

第8回 数理科学ⅢA 6/4(火)

Category A, B ; categories

functor $A \times B$ 直積の category $\xrightarrow{\text{functor}} C$
 $(A, B) \xrightarrow{(f, g)} (A', B')$

natural transformation $A \xrightarrow{f} A'$ bifunctor
 $B \xrightarrow{g} B'$

categories の全体

$H(A, B)$ Category A, B : categories
 A から B への functor の全体

$A \xrightarrow{F} B$ vertical composition

$F = A \rightarrow B$ natural transformation
 $G = B \rightarrow C$
 $H(A, B) \times H(B, C)$

$G \circ F \rightarrow G \circ F'$

$A \xrightarrow{F} B \xrightarrow{G} C$ horizontal composition
 $H(A, C)$

$\alpha \downarrow F' \quad \beta \downarrow G'$
 $\xrightarrow{F''} \xrightarrow{G''} \langle \alpha, \beta \rangle$
 $\langle F, G \rangle$
 $\langle F', G' \rangle$

$\beta * \alpha$

- bi functor

$$\langle \begin{matrix} \alpha \downarrow & \downarrow \beta \\ \alpha' \downarrow & \downarrow \beta' \end{matrix} \rangle \quad (\beta' \circ \beta) * (\alpha' \circ \alpha) = (\beta' * \alpha') \circ (\beta * \alpha)$$

$$= \langle \alpha' \circ \alpha, \beta' \circ \beta \rangle$$

高階の category

1- category

2- category \mathcal{A}

(1) $|A|$ class of objects
 A, B

$A(A, B) = \text{category}$

$C_{ABC} = A(A, B) \times A(B, C) \xrightarrow{b: \text{functor}} A(A, C)$

$U_A = 1 \rightarrow A(A, A)$ functor

概念を生み出す!

C_*

$A(A, B) \times A(B, C) \times A(C, D) \xrightarrow{1 \times C_{BCD}} A(A, B) \times A(B, D)$

$C_{ABCX1} \downarrow$



$C_{ABD} \downarrow$

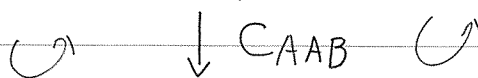
$A(A, C) \times A(C, D) \xrightarrow{\quad\quad\quad} A(A, D)$

C_{ACD} associativity

$A, B \in |A|$

$1 \times A(A, B) \xrightarrow{\quad\quad\quad} A(A, A) \times A(A, B)$
 \parallel
 $A(A, B)$

$\xrightarrow{U_A} A(A, A) \times A(A, B) \xleftarrow{1 \times U_B} A(A, B) \times 1$



$\xrightarrow{1} A(A, B) \xleftarrow{1} \text{unit}$

2-category A, B

$F: A \rightarrow B$ 2-functor $A, A' \in A$

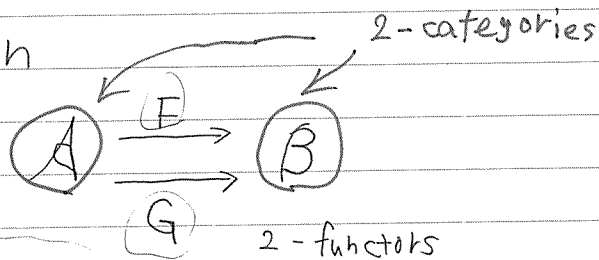
(1) $\forall A \in A$ $FA \in B$
 $F_{A,A}: A(A, A) \rightarrow B(FA, FA)$
 functor

$$\begin{array}{ccc}
 A(A, A') \times A(A', A'') & \xrightarrow{C_{AA'A''}} & A(A, A'') \\
 \downarrow F_{AA'} \times F_{A'A''} & \curvearrowright & \downarrow F_{AA''} \\
 B(FA, FA') \times B(FA', FA'') & \xrightarrow{C_{FA, FA', FA''}} & B(FA, FA'')
 \end{array}$$

$$\begin{array}{ccc}
 1 & \xrightarrow{U_A} & A(A, A) \\
 & \curvearrowright & \downarrow F_{AA} \\
 & \xrightarrow{U_{FA}} & B(FA, FA)
 \end{array}$$

2 - natural transformation

$$\begin{array}{c}
 \downarrow \\
 \theta: F \Rightarrow G \\
 \theta_A: FA \rightarrow GA
 \end{array}$$



$$\begin{array}{ccc}
 A(A, A') & \xrightarrow{F_{AA'}} & B(FA, FA') \quad B(FA, GA) \\
 \downarrow G_{AA'} & \curvearrowright & \downarrow B(1_{FA}, \theta_{A'}) \\
 B(GA, GA') & \xrightarrow{B(\theta_A, 1_{GA'})} & B(FA, GA)
 \end{array}$$

命題

Small 2-categories 2-functors and 2-natural transformations form a 2-category

$f = A \begin{array}{c} \xrightarrow{f} \\ \Downarrow \\ \xrightarrow{g} \end{array} B$ arrow A, B objects \rightarrow 0-cells

$A(A, B)$ 2-cell $\alpha = f \Rightarrow g$

$A \begin{array}{c} \xrightarrow{F} \\ \Downarrow \\ \xrightarrow{G} \end{array} B$ modification

$\Xi_A = \alpha_A \leadsto \beta_A$ 2-cell

$A \begin{array}{c} \xrightarrow{f} \\ \Downarrow \\ \xrightarrow{g} \end{array} A'$

$\Xi_{A'} * F \downarrow = G \downarrow * \Xi_A$

A, B

$x =_A y$

$x \rightarrow y$

∞ -category