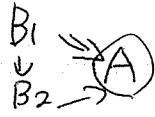


第5回

5/24(火)
数理科学Ⅲ

Category



Simple type theory with equalitise extensional

↔ Cartesian Closed Category

dependent type theory with \dashv $\overset{\text{undecidable}}{\leftrightarrow}$ locally Cartesian Closed Category

intensional $\overset{\text{decidable}}{\leftarrow}$

Category 集合入門

すべては集合である。
一元論

mapping $f: A \rightarrow B$

$\forall a \in A, \exists b \in B$
 $(a, b) \in f$

$A \times B$ 積集合

$f \subseteq A \times B$
 $(a, b) \in f \Rightarrow b = b'$

直和
ALIB
 $(a, f(a))$

集合の作る category (圏) 二元論

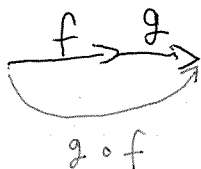
f, g
Category \mathcal{A}
objects

objects
morphisms
(arrows)

morphisms \rightarrow objects $f: \text{dom}(f) \rightarrow \text{cod}(f)$

$\text{dom}(f)$ 定義域
 $\text{cod}(f)$ 値域

$\text{dom}(g) = \text{cod}(f)$
 $\hookrightarrow g \circ f$



$\text{dom}(g \circ f) = \text{dom}(f)$
 $\text{cod}(g \circ f) = \text{cod}(g)$

$$\text{dom}(g) = \text{cod}(f)$$

$$\text{dom}(h) = \text{cod}(g)$$

$$h \circ (g \circ f) = (h \circ g) \circ f \quad \text{結合律}$$

object X $\text{Cod}(id_X) = \text{dom}(id_X) = X$

morphism

$$f: Y \rightarrow X$$

$$id_X \circ f = f$$

$$g: X \rightarrow Z$$

$$g \circ id_X = g$$

群 2項演算

$$\left. \begin{aligned} Z * (Y * X) &= (Z * Y) * X \quad (\text{結合律}) \\ e * X &= X * e = X \quad (\text{単位元}) \end{aligned} \right\} \begin{array}{l} M \\ \text{monoid} \\ N \end{array}$$

$x, \otimes \rightarrow \otimes$ 記号はなんでもいい
 \otimes object
ただ1個

退化

objectが1個

順序集合 (partially ordered set)

$$(M, \leq)$$

$$x \leq x \quad (\text{反射律})$$

~~$$x \leq y \text{ and } y \leq x \Rightarrow x = y \quad (\text{反対称律})$$~~

~~$$x \leq y \text{ and } y \leq z \Rightarrow x \leq z \quad (\text{推移律})$$~~

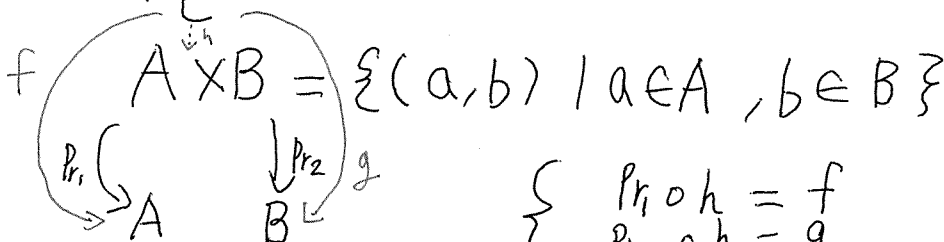
preorder

$$x \leq y \quad x \rightarrow y$$

morphismは高々1つ

\hookrightarrow 退化

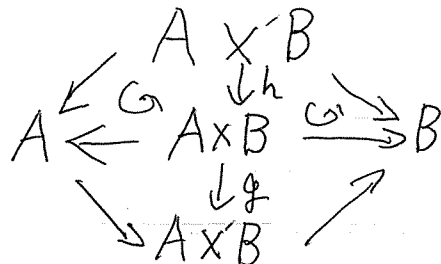
$$A \xrightarrow{f} B \quad B \xrightarrow{g} C$$



図式を可換にするように
hが唯一存在する。

$$\begin{cases} p_1 \circ h = f \\ p_2 \circ h = g \end{cases}$$

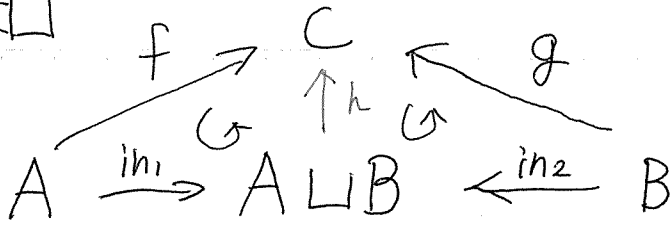
$$\begin{aligned} g \circ f &= id_{A \times B} \\ f \circ g &= id_{A \times B} \end{aligned}$$



機能的

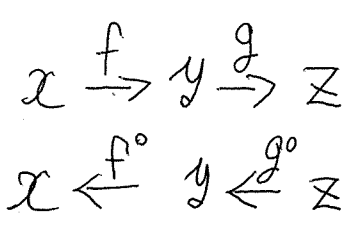
isomorphic

直和



Category $\{x, y\}$

Co product



dual category
 $\text{cod}(f^\circ) = \text{dom}(f)$
 $\text{dom}(f^\circ) = \text{cod}(f)$

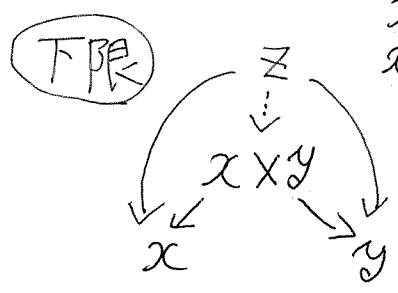
$x_0 \leq y_0$
 $x \leq x$
 $x \geq y$
 $x \leq x$

$y \leq y$
 ~~$x \geq y$~~
 ~~$y \leq y$~~

$f^\circ \circ g^\circ = (g \circ f)^\circ$

preorder

$x \leq y$
 $z \leq x \times y$ 下限



$x \times y \leq x$
 $x \times y \leq y$