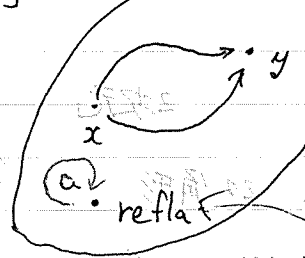
homotopy type theory

$=$ $x, y : X$ type 図形 空間 位相空間論 ×
point

$z_1, z_2 : x = y$ 新しいtype

reflexivity



動かないもの.

xからyへの道の全体

始点が a で 終点も a

$t : [0,1] \to X \quad t(0) = x \quad t(1) = y$

t_2

$z_1 = z_2$

高次 $\Omega(x, a)$ loop

homotopy theory

$t_1 \to t_2 \qquad F : [0,1] \times [0,1] \to X$

$\mathrm{refl}_a = \mathrm{refl}_a$

$F(0, \_) = t_1$

$F(1, \_) = t_2$

$\Omega^2(X, a)$

$\vdots$

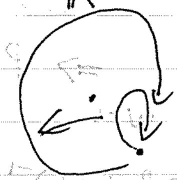
$\Omega^n(X, a)$

Th 2.1.6 (Eckmann-Hilton)

$\Omega(X, a) =$ 基本群(fundamental group)

つぶれる場合 唯1点

R^2 1点を取り除くとはたん



R^3

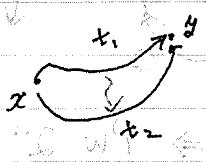
n-type X contractible
a

$x, y : X \qquad x = y$

$x = y$

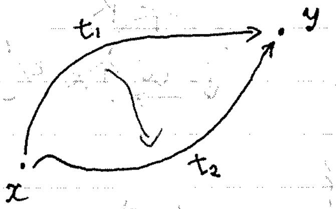
inductive

$t_1, t_2 : x = y$



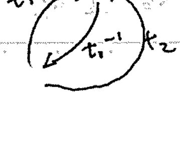
×

$x = y \qquad x, y : X$



$\Omega(X, a)$ つぶれる.

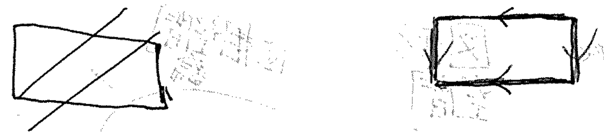
基点のとり方に依らない.



$t_1 = t_1 t_1^{-1} t_2 = t_2$

$\Omega^n(X, a)$ $S^n \to X$

例 7.2.9. $n+1$



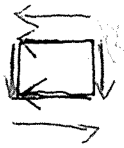
同一視 円環面(ドーナッツ)

閉曲面 2次元

正 $2n$ 角形 $2n$ 個の辺

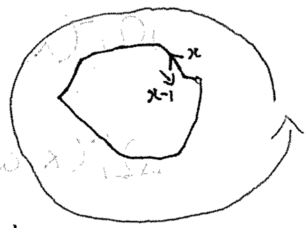
Apprentice

任意に2個づつ対にして同一視



$x, y, z, w, u, \ldots$

$x y z^{-1} y^{-1} u u x z$



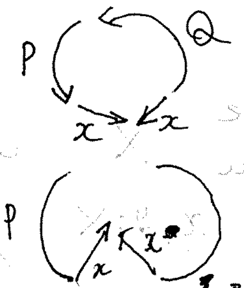
基本変形 I II III IV V P, Q

(I) 文字列をずらす. $x y z^{-1} y^{-1} u u x z \Rightarrow z^{-1} y^{-1} u u x z x y$

(II) $P x x^{-1} Q \Rightarrow PQ$

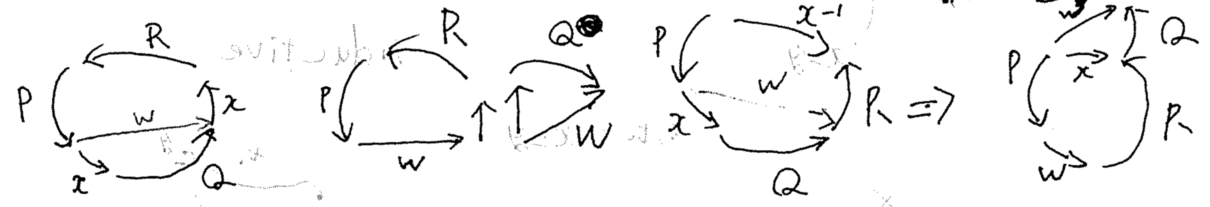
(III) $P x Q R x^{-1} \Rightarrow P w R Q w^{-1}$

$w \quad w^{-1}$

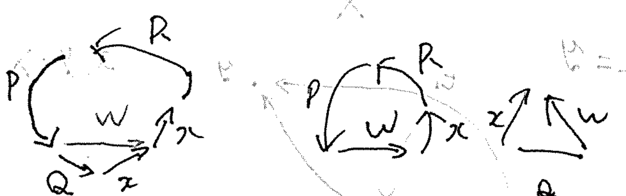


(IV) $P x Q x R \Rightarrow P w w Q^{-1} R$

w



(V) $P Q x x R \Rightarrow P w Q^{-1} w R$



例 (i) $x x^{-1}$ 球になる.

(ii) $x_1 y_1 x_1^{-1} y_1^{-1} x_2 y_2 x_2^{-1} y_2^{-1} \ldots x_k y_k x_k^{-1} y_k^{-1}$

(iii) $x_1 x_1 x_2 x_2 \ldots x_k x_k$

$\{e\}$

$x_1 y_1 x^{-1} y^{-1} \cdots = e$

自由群を割ってる.