10/18(火) 第3回 数理科学IIIB

Taylor展開　基本対称式

$x_1, \dots, x_n$　$n=3$

$C_0^n = 1$

$C_1^n(x_1, \dots, x_n) = x_1 + \cdots + x_n$　1次の基本対称式

$C_2^n(x_1, x_2, x_3) = x_1x_2 + x_1x_3 + x_2x_3$　2次 〃

$C_3^n(x_1, x_2, x_3) = x_1x_2x_3$　3次

Newton の定理

任意の対称式は基本対称式の整式として表わせる。

$n=2$　$x_1 + x_2$

x_1x_2

$x_1^2 + x_2^2 = (x_1+x_2)^2 - 2x_1x_2$

${}_nC_r$　組合せ

$1\ 2\ \cdots\ n\ n+1$

${}_{n+1}C_{r+1} = {}_nC_{r+1} + {}_nC_r$

$$\boxed{C_{r+1}^{n+1}(x_1, \dots, x_{n+1}) = C_{r+1}^n(x_1, \dots, x_n) + x_{n+1}C_r^n(x_1, \dots, x_n)}$$

$$\begin{aligned} f(x+d_1+d_2) &= f(x+d_1) + f'(x+d_1)d_2 \\ &= f(x) + f'(x)d_1 + f'(x)d_2 + f''(x)d_1d_2 \\ &= f(x) + f'(x)\underbrace{(d_1+d_2)}_{C_1^2(d_1,d_2)} + f''(x)\underbrace{d_1d_2}_{C_2^2(d_1,d_2)} \end{aligned}$$

$C_0^n = 1$

$$\begin{aligned} f(x+d_1+d_2+d_3) &= f(x+d_1+d_2) + f'(x+d_1+d_2)d_3 \\ &= f(x) + f'(x)C_1^2(d_1,d_2) + f''(x)C_2^2(d_1,d_2) \\ &\quad + \{f'(x) + f''(x)(d_1,d_2) + f'''(x)C_2^2(d_1,d_2)\}d_3 \end{aligned}$$

Kock - Lawvere の公理　$D = \{d \in \mathbb{R} \mid d^2 = 0\}$

$\varphi : D \to \mathbb{R}$　$(\exists! a \in \mathbb{R})(\forall d \in D)$

$\varphi(d) = \varphi(0) + ad$

$\mathbb{R}$代数　ideal

$\mathbb{R}[X]$　$\mathbb{R}$　$\mathbb{R}$代数としての準同型の全体

$\mathrm{Spec}_{\mathbb{R}}\mathbb{R}[X] = \mathbb{R}$

$\mathbb{R}[X]/(X^2)$　$\mathbb{R}$　$\mathrm{Spec}_{\mathbb{R}}\mathbb{R}[X]/(X^2) = D = D_1$

$a_0 + a_1X + a_2X^2 + \cdots a_nX^n$　$\mathrm{Spec}_{\mathbb{R}}\mathbb{R}[X]/(X^{n+1}) = D_n = \{d \in \mathbb{R} \mid d^{n+1} = 0\}$

D_n：n次の無限小

Lemma　$d_n \in D_n$, $d_m \in D_m \Rightarrow d_n + d_m \in D_{n+m}$

Corollary　$d_1, \dots, d_n \in D \Rightarrow d_1 + d_2 + \cdots + d_n \in D_n$

$\mathbb{R}[X]/(X^2) \longrightarrow \mathrm{Spec}_{\mathbb{R}}\mathbb{R}[X]/(X^2) \longrightarrow \mathbb{R}^{\mathrm{Spec}_{\mathbb{R}}\mathbb{R}[X]/(X^2)} = \mathbb{R}^D$

$\mathrm{Spec}_{\mathbb{R}}\mathbb{R}[X]/(X^2) = D$

$a_0 + a_1X \longmapsto$　$d \longmapsto a_0 + a_1 d$　$\mathbb{R}$代数

$\mathbb{R}$代数としての準同型　同型　巾零無限小

$D_1 = D = \mathrm{Spec}_{\mathbb{R}}\, \mathbb{R}[X]/(X^2)$

$D_2 = \mathrm{Spec}_{\mathbb{R}}\, \mathbb{R}[X]/(X^3)$

$\varphi : D_2 \to \mathbb{R} \quad (\exists!\, a_1 \in \mathbb{R})(\exists!\, a_2 \in \mathbb{R})$

$\varphi(d) = \varphi(0) + a_1 d + a_2 d^2$

$\mathbb{R}[X]/X^3 \longrightarrow D_2 \longrightarrow \mathbb{R}^{D_2}$

$a_0 + a_1 X + a_2 X^2 \longmapsto (\underset{D_2}{\overset{\in}{d}} \longmapsto a_0 + a_1 d + a_2 d^2) \in \mathbb{R}^{D_2}$

準同型

generalized Kock-Lawvere の公理

$\mathbb{R}[X,Y]/(X^2, Y^2) \quad \mathbb{R} \quad \mathrm{Spec}_{\mathbb{R}}\, \mathbb{R}[X,Y]/(X^2, Y^2) = D \times D$

$\mathbb{R}[X,Y]/(X^2, Y^2, XY) \quad \mathrm{Spec} \sim = D(2)$

$= \{ (d_1, d_2) \in D \times D \mid d_1 \cdot d_2 = 0 \}$

$\underbrace{\mathbb{R}[X]/(X)}_{\mathbb{R}} \quad {}_0\mathbb{R} \quad \{0\} = 1$

Weil 代数
ヴェーユ

$\mathbb{R}[X_1, \dots, X_n]/(X_1^{m_1}, X_2^{m_2}, \dots, X_n^{m_n}, \dots)$

$(X_1, \dots, X_n)^m = 0$

$\mathrm{Spec}_{\mathbb{R}} W$

宿 一般的なテイラー展開の証明