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type　　$A \to B$　　AからBへの関数の全体　　$x : A$
A, B

universe　すべての集合が作る集合 ⇒ paradox

category theory　　small
階層的
$A : \mathcal{U}$　　$U_0 : U_1 : U_2 : U_3$
$A : \mathcal{U}_0 \Rightarrow A : U_1$　　U

dependent function type

$A : \text{type}$　　$B : A \to U$　　$\Pi_{x:A} B(x)$　　$\prod_{x:A} B(x)$

$\Pi(x:A), B(x)$　　$B(x) = B$　　$\Phi : B(x)$

$A \to B$　　$f(x) :\equiv \Phi$ for $x : A$

$$\lambda x . \Phi : \prod_{x:A} B(x)$$

$Fin : \mathbb{N} \to \mathcal{U}$
Values are the standard finite sets with elements

$$0_n, 1_n, \ldots, (n-1)_n : Fin(n)$$

$$\Pi_{n:\mathbb{N}} Fin(n+1)$$

$f_{max}(n) :\equiv n_{n+1}$

f_{max}　largest element

A, B　$A \times B$　直積　　dependent pair type
(Σ-types)

$\Sigma_{x:A} B(x)$　　$\sum_{x:A} B(x)$　　$\Sigma(x:A), B(x)$

$$\sum_{x:A} B \equiv A \times B$$

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$\mathbb{2}$ the type of booleans

$0_{\mathbb{2}}$, $1_{\mathbb{2}}$

natural number $\mathbb{N}$ $\qquad 0 : \mathbb{N} \qquad succ : \mathbb{N} \to \mathbb{N}$

Formation
introduction
elimination
computation

$$\frac{\Gamma \ \text{context}}{\Gamma \vdash \mathbb{N} : U} \quad \mathbb{N}\text{ formation}$$

$$\frac{\Gamma \ \text{context}}{\Gamma \vdash 0 : \mathbb{N}} \quad \mathbb{N} - \text{introduction}_1$$

$$\frac{\Gamma \vdash n : \mathbb{N}}{\Gamma \vdash succ(n) : \mathbb{N}} \quad \mathbb{N} - \text{introduction}_2$$

$\Gamma, x : \mathbb{N} \vdash C : U \qquad \Gamma \vdash C_0 : C[0/x] \qquad \Gamma, x : \mathbb{N}$, ~~$y : C_s$~~ :
$y : C \vdash C_s : C[succ(x)/x]$

$$\frac{\Gamma \vdash n : \mathbb{N}}{\Gamma \vdash ind_{\mathbb{N}}(x : C \ , C_0 \ \ x, y, C_s \ , n) : C[n/x]}$$

$$\frac{\Gamma, x : \mathbb{N} \vdash C : U \quad \Gamma \vdash C_0 : C[0/x] \quad \Gamma, x : \mathbb{N}, y : C \vdash C_s : C[succ(x)/x]}{\Gamma \vdash ind_{\mathbb{N}}(x : C, C_0, x, y, C_s, 0) \equiv C_0 : C[0/x]}$$

$\mathbb{N}$ - computation

← CはXにdependしている.

$$\frac{\Gamma, \underline{x} : \mathbb{N} \vdash \underline{C} : U \quad \Gamma \vdash C_0 : C[0/x] \quad \Gamma, x : \mathbb{N}, y : C \vdash C_s : C[succ(x)/x] \quad \Gamma \vdash n : \mathbb{N}}{\Gamma \vdash ind_{\mathbb{N}}(x : C, C_0, x.y.C_s, \underline{succ(n)})}$$

$\equiv C_s[n, ind_{\mathbb{N}}(x : C, C_0, x, y, C_s, n) / x, y] : C[succ(n)/x]$

computations

dependent function types (Π-types)

$$\frac{\Gamma \vdash A : U \qquad \Gamma, x:A \vdash B : U}{\Gamma \vdash \Pi_{x:A} B : U} \quad (\Pi\text{-formation})$$

$$\frac{\Gamma, x:A \vdash b : B}{\Gamma \vdash \lambda(x:A).b : \Pi_{x:A} B} \quad (\Pi\text{- introduction})$$

$$\frac{\Gamma \vdash f : \Pi_{x:A} B \qquad \Gamma \vdash a:A}{\Gamma \vdash f(a) : B[a/x]} \quad (\Pi\text{-elimination})$$

$$\frac{\Gamma, x:A \vdash b : B , \quad \Gamma \vdash a:A}{\Gamma \vdash (\lambda(x:A).b)(a) \equiv b[a/x] : B[a/x]} \quad \Pi \text{ computation}$$

~~de~~ dependent pair types (Σ-types)

$$\frac{\Gamma \vdash A : U \quad \Gamma, x:A \vdash B : U}{\Gamma \vdash \Sigma_{x:A} B : U} \quad (\Sigma\text{-formation})$$

$$\frac{\Gamma, x:A \vdash B : U \quad \Gamma \vdash a:A \quad \Gamma \vdash b : B[a/x]}{\Gamma \vdash (a,b) : \Sigma_{x:A} B} \quad (\Sigma\text{-introduction})$$

$$\frac{\Gamma, z : \Sigma_{x:A} B \vdash C : U \quad \Gamma, x:A, y:B \vdash g : C[(x,y)/z] \quad \Gamma \vdash p : \Sigma_{x:A} B}{\Gamma \vdash \mathrm{ind}_{\Sigma_{x:A} B}(z, C, x, x, y, y, p) : C[p/z]} \quad \Sigma\text{-elimination}$$

~~ex~~ double : $\mathbb{N} \to \mathbb{N}$ recursion

double $(0) :\equiv 0$

double $(\mathrm{succ}(u)) :\equiv \mathrm{succ}(\mathrm{succ}(\mathrm{double}(u)))$

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$double(2) \equiv double(succ(succ(0)))$

$\equiv C_s(succ(0), double(succ(0))$

$\equiv succ(succ(double(succ(0)))$

$\equiv succ(succ(C_s(0, double(0)))$

$\equiv succ(succ(succ(succ(double(0))))$

$\equiv Succ(Succ(Succ(succ(C_0)))) = 4$

addition

$C_0 : \mathbb{N} \to \mathbb{N}$

$C_0(n) :\equiv n$

$C_s : \mathbb{N} \to (\mathbb{N} \to \mathbb{N}) \to (\mathbb{N} \to \mathbb{N})$

$C_s(m, g)(n) \equiv succ(g(n))$

$\mathbb{N} \times \mathbb{N} \to \mathbb{N}$

$add : \mathbb{N} \to (\mathbb{N} \to \mathbb{N})$

$add(0, n) \equiv n$

$add(succ(m), n) \equiv succ(add(m, n))$

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