

universal arrow

第3回

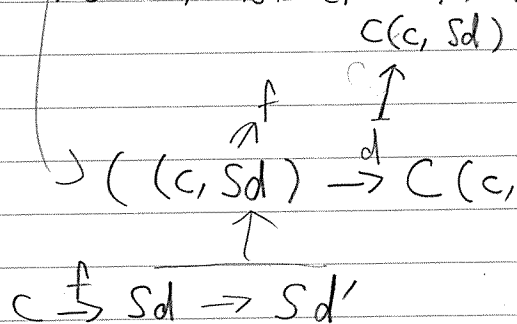
$S = D \rightarrow C$ functor $r = D$ の object
 $c = C$ の object

$\langle r, c \rightarrow Sr \rangle =$ universal arrow from c to S

iff $D(r, d) \cong C(c, Sd)$ bijection

$r \xrightarrow{f'} d$
 $c \xrightarrow{u} Sr \xrightarrow{Sf'} Sd$
 This bijection is natural in d .

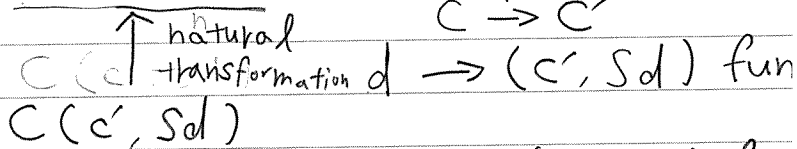
任意の category C の object C について universal arrow from C to S が存在 (トキと仮定)



$D \rightarrow \text{Set}$ functor

$D((Tc), d) \cong C(c, Sd)$

representable (表現可)



$d \rightarrow C(c, Sd)$ func

$$C(c', Sd') \longrightarrow C(c, Sd')$$



$$d \xrightarrow{g} d' Sd'$$

$$c' \xrightarrow{\varphi} Sd$$

$$C(c', Sd) \longrightarrow C(c, Sd) \quad f \uparrow$$

$$C \xrightleftharpoons[S]{T} D$$



Yoneda lemma

Txi3 functor

$$TC \xrightarrow{Tf} TC' \xrightarrow{Tf'} TC'' \quad f: C' \rightarrow C''$$

$$T(f' \circ f) = T(f') \circ T(f)$$

$$D(TC, d) \cong C(c, Sd) \text{ bijection.}$$

$d \in \text{nat} \} \text{ natural}$
 $c \in \text{nat} \}$

adjunction (随伴)

$$S: D \rightarrow C \quad C \xrightleftharpoons[S]{T} D \quad \text{forgetful functor (忘却関数)}$$

$D = \text{線形空間}$ の category $\rightarrow C = \text{集合}$ の category
 $C \rightarrow S_r$

群の category

自由群

adjunction が先にあった場合 X x, y $xyzyx^{-1}y$
 xx^{-1}

$$\text{id}_{Tc} : Tc \rightarrow Tc$$

$d \in \text{Ob } Tc$ をとる。

$$D(Tc, Tc) \cong C(c, STc)$$

universal from c to S
 $c \rightarrow STc$

$$f: Tc \rightarrow d \quad \downarrow D(Tc, f)$$

$$D(Tc, d) \cong C(c, Sd)$$

$\langle Tc, c \rightarrow STc \rangle$

$$D(Tc, d)$$

$$c \rightarrow STc \rightarrow Sf$$

$$D \xleftarrow{T} C$$

universal arrow from T to d

