第8回 数理科学ⅢA 6/14(火)

Category $\mathcal{A}, \mathcal{B}$; categories

functor $\mathcal{A} \times \mathcal{B}$ 直積のcategory $\xrightarrow{\text{functor}} \mathcal{C}$

$(A, B) \xrightarrow{(f,g)} (A', B')$

natural transformation

$A \xrightarrow{f} A'$ bifunctor

$B \xrightarrow{g} B'$

categories の全体

$H(\mathcal{A}, \mathcal{B})$ Category $\mathcal{A}, \mathcal{B}$: categories

$\mathcal{A}$から$\mathcal{B}$へのfunctorsの全体

$$\begin{tikzcd} \mathcal{A} \arrow[r, bend left, "F"] \arrow[r, bend right, "G"'] \arrow[r, phantom, "\Downarrow\alpha"] & \mathcal{B} \end{tikzcd}$$

vertical composition

natural transformation

$F : \mathcal{A} \to \mathcal{B}$ $\quad G : \mathcal{B} \to \mathcal{C}$

$H(\mathcal{A}, \mathcal{B}) \times H(\mathcal{B}, \mathcal{C})$

$G \circ F \to G' \circ F'$

$\boxed{H(\mathcal{A}, \mathcal{C})}$

$$\begin{tikzcd} \mathcal{A} \arrow[r, bend left, "F"] \arrow[r, bend right, "F'"'] \arrow[r, phantom, "\Downarrow\alpha"] & \mathcal{B} \arrow[r, bend left, "G"] \arrow[r, bend right, "G'"'] \arrow[r, phantom, "\Downarrow\beta"] & \mathcal{C} \end{tikzcd}$$

horizontal composition

$\beta * \alpha$

$\alpha' \downarrow$ F'' $\quad \beta' \downarrow$ G''

$\langle F, G \rangle$

$\langle \alpha, \beta \rangle$

$\langle F', G' \rangle$

- bifunctor

$$\left\langle \begin{matrix} \alpha \downarrow & \downarrow \beta \\ \alpha' \downarrow & \downarrow \beta' \end{matrix} \right\rangle \quad (\beta' \circ \beta) * (\alpha' \circ \alpha) = (\beta' * \alpha') \circ (\beta * \alpha)$$

$= \langle \alpha' \circ \alpha, \beta' \circ \beta \rangle$

高階のcategory

1- category

2- category $\mathbb{A}$

(1) $|\mathbb{A}|$ class of objects

A, B

例 漢字

造語

恐れる 畏れる

怖れる 懼れる

$$\mathbb{A}(A,B) : \text{category}$$

$$C_{ABC} : \mathbb{A}(A,B) \times \mathbb{A}(B,C) \xrightarrow{\text{b: functor}} \mathbb{A}(A,C)$$

$$u_A : 1 \longrightarrow \mathbb{A}(A,A) \quad \text{functor}$$

概念を生み出す!

C_*

$$\begin{tikzcd}
\mathbb{A}(A,B) \times \mathbb{A}(B,C) \times \mathbb{A}(C,D) \arrow[r, "1 \times C_{BCD}"] \arrow[d, "C_{ABC} \times 1"'] \arrow[dr, phantom, "\circlearrowright"] & \mathbb{A}(A,B) \times \mathbb{A}(B,D) \arrow[d, "C_{ABD}"] \\
\mathbb{A}(A,C) \times \mathbb{A}(C,D) \arrow[r, "C_{ACD}"'] & \mathbb{A}(A,D)
\end{tikzcd}$$

associativity

$A, B \in |\mathbb{A}|$

$$\begin{tikzcd}
1 \times \mathbb{A}(A,B) \overset{\shortparallel}{=} \mathbb{A}(A,B) \arrow[r, "u_A \times 1"] \arrow[dr, "1"'] & \mathbb{A}(A,A) \times \mathbb{A}(A,B) \arrow[d, "C_{AAB}"] \arrow[dl, phantom, "\circlearrowright"] \arrow[dr, phantom, "\circlearrowright"] & \mathbb{A}(A,B) \times 1 \arrow[l, "1 \times u_B"'] \arrow[dl, "1"] \\
& \mathbb{A}(A,B) &
\end{tikzcd}$$

unit

2-category $\mathcal{A}, \mathcal{B}$

$F: \mathcal{A} \to \mathcal{B}$ 2-functor $A, A' \in \mathcal{A}$

(1) $\forall A \in \mathcal{A}$ $FA \in \mathcal{B}$

$F_{A,A'}: \mathcal{A}(A,A') \to \mathcal{B}(FA,FA')$ functor

$$\begin{tikzcd}
\mathcal{A}(A,A') \times \mathcal{A}(A',A'') \arrow[r, "C_{AA'A''}"] \arrow[d, "F_{AA'} \times F_{A'A''}"'] \arrow[dr, phantom, "\circlearrowright"] & \mathcal{A}(A,A'') \arrow[d, "F_{AA''}"] \\
\mathcal{B}(FA,FA') \times \mathcal{B}(FA',FA'') \arrow[r, "C_{FA,FA',FA''}"'] & \mathcal{B}(FA,FA'')
\end{tikzcd}$$

$$\begin{tikzcd}
1 \arrow[r, "u_A"] \arrow[dr, "u_{FA}"'] \arrow[r, phantom, shift right=4ex, "\circlearrowright"] & \mathcal{A}(A,A) \arrow[d, "F_{AA}"] \\
 & \mathcal{B}(FA,FA)
\end{tikzcd}$$

2 - natural transformation

2-categories

$\theta: F \Rightarrow G$

$\theta_A \quad FA \to GA$

$$\begin{tikzcd}
\mathcal{A} \arrow[r, shift left, "F"] \arrow[r, shift right, "G"'] & \mathcal{B}
\end{tikzcd}$$

2-functors

$$\begin{tikzcd}
\mathcal{A}(A,A') \arrow[r, "F_{AA'}"] \arrow[d, "G_{AA'}"'] \arrow[dr, phantom, "\circlearrowright"] & \mathcal{B}(FA,FA') \arrow[d, "\mathcal{B}(1_{FA}, \theta_{A'})"] & \mathcal{B}(FA,GA) \\
\mathcal{B}(GA,GA') \arrow[r, "\mathcal{B}(\theta_A, 1_{GA'})"'] & \mathcal{B}(FA,GA) &
\end{tikzcd}$$

命題

Small 2-categories 2-functors and
2-natural transformations form a 2-category

$f: A \overset{f}{\underset{g}{\rightrightarrows}} B$ arrow A, B objects — 0-cells

$\mathcal{A}(A,B)$ 2-cell $\alpha : f \Rightarrow g$

$$\begin{tikzcd} \mathcal{A} \arrow[r, bend left, "F"] \arrow[r, bend right, "G"'] \arrow[r, phantom, "\alpha,\beta \Downarrow"] & \mathcal{B} \end{tikzcd}$$ modification

$\Xi_A : \alpha_A \rightsquigarrow \beta_A$ 2-cell

$$\begin{tikzcd} A \arrow[r, bend left, "f"] \arrow[r, bend right, "g"'] \arrow[r, phantom, "\alpha \Downarrow"] & A' \end{tikzcd}$$

$\Xi_{A'} * F_\alpha = G_\alpha * \Xi_A$

A, B $x \,\Xi_A\, y$ $x \to y$

∞- category