

Category 第7回

6/7(火) 数理学ⅢA

functor

$$A \xrightarrow{F} B \xrightarrow{G} C$$

natural transformation

合成 $G \circ F$

object category } category
 morphism functors }

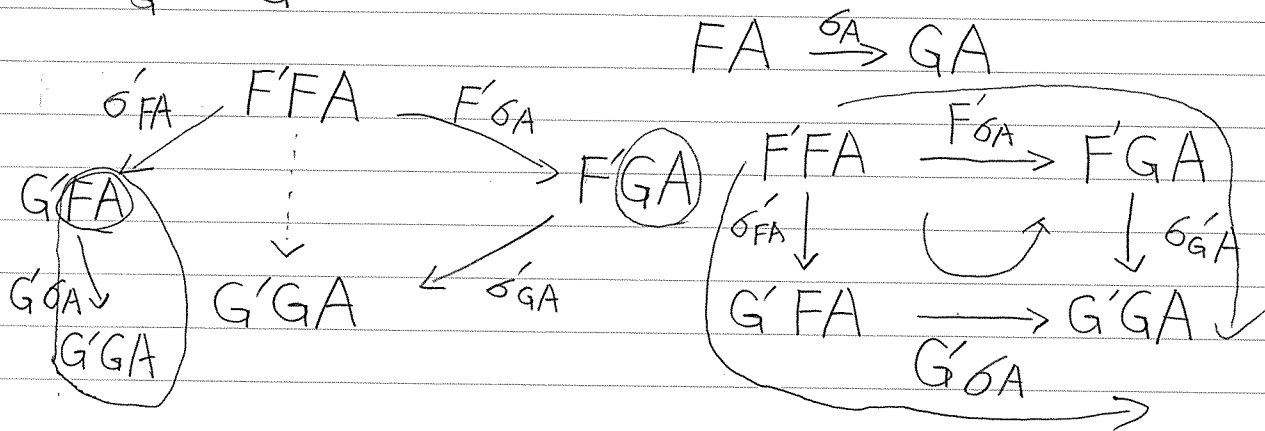
$$A \begin{array}{c} \xrightarrow{F} \\ \xrightarrow{G} \downarrow \sigma \\ \xrightarrow{H} \downarrow \tau \end{array} B \quad F \xrightarrow{\sigma} G \xrightarrow{\tau} H \quad f: A_1 \rightarrow A_2 \text{ in } A$$

vertical composition

$$\begin{array}{ccc} \tau_{A_1} \circ \sigma_{A_1} & & \tau_{A_2} \circ \sigma_{A_2} \\ \downarrow \sigma_{A_1} & \begin{array}{c} F A_1 \xrightarrow{Ff} F A_2 \\ \downarrow \sigma_{A_1} \quad \uparrow \downarrow \sigma_{A_2} \\ G A_1 \xrightarrow{Gf} G A_2 \\ \downarrow \tau_{A_1} \quad \uparrow \downarrow \tau_{A_2} \\ H A_1 \xrightarrow{Hf} H A_2 \end{array} & \downarrow \sigma_{A_2} \\ \tau_{A_1} \circ \sigma_{A_1} & & \tau_{A_2} \circ \sigma_{A_2} \end{array}$$

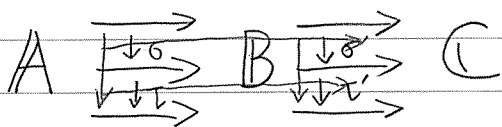
horizontal composition

$$A \xrightarrow{F} B \xrightarrow{F'} C \quad F' \circ F \quad \xrightarrow{\sigma' \circ \sigma} \quad G' \circ G$$



$$\begin{array}{ccc}
 F'FA \rightarrow G'GA & & A \xrightarrow{f} B \text{ in } A \\
 F'f \downarrow \quad \text{(?)} \quad \downarrow G'gf & & \\
 F'FB \rightarrow G'GB & & \\
 \\
 FA \xrightarrow{g_A} GA & & F'FA \xrightarrow{F'g_A} F'GA \xrightarrow{g'_A} G'GA \\
 Ff \downarrow \quad \cup \quad \downarrow Gf & & F'f \downarrow \quad \cup \quad \downarrow FGf \quad \cup \quad \downarrow G'gf \\
 FB \xrightarrow{g_B} GB & & F'FB \rightarrow F'GB \xrightarrow{g'_B} G'GB
 \end{array}$$

horizontal composition



vertical \rightarrow horizontal
 \cup ?
 horizontal \rightarrow vertical