第12回

homology 代数 ⇒ 計算機科学

(1) Universal arrows

(2) The Yoneda lemma

森田紀一
Mrita dua

Prop　　functor $S: D \to C$

$\langle r, u : c \to Sr \rangle$　　universal from c to S

$f': r \to d \Rightarrow c \xrightarrow{u} Sr \xrightarrow{Sf'} Sd$

$D(r,d) \cong C(c,$

natural in d

Definition　　$hom_D(x,y)$ 集合　x,y objects of

D : category having small hom-sets

$k: D \to \underline{Set}$　　functor　　r: object of D

$k(x) \cong D(r,x)$　　$k \cong D(r, \_)$

$k(f) \downarrow$　　$\downarrow D(r,f)$

$k(y) \cong D(r,y)$

bijection

natural isomorphis

$\underline{D(x,y)} = hom_D(x,y)$

representable (表現可

representing object 表現する対象

$x \mapsto D(r \xrightarrow{g} x)$

$\downarrow D(r,f)$

$\underline{D(r,y)}$

$r \xrightarrow{g} x$

$\downarrow f$

y

f

x

$\uparrow D$

y

Lemma (Yoneda) D : category with small hom

$k : D \to Set$ functor

r : object in D

$Nat(D(r,-), k) \cong kr$

証明

$D(r,f)(1_r) = f$

$r \xrightarrow{1_r} r$ $D(r,r) \longrightarrow kr$

$D(r,f) \downarrow$ $\downarrow kf$

$D(r,x) \longrightarrow kx$

$f : r \to x$

$D(r,x) \xrightarrow{D(r,h)} D($

$s \xrightarrow{g} r \xrightarrow{f} x$

$h : x \to y$

$D(s,x) \to D($

$D(s,h)$

基底

Corollary

$D \to Set$

D category with small hom-sets

r, s : objects in D

$D(r,-)$

$D(r,r)$

$D(r,x)$

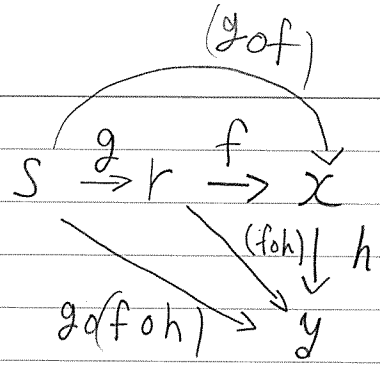
$D(s,-)$

$D(s,r)$

$s \xrightarrow{g} r$

$\searrow \downarrow$

x



universal arrow

$S : D \to C$ $\quad c \to Sr$ $\quad c$ object in

$D(r,d) \cong C(c, Sd)$ bijection

$r \xrightarrow{f'} d$

category C の任意の object c に対して 必ず universal arrow が存在する場合.

$c \to Sr \xrightarrow{Sf'} Sd$

$c \xrightarrow{h} c' \to Sr'$

$D(r,d) \cong C(c, Sd)$

$\uparrow$

$D(r',d) \cong C(c', Sd)$

$c \xrightarrow{f} c' \xrightarrow{g} Sd$

T : functor

$T(f' \circ f) = T(f') \circ T(f)$

$r \xrightarrow{h} d$

$c \xrightarrow{f} c' \to Sr' \xrightarrow{Sk} Sd$

$k = T(f)$

$c \to Sr$

$f \downarrow \quad \curvearrowright \quad \downarrow$

$c' \to Sr'$

$r = T(c)$

$r' = T(c')$