位相幾何学

20c

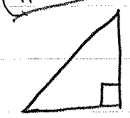
数学観　群　環 } 抽象代数学

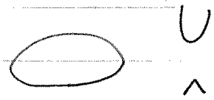
図形

19c 集合論 (Cantor)

経験的 事実として知られている.

三平方の定理





位相空間

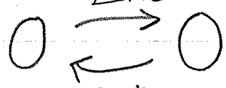
前2000

メソポタミア

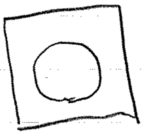
ピタゴラス (前6c)

トポロジー　岩波　1972

田村一郎　600円



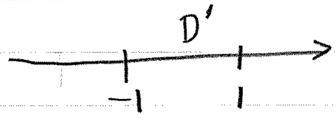
同相

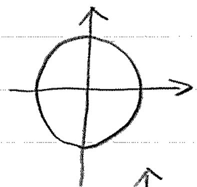


$$D^n = \{(x^1, \dots, x^n) \in \mathbb{R}^n \mid (x^1)^2 + \dots + (x^n)^2 \leqq 1\}$$

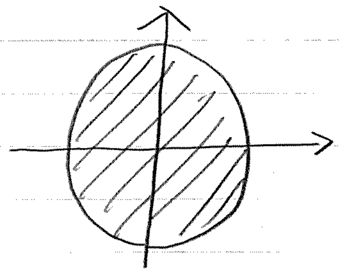
n次元球体

$$S^n = \{(x^1, \dots, x^{n+1}) \in \mathbb{R}^{n+1} \mid (x^1)^2 + \dots + (x^{n+1})^2 = 1\}$$

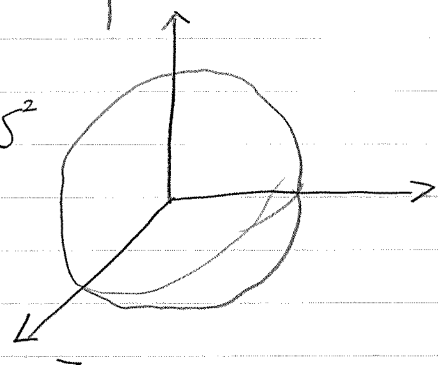




$n=2$　D^2



S^2



$$D_+^n = \{(x^1, \dots, x^{n+1}) \in S^n \mid x^{n+1} \geqq 0\}$$

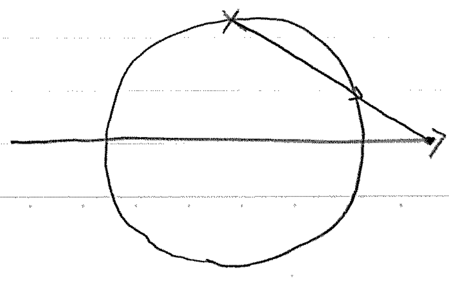
$$D_-^n = \{(x^1, \dots, x^{n+1}) \in S^n \mid x^{n+1} \leqq 0\}$$

D^n 同相

例

S^n　$(0, 0, \dots, 0, 1) = a$

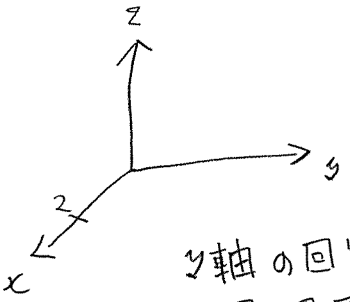
$S^n - \{a\}$ と $\mathbb{R}^n$ は同相



$\mathbb{R}^3$ (x,y,z)

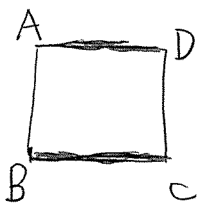
$\{(x,y,0) \mid (x-2)^2+y^2=1\}$

$(2,0)$中心 半径1 } 円



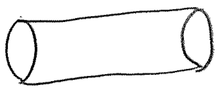
y軸の回りに回転

$S^1 \times S^1$ T 円環面(トーラス) torus



正方形 同一視

ADとBCを糊つけ



円筒

さらに ABとDCを糊つけ

商空間

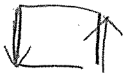
A $(0,1)$ B $(0,0)$ C $(1,0)$ D $(1,1)$

$I \times I = \{(x,y) \mid 0 \leq x \leq 1,\ 0 \leq y \leq 1\}$

$(x,y) \sim (x,y)$ $(x,0) \sim (x,1)$ $(x,1) \sim (x,0)$ $(0 \leq x \leq 1)$

$(0,y) \sim (1,y)$ $(1,y) \sim (0,y)$ $(0 \leq y \leq 1)$ 同値関係

$I \times I$ $(0,y)$と$(1,1-y)$を同一視



Mobiusaの帯

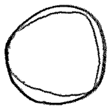
$S^1 \times I$ $((x,y),0)$と$((-x,y),1)$を同一視

Kleinの壺

例) 2つのn次元体球体

D S^n S^2

D^2

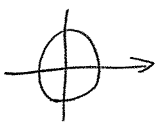


4次元

D^4

S^n $(x^1, \ldots, x^{n+1})$ $(-x^1, -x^2, \ldots, x^{n+1})$ 同一視

P^1 S^1



n次元 射影空間 P^n

projective