

第3回 数理科学III B

Taylor展開

基本対称式

x_1, \dots, x_n

$C_0^n = 1$

$n=3$

$C_1^n(x_1, \dots, x_n) = x_1 + \dots + x_n$ 1次の基本対称式

$C_2^n(x_1, x_2, x_3) = x_1x_2 + x_1x_3 + x_2x_3$ 2次

$C_3^n(x_1, x_2, x_3) = x_1x_2x_3$ 3次

Newtonの定理

任意の基本対称式は基本対称式の整式として表せる。

$n=2$

$x_1 + x_2$

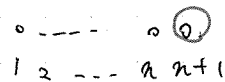
x_1x_2

$x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1x_2$

nCr

組合せ

$\underline{n+1Cr+1} = \underline{nCr+1} + \underline{nCr}$



$C_{r+1}^{n+1}(x_1, \dots, x_{n+1}) = C_{r+1}^n(x_1, \dots, x_n) + x_{n+1} C_r^n(x_1, \dots, x_n)$

$f(x+d_1+d_2) = f(x+d_1) + f'(x+d_1)d_2$
 $= f(x) + f'(x)d_1 + f'(x)d_2 + f''(x)d_1d_2$
 $= f(x) + f'(x) \underbrace{(d_1+d_2)}_{C_1^2(d_1, d_2)} + f''(x) \underbrace{d_1d_2}_{C_2^2(d_1, d_2)}$

$C_0^n = 1$

$f(x+d_1+d_2+d_3) = f(x+d_1+d_2) + f'(x+d_1+d_2)d_3$
 $= f(x) + f'(x)C_1^2(d_1, d_2) + f''(x)C_2^2(d_1, d_2)$
 $+ \{f'(x) + f''(x)(d_1+d_2) + f'''(x)C_2^2(d_1, d_2)\}d_3$

Kock - Lawvereの公理

$D = \{d \in \mathbb{R} \mid d^2 = 0\}$

$\varphi: D \rightarrow \mathbb{R} \quad (\exists! a \in \mathbb{R}) (\forall d \in D)$

$\varphi(d) = \varphi(0) + ad$

\mathbb{R} 代数
 $\mathbb{R}[x]$

ideal

\mathbb{R}

\mathbb{R} 代数としての準同型の全体

$\text{Spec}_{\mathbb{R}} \mathbb{R}[x]$

$\mathbb{R}[x]/(x^2)$

\mathbb{R}

$\text{Spec}_{\mathbb{R}} \mathbb{R}[x]/(x^2) = D = \{0\}$

$a_0 + a_1x + a_2x^2 + \dots + a_nx^n$

$\text{Spec}_{\mathbb{R}} \mathbb{R}[x]$

$\mathbb{R}[x]/(x^{n+1}) = D_n = \{d \in \mathbb{R} \mid d^{n+1} = 0\}$

$D_n = n$ 次の無限小

Lemma $d_n \in D_n, d_m \in D_m \Rightarrow d_n + d_m \in D_{n+m}$

Corollary $d_1, \dots, d_n \in D \Rightarrow d_1 + d_2 + \dots + d_n \in D_n$

$\mathbb{R}[x]/(x^2)$

$\text{Spec}_{\mathbb{R}} \mathbb{R}[x]/(x^2)$

$\mathbb{R}^{\text{Spec}_{\mathbb{R}} \mathbb{R}[x]/(x^2)} = \mathbb{R}^D$

$a_0 + a_1x$

D

$d \mapsto a_0 + a_1d$

\mathbb{R} 代数

\mathbb{R} 代数としての準同型 同型 中零無限小

$$D_1 = D = \text{Spec}_{\mathbb{R}} \mathbb{R}[x]/(x^2)$$

$$D_2 = \text{Spec}_{\mathbb{R}} \mathbb{R}[x]/(x^3) \quad \varphi: D_2 \rightarrow \mathbb{R} \quad (\exists! a_1 \in \mathbb{R}) (\exists! a_2 \in \mathbb{R})$$

$$\varphi(d) = \varphi(0) + a_1 d + a_2 d^2$$

$$\mathbb{R}[x]/x^3 \rightarrow D_2 \rightarrow \mathbb{R}^{D_2}$$

$$a_0 + a_1 x + a_2 x^2 \mapsto (d \mapsto a_0 + a_1 d + a_2 d^2) \in \mathbb{R}^{D_2}$$

\uparrow
 D_2

準同型

generalized Kock-Lawvere の公理

$$\mathbb{R}[x, y]/(x^2, y^2) \quad \text{Spec}_{\mathbb{R}} \mathbb{R}[x, y]/(x^2, y^2) = D \times D$$

$$\mathbb{R}[x, y]/(x^2, y^2, xy) \quad \text{Spec} \sim = D(2) = \{(d_1, d_2) \in D \times D \mid d_1 \cdot d_2 = 0\}$$

$$\mathbb{R}[x]/(x) \quad \circlearrowleft \mathbb{R} \quad \{0\} = 1$$

Weil 代数 $\forall i=1, \dots, n$

$$\mathbb{R}[x_1, \dots, x_n]/(x_1^{m_1}, x_2^{m_2}, \dots, x_n^{m_n})$$

$$(x_1, \dots, x_n)^m = 0$$

$$\text{Spec}_{\mathbb{R}} W$$

宿 - 一般的な行-展開の証明