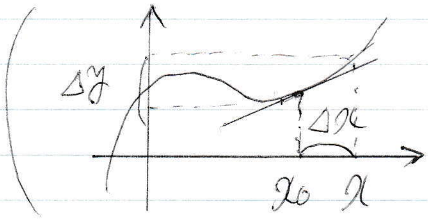
微積分 第5回

微分

曲がっているものを
まっすぐなものでおきかえる。



$f: \mathbb{R} \to \mathbb{R}$

$\Delta x = x - x_0$

$\Delta y = f(x) - f(x_0)$

$\Delta y = g(\Delta x)$ 複雑

線型写像

接線

比例関数

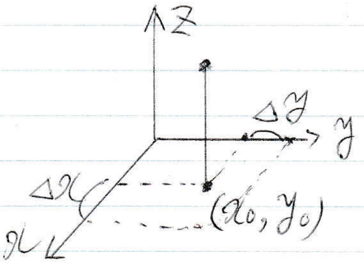
1次元の線型写像

比例定数

特徴づけられる

$f(x)$ 微分係数

$f: \mathbb{R}^2 \to \mathbb{R}$



$\Delta z = g(\Delta x, \Delta y)$

線型写像
行列で表される

$$\underset{1\times2}{[a\ b]}\underset{2\times1}{\begin{pmatrix}\Delta x\\ \Delta y\end{pmatrix}} = \underset{1\times1}{a\Delta x + b\Delta y}$$

$$a = \frac{\partial f}{\partial x}(x_0, y_0)$$

$$b = \frac{\partial f}{\partial y}(x_0, y_0)$$

$f: \mathbb{R}^2 \to \mathbb{R}^2$

点(x_0, y_0)で微分すると

$\mathbb{R}^2 \to \mathbb{R}^2$への線型写像

2×2行列で表される

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} = \begin{pmatrix} a_{11}\Delta x + a_{12}\Delta y \\ a_{21}\Delta x + a_{22}\Delta y \end{pmatrix}$$

決定するには…

$$f(x, y) = \begin{pmatrix} f_1(x, y) \\ f_2(x, y) \end{pmatrix}$$

$f_1 : \mathbb{R}^2 \to \mathbb{R}$

$f_2 : \mathbb{R}^2 \to \mathbb{R}$

$$a_{11} = \frac{\partial f_1}{\partial x}(x_0, y_0) \qquad a_{12} = \frac{\partial f_1}{\partial y}(x_0, y_0)$$

$$a_{21} = \frac{\partial f_2}{\partial x}(x_0, y_0) \qquad a_{22} = \frac{\partial f_2}{\partial y}(x_0, y_0)$$

合成関数の微分

(数Ⅲ)

$f : \mathbb{R} \to \mathbb{R}$

$g : \mathbb{R} \to \mathbb{R}$

$$(g \circ f)(x) = g(f(x))$$

$$(g \circ f)'(x) = g'(f(x)) \cdot f'(x)$$

$f : \mathbb{R}^2 \to \mathbb{R}^2, \quad g : \mathbb{R}^2 \to \mathbb{R}^2$

$u = f_1(x, y) = u(x, y) \qquad g_1(u, v)$
$v = f_2(x, y) = v(x, y) \qquad g_2(u, v)$

合成関数 $(g \circ f)(x, y) = g(f(x, y))$

微分
$g_1(f(x, y))$
$g_2(f(x, y))$

$(x_0, y_0) \mapsto (u_0, v_0)$

(1,1)成分計算

$$\begin{bmatrix} \dfrac{\partial (g_1 \circ f)}{\partial x} & \dfrac{\partial (g_1 \circ f)}{\partial y} \\ \dfrac{\partial (g_2 \circ f)}{\partial x} & \dfrac{\partial (g_2 \circ f)}{\partial y} \end{bmatrix}$$

$$\boxed{\frac{\partial g_1}{\partial u}(u_0, v_0) \frac{\partial f_1}{\partial x}(x_0, y_0) + \frac{\partial g_1}{\partial v}(u_0, v_0) \frac{\partial f_2}{\partial x}(x_0, y_0)}$$

(1,1)成分

$$\underbrace{\begin{bmatrix} \dfrac{\partial g_1}{\partial u} & \dfrac{\partial g_1}{\partial v} \\ \dfrac{\partial g_2}{\partial u} & \dfrac{\partial g_2}{\partial v} \end{bmatrix}}_{g'(u_0, v_0)} \underbrace{\begin{bmatrix} \dfrac{\partial u}{\partial x} & \dfrac{\partial u}{\partial y} \\ \dfrac{\partial v}{\partial x} & \dfrac{\partial v}{\partial y} \end{bmatrix}}_{f'(x_0, y_0)}$$

線型化

合成

$(g \circ f)'(x, y) = g'(f_1(x_0, y_0), f_2(x_0, y_0)) \cdot f'(x_0, y_0)$

$\mathbb{R}^2 \to \mathbb{R}^2$ への線型写像 $\qquad \mathbb{R}^2 \to \mathbb{R}^2$ への線型写像

No. 4

Date

問題. 合成関数の微分 $\dfrac{dz}{dt}$ を計算せよ.

(1) $z = x^2 + 2xy^2 + 3y$

$x = t^2 - t + 2$

$y = 2t + 1$

$t \longrightarrow \begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow z$

(2) $z = \sin(xy)$

$x = t + e^t$

$y = te^{-t}$

$f : \mathbb{R} \longrightarrow \mathbb{R}$ $\qquad D = \{ d \in \mathbb{R} \mid d^2 = 0 \}$

$x \in \mathbb{R}$

公理

$\varphi : D \longrightarrow \mathbb{R}$

$\exists! a \in \mathbb{R}$

$\forall d \in D \quad \varphi(d) = \varphi(0) + ad \qquad$ dの1次式

$\varphi : d \in D \longmapsto f(x+d) \in \mathbb{R}$

$$\underbrace{\varphi(d)}_{f(x+d)} = \underbrace{\varphi(0)}_{f(x)} + ad$$

$$f(x+d) = f(x) + \underbrace{a}_{f'(x)} d$$

$f : D \to \mathbb{R}^m$ $\quad \begin{pmatrix} f_1 \\ \vdots \\ f_m \end{pmatrix}$ $\quad f_i : D \to \mathbb{R}$

$\exists ! a_i \in \mathbb{R}, \quad f_i(d) = f_i(0) + a_i d$

$$f(d) = \begin{pmatrix} f_1(d) \\ \vdots \\ f_m(d) \end{pmatrix} = \begin{pmatrix} f_1(0) + a_1 d \\ \vdots \\ f_m(0) + a_m d \end{pmatrix}$$

$$= \begin{pmatrix} f_1(0) \\ \vdots \\ f_m(0) \end{pmatrix} + \begin{pmatrix} a_1 \\ \vdots \\ a_m \end{pmatrix} d$$

$$= f(0) + a\, d$$

公理から
導かれた。

$f : \mathbb{R}^n \to \mathbb{R}^m$
$x \in \mathbb{R}^n$
$a \in \mathbb{R}^n$

$$d \in D \longmapsto f(x + ad) \in \mathbb{R}^m$$

$$f(x + ad) = f(x) + \underset{\substack{\in \mathbb{R}^m \\ f'(x)(a)}}{?}\, d$$

$$\begin{cases} f'(x)(a_1 + a_2) = f'(x)(a_1) + f'(x)(a_2) \\ f'(x)(\alpha a) = \alpha f'(x)(a) \end{cases}$$

$f'(x)$ が $\mathbb{R}^n$ から $\mathbb{R}^m$ への線型写像.
(次回)