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Date '16 . 7 . 27

微積分演習 第15回

(4) の解答.

$$A = \begin{pmatrix} -1 & 2 \\ -1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$|tE - A| = \begin{vmatrix} t+1 & -2 \\ 1 & t-1 \end{vmatrix}$$
$$= (t+1)(t-1) + 2$$
$$= t^2 + 1$$

固有値 $t = \pm i$

$$A = P^{-1} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} P$$

$$e^{tA} = P^{-1} \underline{e^{t\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}}} P$$

$$e^{ti} = \cos t + i \sin t$$

$$\begin{cases} x = C_{11} \cos t + C_{12} \sin t \\ y = C_{21} \cos t + C_{22} \sin t \end{cases}$$ の形

$$x' = -C_{11} \sin t + C_{12} \cos t = -x + 2y$$
$$y' = -C_{21} \sin t + C_{22} \cos t = -x + y$$

$$C_{12} + C_{11} - 2C_{21} = 0$$
$$-C_{11} + C_{12} - 2C_{22} = 0$$
$$\longrightarrow \quad C_{12} = C_{21} + C_{22}, \quad C_{11} = C_{21} - C_{22}$$

$$\begin{cases} x = (C_{21} - C_{22}) \cos t + (C_{21} + C_{22}) \sin t \\ y = C_{21} \cos t + C_{22} \sin t \end{cases}$$

周期解

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

逆行列を求める. $B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$ とする。

$$AB = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} a_{11}b_{11}+a_{12}b_{21} & a_{11}b_{12}+a_{12}b_{22} \\ a_{21}b_{11}+a_{22}b_{21} & a_{21}b_{12}+a_{22}b_{22} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = E$$

$$\begin{cases} a_{11}b_{11}+a_{12}b_{21} = 1 & \cdots\cdots ① \\ a_{11}b_{12}+a_{12}b_{22} = 0 & \cdots\cdots ② \\ a_{21}b_{11}+a_{22}b_{21} = 0 & \cdots\cdots ③ \\ a_{21}b_{12}+a_{22}b_{22} = 1 & \cdots\cdots ④ \end{cases}$$

$|AB| = |E| = 1$

$|A|\,|B| = 1$

よって $|A| \neq 0$

①$\times a_{21}$ − ③$\times a_{11}$ をすると.

$$\begin{array}{rl} & a_{21}a_{11}b_{11} + a_{21}a_{12}b_{21} = a_{21} \\ -) & a_{11}a_{21}b_{11} + a_{11}a_{22}b_{21} = 0 \\ \hline & a_{21}a_{12} - a_{11}a_{22} = a_{21} \end{array}$$

$$b_{21} = \frac{-a_{21}}{a_{11}a_{22}-a_{12}a_{21}}$$

①$\times a_{22}$ − ③$\times a_{12}$ をすると.

$$(a_{11}a_{22}-a_{12}a_{21})\,b_{11} = a_{22}$$

$$b_{11} = \frac{a_{22}}{a_{11}a_{22}-a_{12}a_{21}}$$

このようにして計算すると.

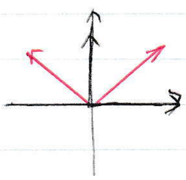
$$B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \frac{1}{a_{11}a_{22}-a_{12}a_{21}} \begin{pmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{pmatrix}.$$

・座標の変換

$a = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \in \mathbb{R}^2$

$e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ を使って.

一通りに $a = a_1 e_1 + a_2 e_2$ と書ける.



ところで. $\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ を使っても

$a = \bigcirc \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \bigcirc \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ と書ける.

同一直線上にない 2つのベクトル であればよい

基底

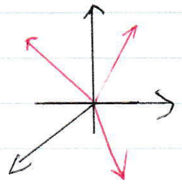
$\boxed{\text{1次独立　線型独立}}$

$\mathbb{R}^3$の場合

$a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \in \mathbb{R}^3$

$e_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$. $e_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$. $e_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ を使って

一通りに $a = a_1 e_1 + a_2 e_2 + a_3 e_3$ と書ける.



他の. 同一平面上にない 3つのベクトル であれば.

一通りに書ける。　基底

$\mathbb{R}^4, \cdots$ なども同様に考えられる

- 基底のとりかえ

$\mathbb{R}^2$のとき. (a_1, a_2) 基底
(b_1, b_2) 〃 とする。

$b_1 = \alpha_{11} a_1 + \alpha_{21} a_2$
$b_2 = \alpha_{12} a_1 + \alpha_{22} a_2$ と表せる $(\alpha_{11}, \alpha_{12}, \alpha_{21}, \alpha_{22} \in \mathbb{R})$

$$(b_1\ b_2) = (a_1\ a_2)\underbrace{\begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix}}_{A}$$ 基底の交換の行列

同じように
$$(a_1\ a_2) = (b_1\ b_2)\underbrace{\begin{pmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{pmatrix}}_{B}$$

$$(a_1\ a_2) = (a_1\ a_2)AB$$

$$AB = E.$$

- 線型写像を表す行列

線型写像 $\varphi : \mathbb{R}^2 \to \mathbb{R}^2$

$$\begin{cases} \varphi(a_1 + a_2) = \varphi(a_1) + \varphi(a_2) \\ \varphi(\alpha a) = \alpha \varphi(a) \end{cases}$$

基底 e_1, e_2 を決める。
$a = a_1 e_1 + a_2 e_2$ と書ける。$(a_1, a_2 \in \mathbb{R})$

$$\begin{aligned} \varphi(a) &= \varphi(a_1 e_1 + a_2 e_2) \\ &= a_1 \varphi(e_1) + a_2 \varphi(e_2) \end{aligned}$$ 線型性

線型写像は $\varphi(e_1), \varphi(e_2)$ が決まれば、決まる。

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$$\varphi(e_1) = \alpha_{11} e_1 + \alpha_{21} e_2$$
$$\varphi(e_2) = \alpha_{12} e_1 + \alpha_{22} e_2$$

$$(e_1\ e_2)\begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix}$$ ……… φを基底e_1.e_2で表す行列