to \mathbb{R}^{\mathrm{Spec}_{\mathbb{R}} W}$　canonical homomorphism of $\mathbb{R}$-algebras

$f(x)$

$\lambda_{x \in W} \lambda_{f \in \mathrm{Spec}_{\mathbb{R}} W}$　同型

一般化された
Kock-Lawvere の公理

Weil代数

finite limit diagram

$\mathrm{Spec}_{\mathbb{R}}$

$C \to A$, $C \to B$, $C \to A \times B$;　$A \leftarrow A \times B \to B$

quasi-colimit diagram

$D \to A \times_C B \to A$, $A \times_C B \to B$, $A \xrightarrow{f} C$, $B \xrightarrow{g} C$

$A \times_C B = \{(a,b) \in A \times B \mid f(a) = g(b)\}$

$\mathbb{R}^{\bigcirc}$

反変 contravariant

1　$W = \mathbb{R} \simeq \mathbb{R}[x]/(x)$

$\{0\} = 1 \to D$, $1 \to D$, $D \xrightarrow{i_2} D(2)$, $D \xrightarrow{i_1} D(2)$

$\mathrm{Spec}_{\mathbb{R}} W = 0$

$D = \{d \in \mathbb{R} \mid d^2 = 0\}$

$i_1 : d \in D \mapsto (d, 0)$

$i_2 : d \in D \mapsto (0, d)$

quasi-colimit diagram

$D = \mathrm{spec}_{\mathbb{R}} \mathbb{R}[x]/(x^2)$

limit

$b_0 = a_0$

$\mathbb{R}^1 \leftarrow \mathbb{R}^D \ni f$

$g \in \mathbb{R}^D \leftarrow \mathbb{R}^{D(2)}$

$f(d) = a_0 + a_1 d$

$g(d) = b_0 + b_1 d$

$c_1 = b_1$
$c_2 = a_1$

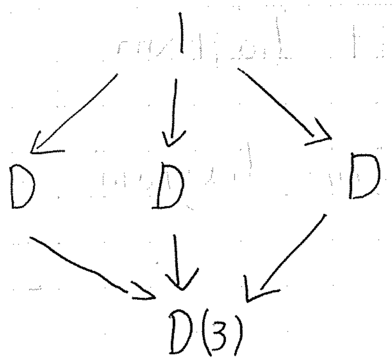
$c_0 + c_1 d$

$a_0 = b_0 = c_0$ のとき等しい。

$h(d_1, d_2) = c_0 + c_1 d_1 + c_2 d_2$

$\downarrow$

$c_0 + c_1 d$



$D(3) = \{(d_1,d_2,d_3)\in D^3 \mid d_1d_2 = d_1d_3 = d_2d_3 = 0\}$

$D\times D\times D \underset{id\times m}{\overset{m\times id}{\rightrightarrows}} D\times D \xrightarrow{m} D$ multiplication

$m : (d_1,d_2)\in D\times D \mapsto d_1d_2 \in D$

$m\times id \quad (d_1,d_2,d_3) \longmapsto (d_1d_2, d_3)$

$id\times m \quad \longmapsto (d_1, d_2d_3)$

$\mathbb{R}^{D\times D\times D} \underset{\mathbb{R}^{id\times m}}{\overset{\mathbb{R}^{m\times id}}{\leftleftarrows}} \mathbb{R}^{D\times D} \xleftarrow{\mathbb{R}^m} \mathbb{R}^D$ botbid equalizer

$\mathbb{R}^{D\times D} \ni$

$b_0 = a_0 \quad b_1 = a_{12}$

$f(d_1,d_2) = a_0 + a_1d_1 + a_2d_2 + a_{12}d_1d_2$

$\leftarrow a_0 + a_1d_1d_2 + a_2d_3 + a_{12}d_1d_2d_3$

$\quad \parallel$

$\leftarrow a_0 + a_1d_1 + a_2d_2d_3 + a_{12}d_1d_2d_3$

$\Downarrow$

$a_1 = a_2 = 0$

宿

(I) $D \overset{i_1,\ i_2,\ 0}{\rightrightarrows} D\times D \xrightarrow{m} D$

$i_1(d) = (d,0)$

$i_2(d) = (0,d)$

$0(d) = (0,0)$

(II) $D \underset{0}{\overset{i}{\rightrightarrows}} D_2 \xrightarrow{q} D$, $\quad D_2 = \mathbb{R}[x]/(x^3)$

i：そのまま

0： 0を対応

$q(d) = d^2$

(III) $D\times D \underset{\tau}{\overset{id}{\rightrightarrows}} D\times D \xrightarrow{+} D_2$

$(d_1,d_2) \to d_1+d_2$

$\tau(d_1,d_2) = (d_2,d_1)$