

第4回

8/2(火)

Adjointness (随伴)

Definition A, X categories

An adjunction from X to A

triple $\langle F, G, \varphi \rangle = X \rightarrow A$

F, G functors $X \begin{matrix} \xrightarrow{F} \\ \xleftarrow{G} \end{matrix} A$

$\varphi = \varphi_{x,a} = A(Fx, a) \cong X(x, Ga)$ bijection
 natural in x and a
 $(x \in X, a \in A)$

$$x = Ga \quad A(FGa, a) \cong X(Ga, Ga)$$

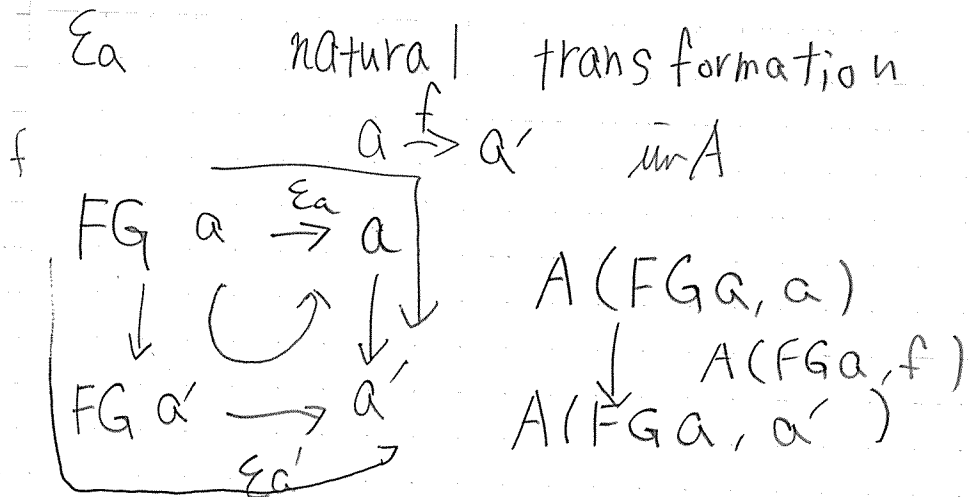
\downarrow
 id_{Ga}

$\varepsilon_a = FG a \rightarrow a$

$$A(Fx, Fx) \cong X(x, GFx) \quad a = Fx$$

\downarrow
 id_{Fx}

$\eta_x = x \rightarrow GFx$



idea

$$\epsilon_a \in A(FG a, a) \quad A(FG a, a') \xrightarrow{A(FG f, a')} A(FG a', a')$$

$$A(FG a, f) \downarrow \quad \downarrow x(G a, G f)$$

$$A(FG a, a') \cong x(G a, G a')$$

$$A(FG f, a') \uparrow \quad \uparrow x(G f, G a')$$

$$\epsilon_{a'} \in A(FG a', a') \cong x(G a', G a')$$

$id_{G a'}$ natural transformation

$$X \begin{array}{c} \xrightarrow{F} \\ \xleftarrow{G} \end{array} A$$

$$G \xrightarrow{\eta_G} GFG \xrightarrow{G\varepsilon} G$$

$$A \begin{array}{c} \xrightarrow{id_{FGa}} \\ \cong \end{array} X \begin{array}{c} \xrightarrow{\eta_{Ga}} \\ \cong \end{array} (Ga, GFGa)$$

$$\begin{array}{ccc} \downarrow A(FGa, \varepsilon_a) & & \downarrow X(Ga, G\varepsilon_a) \\ A(FGa, a) & \cong & X(Ga, Ga) \end{array}$$

$$F \xrightarrow{F\eta} \underline{FGF} \xrightarrow{\varepsilon_F} F$$

$$X(GFx, GFx) \begin{array}{c} \xrightarrow{id_{GFx}} \\ \cong \end{array} A(Fx, FGFx)$$

$$\begin{array}{ccc} \downarrow & & \downarrow A(Fx, \varepsilon_a Fx) \\ X(GFx, x) & \cong & A(Fx, Fx) \\ & & \downarrow id_{Fx} \end{array}$$

$$A(Fx, a) \cong X(x, Ga)$$

$$\eta: I_X \rightarrow GF \quad \left\{ \begin{array}{l} \text{natural} \\ \text{transformation} \end{array} \right.$$

$$\varepsilon = FG \rightarrow IA \quad \underline{GF}$$

$$X \begin{array}{c} \xrightarrow{F} \\ \xleftarrow{G} \end{array} A$$

逆写像

$$Fx \xrightarrow{f} a$$

$$x \xrightarrow{\eta_x} GFx \xrightarrow{GF} Ga$$

$$x \xrightarrow{g} Ga$$

$$Fx \xrightarrow{Fg} FGa \xrightarrow{\varepsilon_a} a$$

$$\begin{array}{ccccc} Fx & \rightarrow & FGFx & \xrightarrow{FGf} & FGa \\ \downarrow \varepsilon_{Fx} & & \downarrow & \curvearrowright & \downarrow \varepsilon_a \\ Fx & \xrightarrow{f} & & & a \end{array}$$

$$\begin{array}{c} A(Fx, a) \cong X(x, G) \\ \downarrow \\ A(Fx, a') \cong G, a' \end{array}$$