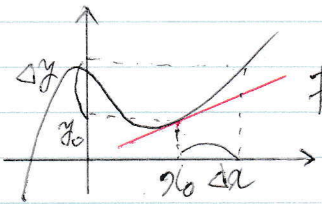
No. 1
Date '16. 5. 23

# 微積分第4回

## 微分

基本的なidea
曲がっているものを
まっすぐなもので置き換えよう




接線

$$\Delta y = f(x_0+\Delta x) - f(x_0)$$
$$= g(\Delta x)$$

傾き
微分係数 $f'(x_0)$

接線
$$\Delta y = \underset{\text{定数}}{a}\,\Delta x$$

多変数
2変数　接平面
3変数　代数的に扱う

$f : \mathbb{R} \to \mathbb{R}$

$f(x) = ax$　比例関数

$$\begin{cases} f(x+y) = f(x) + f(y) \\ f(\alpha x) = \alpha f(x) \quad (\alpha \in \mathbb{R}) \end{cases}$$

線形(型)関数という.

一般化
$f : \mathbb{R}^n \to \mathbb{R}^m$
n次列ベクトル　m次列ベクトル

$$\begin{cases} f(x+y) = f(x) + f(y) \\ f(\alpha x) = \alpha f(x) \end{cases}$$

線形関数

和 $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} x_1+y_1 \\ x_2+y_2 \end{pmatrix}$

スカラー倍 $\alpha \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \alpha x_1 \\ \alpha x_2 \end{pmatrix}$

$$\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix},\ \mathbf{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix},\ \cdots,\ \mathbf{e}_n = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}$$ を用いて

$$x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = x_1\mathbf{e}_1 + x_2\mathbf{e}_2 + \cdots + x_n\mathbf{e}_n$$ と表せる.

$$\begin{aligned} f(x) &= f(x_1\mathbf{e}_1 + x_2\mathbf{e}_2 + \cdots + x_n\mathbf{e}_n) \\ &= f(x_1\mathbf{e}_1) + \cdots + f(x_n\mathbf{e}_n) \\ &= x_1 \underbrace{f(\mathbf{e}_1)}_{a_1} + \cdots + x_n \underbrace{f(\mathbf{e}_n)}_{a_n} \end{aligned}$$ とする.

$$x_1 a_1 + \cdots + x_n a_n$$

$$a_1 = \begin{pmatrix} a_{11} \\ \vdots \\ a_{m1} \end{pmatrix} \qquad a_n = \begin{pmatrix} a_{1n} \\ \vdots \\ a_{mn} \end{pmatrix}$$

$$[a_1 \cdots a_n] = m\text{個}\left\{ \overbrace{\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}}^{n\text{個}} \right.$$ $m \times n$ 行列

$$f(x) = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

$$= \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \end{bmatrix}$$

線形写像は
行列で表現できる

例 $m=n=2$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{pmatrix} 5 \\ 6 \end{pmatrix} = \begin{pmatrix} 1\times 5 + 2\times 6 \\ 3\times 5 + 4\times 6 \end{pmatrix} = \begin{pmatrix} 17 \\ 39 \end{pmatrix}$$

$m=n=1$

(小学校) 1次元の線形代数

$f : \mathbb{R} \to \mathbb{R}$

1×1の行列 $[a](x) = (ax)$

---

$f, g : \mathbb{R}^n \to \mathbb{R}^m$ 線形とする.

$(f+g)(x) = f(x) + g(x)$ と定義する.

$(f+g)$も線形である.

$$\begin{aligned} (f+g)(x+y) &= f(x+y) + g(x+y) \\ &= f(x) + f(y) + g(x) + g(y) \\ &= \underline{f(x) + g(x)} + \underline{f(y) + g(y)} \\ &= (f+g)(x) + (f+g)(y) \end{aligned}$$

report I

$\alpha \in \mathbb{R}$ $(f+g)(\alpha x) = \alpha (f+g)(x)$ を証明せよ

$(\alpha f)(x) = \alpha f(x)$ と定義する.

$(\alpha f)$も線形である

$$\begin{aligned} (\alpha f)(x+y) &= \alpha f(x+y) \\ &= \alpha \{ f(x) + f(y) \} \\ &= \alpha f(x) + \alpha f(y) \\ &= (\alpha f)(x) + (\alpha f)(y) \end{aligned}$$

report II

$(\alpha f)(\beta x) = \beta (\alpha f)(x)$ を証明せよ.

No. 8

Date

$f, g : \mathbb{R}^n \to \mathbb{R}^m$ 線形

$f \longleftrightarrow A$ $m \times n$ 行列が対応

$g \longleftrightarrow B$

$f+g \longleftrightarrow \bigcirc$

$x = x_1 e_1 + \cdots\cdots + x_n e_n$

$$(f+g)(x) = x_1 \underline{(f+g)(e_1)} + \cdots\cdots + x_n \underline{(f+g)(e_n)}$$

$$\left[ (f+g)(e_1) \ \cdots\cdots \ (f+g)(e_n) \right]$$

$$\left[ f(e_1)+g(e_1) \ \cdots\cdots \ f(e_n)+g(e_n) \right]$$

$$\underset{A}{\left[ f(e_1) \cdots f(e_n) \right]} + \underset{B}{\left[ g(e_1) \cdots g(e_n) \right]}$$

$n = m = 2$

$$f \longleftrightarrow A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

$$g \longleftrightarrow B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$$

$$f+g \longleftrightarrow A+B = \begin{pmatrix} a_{11}+b_{11} & a_{12}+b_{12} \\ a_{21}+b_{21} & a_{22}+b_{22} \end{pmatrix}$$

$$\alpha f \longleftrightarrow \alpha A = \begin{pmatrix} \alpha a_{11} & \alpha a_{12} \\ \alpha a_{21} & \alpha a_{22} \end{pmatrix}$$

$f: \mathbb{R}^n \to \mathbb{R}^m$, $g: \mathbb{R}^m \to \mathbb{R}^l$ 線形とする

合成 $(g \circ f)(x) = g(f(x))$ も線形である

$$
\begin{aligned}
(g \circ f)(x+y) &= g(f(x+y)) \\
&= g(f(x)+f(y)) \\
&= g(f(x)) + g(f(y)) \\
&= (g \circ f)(x) + (g \circ f)(y)
\end{aligned}
$$

report III

$(g \circ f)(\alpha x) = \alpha (g \circ f)(x)$ を証明せよ

---

$f \longleftrightarrow A$ $m \times n$ 行列

$g \longleftrightarrow B$ $l \times m$ 行列

$g \circ f \longleftrightarrow \bigcirc$

$l = m = n = 2$ とする.

$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$, $B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$

$f(x) = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ $g(y) = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$

$$
\begin{aligned}
(g \circ f)(x) &= g(f(x)) \\
&= g\left( \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \right) \\
&= g \begin{pmatrix} a_{11}x_1 + a_{12}x_2 \\ a_{21}x_1 + a_{22}x_2 \end{pmatrix} \\
&= \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \begin{pmatrix} a_{11}x_1 + a_{12}x_2 \\ a_{21}x_1 + a_{22}x_2 \end{pmatrix} \\
&= \begin{pmatrix} b_{11}(a_{11}x_1 + a_{12}x_2) + b_{12}(a_{21}x_1 + a_{22}x_2) \\ b_{21}(a_{11}x_1 + a_{12}x_2) + b_{22}(a_{21}x_1 + a_{22}x_2) \end{pmatrix}
\end{aligned}
$$

No. 6

Date

$$= \begin{pmatrix} (b_{11}a_{11}+b_{12}a_{21})x_1+(b_{11}a_{12}+b_{12}a_{22})x_2 \\ (b_{21}a_{11}+b_{22}a_{21})x_1+(b_{21}a_{12}+b_{22}a_{22})x_2 \end{pmatrix}$$

$$= \begin{bmatrix} b_{11}a_{11}+b_{12}a_{21} & b_{11}a_{12}+b_{12}a_{22} \\ b_{21}a_{11}+b_{22}a_{21} & b_{21}a_{12}+b_{22}a_{22} \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \qquad A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$