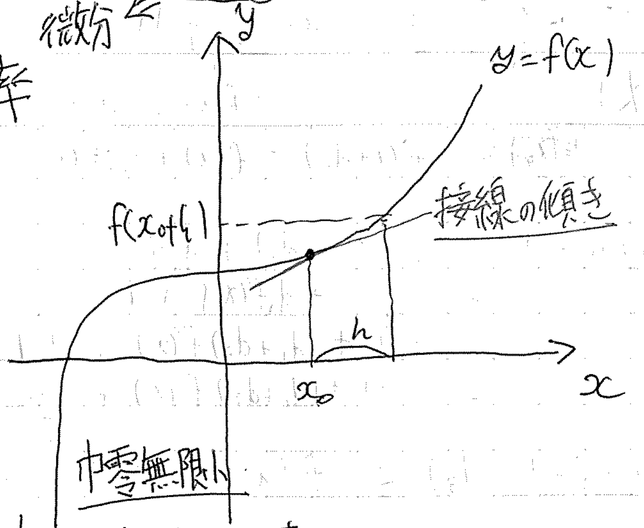
10/4(火) # 第1回

微分 ⇐ 極限 　19c 以降

平均の変化率

$$f'(x_0) = \lim_{h\to 0} \boxed{\frac{f(x_0+h)-f(x_0)}{h}}$$

ε-δ



巾零無限小

17C Newton, Leibniz

18C Euler, Lograngeh

hが十分小さい時

$h^2=0 \Rightarrow h=0$

$$f(x_0+h)-f(x_0) = f'(x_0)h$$

$$f(x_0+h) = f(x_0) + f'(x_0)h$$

$$D=\{d\in\mathbb{R} \mid d^2=0\} \supsetneq \{0\}$$

$$f(x+d) = f(x) + a\,d \quad (\forall d\in D)$$

$\exists! a$ 　linear

$f'(x)$ と書く

$$(f+g)' = f'+g'$$

$$\lim_{h\to 0}\frac{f(x+h)+g(x+h)-\{f(x)+g(x)\}}{h} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} + \lim_{h\to 0}\frac{g(x+h)-g(x)}{h}$$

$$f(x+d)+g(x+h) = f(x)+f'(x)d+g(x)+g'(x)d$$

$$= f(x)+g(x)+d\{f'(x)+g'(x)\}$$

$$(fg)' = f'g+fg' \quad (\text{Leibniz})$$

$$\lim_{h\to 0}\frac{f(x+h)g(x+h)-f(x)g(x)}{h}$$

$$= \lim_{h\to 0}\frac{f(x+h)g(x+h)-f(x)g(x+h)+f(x)g(x+h)-f(x)g(x)}{h}$$

$$= \lim_{h\to 0}\frac{(f(x+h)-f(x))g(x+h)}{h} + \lim_{h\to 0}\frac{f(x)(g(x+h)-g(x))}{h}$$

$$= \lim_{h\to 0} g(x+h)\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} + f(x)\lim_{h\to 0}\frac{g(x+h)-g(x)}{h}$$

$$= g(x)f'(x)+f(x)g'(x)$$

展開

$$f(x+d)g(x+d) = \{f(x)+f'(x)d\}\{g(x)+g'(x)d\}$$

$$= f(x)g(x)+d\{\underline{f'(x)g(x)+f(x)g'(x)}\} + \underset{\parallel\atop 0}{d^2}f'(x)g'(x)$$

定値

$f(x)=C$ 　$\underline{f'(x)=0}$

$$f(x+d)-f(x) = C-C = 0 = 0d$$

$\underline{f'(x)=1}$

$f(x)=x$ 　$f(x+d)-f(x) = x+d-x = d = 1d$

$f(x)=x^n$ 　$f(x+d)-f(x) = \underline{(x+d)^n}-x^n$

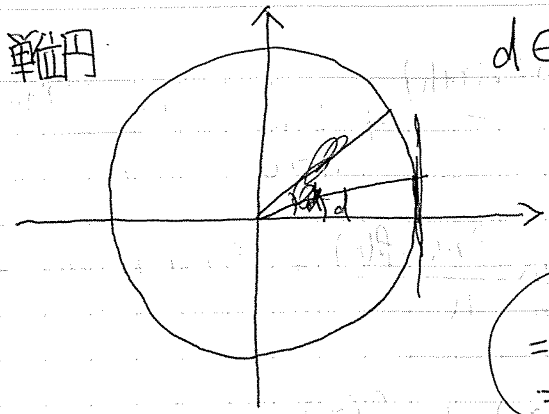
2項定理

$= \underline{nx^{n-1}}$ 　　$x^n+nx^{n-1}d,\ x^{n-2}\underset{\parallel\atop 0}{d^2}$

$$f'(x) = nx^{n-1}$$

三角関数　単位円

ラジアン



$d \in D$

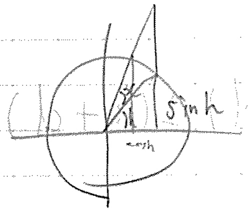
$d^2=0$ となる程小さいdのとこ

$\sin d = d$

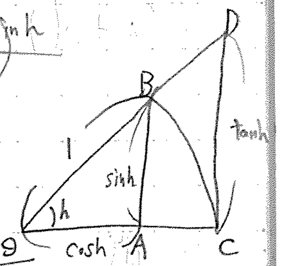
$\cos d = 1$

$\sin^2 d + \cos^2 d$
$= d^2 + 1$
$= 1$

$\sin(x+d) = \sin x \cos d + \cos x \frac{\sin d}{d}$
$= \sin x + d\cos x$

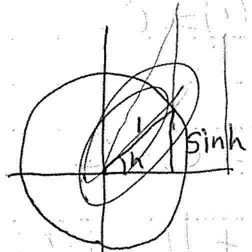


$\cos(x+d) = \cos x \cos d - \sin x \sin d$
$= \cos x - d \sin x$



$\lim_{h \to 0} \frac{\sin h}{h} = 1$

指数関数 $(e^x)' = e^x$



$|OA| = \cos h$
$|AB| = \sin h$
$|CD| = \tan h$
$|\triangle OAB| = \frac{1}{2}\cos h \sin h$
$|\triangle OCD| = \frac{1}{2}\tan h$

$\frac{1}{2}\cos h \sin h < |\underbrace{扇形OCB}_{\frac{h}{2}}| < \frac{1}{2}\tan h$

$\to \cos h \sin h < h < \frac{\sin h}{\cos h}$

$\Rightarrow \cos h < \frac{\sin h}{h} < \frac{1}{\cos h}$

$\lim_{h\to 0} \frac{\sin h}{h} = 1$

$e = \lim_{h \to 0}\left(h + \frac{1}{h}\right)^h$

$g(x) = e^x$

$e = 10^{\log_{10} e}$

$f(x) = 10^x$

$f(d) = 1 + \underline{a}\, d$

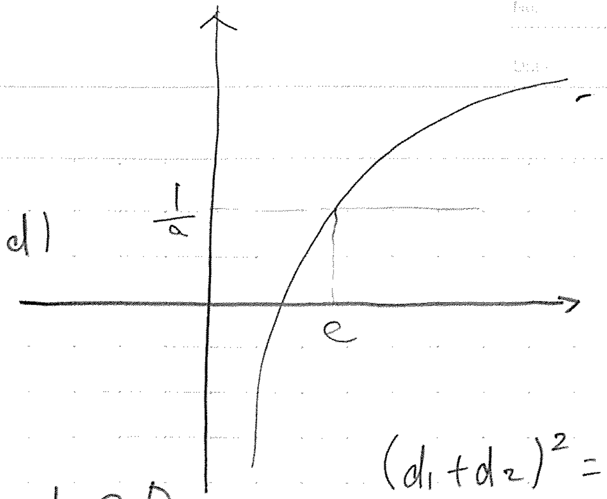
$g(x) = (10^{\log_{10} e})^x$
$= 10^{x(\log_{10} e)}$

$g(d) = 10^d = 10^{d(\log_{10} e)}$
$= 1 + a d(\log_{10} e)$
$= 1 + d\, a(\log_{10} e)$

$\fallingdotseq \frac{1}{a}$

$g'(0) = 1$

$e^{x+d} = e^x e^d = e^x(1+d)$
$= e^x + d e^x$



$(d_1+d_2)^2 = d_1^2 + d_2^2 + 2d_1d_2$
$= 2d_1d_2$

$d_1, d_2 \in D \Rightarrow d_1 + d_2 \in D$

$f(x+d_1) = f(x) + f'(x)d_1$

$f(x+d_1+d_2) = f(x+d_1) + f'(x+d_1)d_2$
$= f(x) + f'(x)d_1 + \{f'(x) + f''(x)d_1\}d_2$
$= f(x) + f'(x)(d_1+d_2) + f''(x)d_1d_2$
$= f(x) + f'(x)(d_1+d_2) + f''(x)\frac{(d_1+d_2)^2}{2}$

$f(x+d_1+d_2+d_3) = f(x+d_1+d_2) + f'(x+d_1+d_2)d_3$
$= \{f(x) + f'(x)(d_1+d_2) + f''(x)d_1d_2\} + \{f'(x) + f''(x)(d_1+d_2) + f'''(x)d_1d_2\}d_3$
$= f(x) + f'(x)(d_1+d_2+d_3) + f''(x)(d_1d_2 + d_1d_3 + d_2d_3) + f'''(x)d_1d_2d_3$
$= f(x) + f'(x)(d_1+d_2+d_3) + f''(x)\frac{(d_1+d_2+d_3)^2}{2} + f'''(x)\frac{(d_1+d_2+d_3)^3}{3!}$

宿　~~[illegible]~~ 4と5の場合を解け！