Marketing Flexibility for the Management of Shopping Centers: Optimal Allocation of Sales Campaign Days and Campaign Budget for Maximizing Expected Profit

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Abstract

In retail business, a sales campaign is typically organized in one or two segments of consecutive days over a certain period, so as to maximize the expected total sales by organizing a sales campaign in such a way that, good-sales-days (GSD) of the previous year would be designated as sales campaign days in the future period with the expectation that the campaign effect could enhance the potential of GSDs further. However, there is no theoretical foundation to claim that it would be better to organize a sales campaign in such a way. This thesis challenges these common practices, based on the Marketing Flexibility concept, the results show that it could be more profitable to assign sales campaign days in a more flexible manner rather than in segments of consecutive days. To the best knowledge of the researcher, the problem of optimally allocating sales campaign days over a certain period, e.g. the winter and fall seasons, has not been addressed in the literature. The purpose of this thesis is to fill this gap by developing a mathematical model to optimize returns in an SC by optimally reallocating sales campaign days based on the marketing flexibility concept.

In the business practice of a Shopping Center (SC), one year is decomposed into 4 seasons: Spring (March through May), Summer (June through August), Fall (September through November) and Winter (December through February). Researchers usually study one or more seasons, as in (Pauwels, 2007; Poel et al., 2004; Arnold et al., 1983). In examining the performance of a sales campaign for an SC, the literature guides one to consider two main elements: the total sales and the number of the purchase transactions for the entire SC, as in Oliver and Swan (1989), Noordewier et al. (1990), and Parsons (2003).

In this thesis, a machine learning technique is employed to estimate whether or not a day is a GSD, this indicator function is composed from total sales and number of purchase transactions. For notational convenience, the set of days involved in the learning dataset (LD) is denoted by D_{LD}, and in the testing dataset (TD) is denoted by D_{TD}. The datasets LD and TD comprise the following elements; 1) the total sales of the $i-th$ day, denoted by $s(i)$, $i \in D_{LD} \cup D_{TD}$, for the entire SC, 2) the number of purchase transactions of the $i-th$ day, denoted by $t(i)$, for the entire SC, and 3) the campaign flag indicating if the $i-th$ day was under the sales campaign, denoted by $I_{CAMP}(i) = 1$, or $I_{CAMP}(i) = 0$, otherwise.

We consider the optimization problem of maximizing the total expected sales over a certain future period, by optimally reallocating N sales campaign days over a future period of M days. This optimization problem consists of four stages, succinctly described as follows; in Stage I, for day $i \in D_{LD}$, one determines the two indicator functions, $I_{CAMP}(i)$ for the sales campaign days, and $\hat{I}_{GOOD:S_0T_0}(i)$ for the GSDs, where S_0 is a numerical threshold level or the decile cut-off point in $s(i)$ and T_0 is defined similarly for $t(i)$. The numerical threshold levels S_0 and T_0 obtained from *LD*, are used to similarly determine $\hat{I}_{GOOD:S_0T_0:TD}(j)$, for $j \in D_{TD}$.

In Stage II, a logistic regression model is developed, given the campaign day assignment vector, denoted by $\underline{d} = [d(1), \cdots, d(j), \cdots, d(M)] \in \{0,1\}^M$ for $j \in D_{TD}$, where $d(j) = 1$ if day j is selected to be a sales campaign day, and $d(j) = 0$, otherwise, and by using the estimated coefficients of the explanatory variables of the logistic regression equation, one can estimate the likelihood value for day $j \in D_{TD}$ to be a GSD, denoted by $\rho_{GOOD}(j)$. The corresponding confusion matrix is employed to find the threshold level, denoted by ρ_{GOOD}, so as $\hat{I}_{GOOD}(j) = 1$ when $\rho_{GOOD}(j) \geq \rho_{GOOD}$ and $\hat{I}_{GOOD}(j) = 0$, otherwise. Consequently, one can determine whether or not a day is a GSD by specifying ${\rho^*}_{GOOD}$ associated with maximum Precision subject to Recall ≥ 0.75 obtained from the confusion matrix of the best logistic regression model.

The logistic regression models for both the winter and fall seasons contain the following significant variables in common: 1) Weekend flag: Saturday and Sunday, 2) Week_1: the first week (7 days) of the month, 3) LY_Transactions: the number of purchase transactions of the same day of the month of the last year, 4) Non-national and national holidays for winter and fall, respectively, in addition to, 5) Campaign flags for each season. The common measures for assessing the appropriateness of the likelihood value to estimate whether or not a day is a GSD is obtained from the confusion matrix, and given by Recall, Precision and Accuracy. This value is determined by considering the optimization problem of maximizing Precision subject to Recall ≥ 0.75.

In Stage III, we turn our attention to the issue of how to estimate the expected total sales for day $j \in D_{TD}$, given $\underline{d}$ and $\hat{I}_{GOOD}(j) = 1$ or 0. Based on $I_{CAMP}(i)$ and $\hat{I}_{GOOD:S_0T_0}(i)$ for $i \in D_{LD}$, four values of average total sales are computed from *LD*, denoted by $\hat{s}_{(m,n)}$, where $m = I_{CAMP}(i)$ and $n = \hat{I}_{GOOD:S_0T_0}(i)$, $m, n \in \{0,1\}$. Based on $\hat{s}_{(m,n)}$, one can estimate the expected total sales for day j, denoted by $\hat{r}_{(m,n)}$ where $m = d(j)$ and $n = \hat{I}_{GOOD}(j), m, n \in \{0,1\}$. Subsequently, the total expected sales over the entire future period, denoted by $\hat{R}(\underline{d})$ can then be computed. In order to test the validity of this approach, one computes the relative accuracy of the total expected sales, $\hat{R}(\underline{d})$, and the actual aggregate total sales over that period, denoted by $R(\underline{I}_{CAMP})$.

In order to test the validity of this systematic approach for estimating expected total sales per day, the formula for computing total expected sales is used with actual campaign days in *TD*, and then compared with the actual total sales of that period, achieving a relative accuracy of less than 2% in both seasons (1.72% and 1.40% for winter and fall, respectively).

In Stage IV, given $\hat{r}_{(m,n)}$, we formulate the problem of optimally reallocating sales campaign days, specified by the campaign day assignment vector, $\underline{d}$ subject to $\sum_{j=1}^{M} d(j) \leq N$ so as to maximize the total expected sales. To assess the impact of this flexibility approach, one compares the optimal solution, $\hat{R}(\underline{d}^*)$ against the actual total sales, $R(\underline{I}_{CAMP})$, obtained from traditionally organizing sales campaign days in segments of consecutive days.

Two extensions of this optimization problem are further considered and treated separately. In the first extension, by introducing the campaign cost per day B_0, the objective function is modified to maximize the expected profit, denoted by $\hat{P}(\underline{d})$, rather than the total expected sales. This is achieved by optimally reallocating sales campaign days, specified by $\underline{d}$ subject to $\sum_{j=1}^{M} d(j) \leq N$. In the second extension, the campaign budget per day is enhanced to $B = B_0 + \Delta_B$, where Δ_B is the campaign budget increase. The optimal expected profit for this extension is achieved by incorporating both the campaign budget increase and the campaign day assignment vector as decision variables of the optimization problem. The campaign day assignment vector is specified here by $\underline{d}_{\Delta_B} = [d_{\Delta_B}(1), \cdots, d_{\Delta_B}(j), \cdots, d_{\Delta_B}(M)] \in \{0,1\}^M$. In order to formulate this optimization problem, the expected total sales per day should be estimated. For this purpose, one determines whether or not a day is a GSD, denoted in this extension by $\hat{I}_{GOOD:\Delta_B}(j)$, and defined similarly as $\hat{I}_{GOOD}(j)$.

For the second extension, a new model for estimating expected total sales per day is developed. Under the effect of the campaign budget increase, it is natural to assume that the expected total sales per day would be increased with the effect of diminishing returns. Accordingly, one defines the function $g(x)$ to be an increasing concave function of x expressing the strengthening effect of Δ_B on the expected total sales per day, where $g(0) = 1$, and $lim_{\Delta_B \to \infty}\, g(x) = 1 + \frac{a}{b}$. This strengthening effect is subject to the following conditions: 1) whether the sales campaign day $d^*(j) = 0$ under $\Delta_B = 0$ with $\hat{s}_{(0,l)}$, $l = \hat{I}_{GOOD}(j)$, switches to ${d_{\Delta_B}}^*(j) = 1$ under $\Delta_B > 0$. In such case, the expected total sales, denoted by $\hat{r}_{(0,l)\to(1,n)}$, is estimated by $\hat{s}_{(0,l)} + \{(\hat{s}_{(1,n)} - \hat{s}_{(0,l)}) \times g_s(\Delta_B)\}$, where $n =$

$\hat{I}_{GOOD:\Delta_B}(j)$. It is also subject to 2) whether $d^*(j) = d_{\Delta_B}{}^*(j) = 1$, in which the expected total sales per day, denoted by $\hat{r}_{(1,l)\to(1,n)}$, would be estimated by $\hat{s}_{(1,n)} \times g_{\neg s}(\Delta_B)$, and finally, 3) it is natural to assume no effect of the campaign budget increase on day $d_{\Delta_B}{}^*(j) = 0$.

In order to solve the optimization problem for maximizing expected profit, $\hat{P}\left(\underline{d}_{\Delta_B}{}^*,\ \Delta_B{}^*\right)$, one needs to estimate the values of the parameters (a_s, b_s) and $(a_{\neg s}, b_{\neg s})$ defining the functions $g_s(\Delta_B)$ and $g_{\neg s}(\Delta_B)$, respectively. For this purpose the partial derivative approach of sensitivity analysis is employed to examine the behavior of the system with different increments of the parameters a and b. By estimating the values of the parameters (a_s, b_s) and $(a_{\neg s}, b_{\neg s})$, the optimal expected profit, denoted by $\hat{P}\left(\underline{d}_{\Delta_B}{}^*,\ \Delta_B{}^*\right)$, can then be achieved by the optimal allocation of sales campaign days and the campaign budget. Finally, In order to assess the impact of the optimal allocation of sales campaign days against that of the optimal campaign budget decision, one compares the optimal expected profit $\hat{P}(\underline{d}^*)$ under $\Delta_B = 0$ against $\hat{P}\left(\underline{d}_{\Delta_B}{}^*,\ \Delta_B{}^*\right)$ with $\Delta_B > 0$.

Through numerical examples, the proposed model demonstrated the power of marketing flexibility. The optimization problem for maximizing total expected sales for the winter season yielded an optimal total expected sales of ¥ 385.78 million amounting to 7% increase from actual total sales over the future winter period. This optimal value is achieved by reallocating 36 sales campaign days over that period. In respect to the fall season, the optimization problem yielded ¥ 355.16 million amounting to 4.47% increase from actual total sales by optimally reallocating 19 sales campaign days. We note here that, actual sales campaign days were 36 and 19 for the winter and fall seasons, respectively. The results imply that, by mere reorganization of sales campaign days freely rather than in segments of consecutive days, the total expected sales is expected to increase with no additional cost.

Furthermore, we compare the effect of the optimal allocation of sales campaign days only against that of reallocating both sales campaign days and the campaign budget on expected profit. The results of the winter season indicated that, optimal expected profit increased by 7.84% from actual profit by optimally reallocating sales campaign days only. However, by optimally reallocating both sales campaign days and the campaign budget, optimal expected profit increased by 9.95% from actual profit. This implies that, the optimal campaign budget is responsible for only (9.95 – 7.84 = 2.26%) of the improvement in optimal expected profit. The numerical example of the fall season provided similar evidence. By optimally reallocating both sales campaign days and the campaign budget, optimal expected profit increased by 6.58% from actual profit. Comparing this result with the 4.79% increase rate from actual profit, achieved by optimally reallocating sales campaign days only, the optimal campaign budget would be responsible for only (6.58 – 4.79 = 1.79%).

In both numerical examples, the optimal campaign budget was responsible for about 2% only of the improvement in optimal expected profit, while the optimal allocation of sales campaign days was responsible for about double this amount in the fall season (4.79%) and more than triple this amount in the winter season (7.84%). This result is consistent with that reported by Fischer et al., (2011), they state that, profit improvement from better allocation across products or regions is much higher than that from improving the overall budget. Similarly, one can state that, optimal allocation of sales campaign days achieves better improvement in optimal expected profit than that achieved by only improving the overall budget.

The proposed approach would be quite useful for the management of an SC, where different stores in one place can organize common sales campaigns to share the advantages of implementing a marketing flexibility-based strategy. To effectively allocate resources, optimal allocation of sales campaign days is recommended to maximize returns. For further improvement, the campaign budget could be optimally allocated along with the sales campaign days. These recommendations challenge the common business practices of improving the overall budget of a sales campaign to further boost its effectiveness. For this approach to be implemented efficiently, it is recommended for the management of the SC to share the timetable of scheduled campaign days with its customers. With the advent of smart phones, reaching out to customers has never been easier. Visitors of the SC can be kept informed through traditional channels of communication and advertising as well.

The structure of this thesis is as follows. Chapter 1 states the purpose of this thesis and provides a succinct summary of the prevalent literature revolving around the topic of SCs and the concept of flexibility. It focuses on three different perspectives: the evolution of SCs, the evolution of research on SC, and the flexibility concept. To summarize the literature review, we focus on three different perspectives: the evolution of SCs, the evolution of research on SC, and the flexibility concept. In the evolution of SCs, the history of the development of SCs, and the context of their advancements were described. The history of the birth of the western-style SC in Japan was also discussed following the line of research in Tsutsui (2009). In the evolution of research on SC, the common business practices prevalent in the management of sales campaigns in SCs were discussed. One of the most crucial points noted in this connection, was the use of data accumulated through the POS system for analysis to develop marketing strategies and to achieve business excellence. In this regard, and because of the complexities involved in the management of the SC business in comparison to that of a single store, much more flexibility would be needed to enhance the profitability of an SC. The three main concepts of flexibility which were discussed here are: economic, organizational and business process flexibility. Understanding these different types of flexibility can facilitate the achievement of flexibility in the context of the management of SCs.

In Chapter 2, the dataset is described and the outliers are cleaned. Next, the mathematical model for the optimization problem of maximizing total expected sales is formulated and implemented on the winter season. Two main issues are addressed in this chapter as part of the mathematical model:1) how to determine whether or not a day is a GSD, and 2) how to estimate the expected total sales for that day, provided that, an allocation of N campaign days over that future period is decided. Two further extensions of this optimization problem are considered and treated separately in the next chapter.

Chapter 3 is devoted to the optimization problems of maximizing expected profit. By introducing the standard campaign budget, the optimization problem is modified to maximize expected profit rather than total expected sales and implemented on the winter season. In the second extension of the optimization problem; by enhancing the campaign budget per day, the campaign budget increase along with the campaign day assignment vector are both considered as decision variables of the optimization problem. In order to express the effect of the campaign budget increase over the expected total sales per day, a strictly increasing concave function is defined to express the campaign effect under an enhanced campaign budget. The chapter also contains general properties of the formal concave function and the total expected sales.

In Chapter 4, the mathematical models described in Chapter 2 and 3 are implemented on the fall season. And finally, Chapter 5 contains the conclusion and discussion. This chapter also covers limitations and possible future work.

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Notation

$\boldsymbol{D_{LD}}$	The set of days in the Learning Dataset (*LD*)
$\boldsymbol{D_{TD}}$	The set of days in the Testing Dataset (*TD*)
$\boldsymbol{i}$	The $i-th$ day, where $i \in D_{LD}$
$\boldsymbol{j}$	The $j-th$ day, where $j \in D_{TD}$
$\boldsymbol{s(i)}$	The total sales of the $i-th$ day, $i \in D_{LD} \cup D_{TD}$, for the entire SC
$\boldsymbol{t(i)}$	The number of purchase transactions of the $i-th$ day, $i \in D_{LD} \cup D_{TD}$ for the entire SC
$\boldsymbol{I_{CAMP}(i)}$	The campaign flag indicating if the $i-th$ day was under the sales campaign, denoted by $I_{CAMP}(i) = 1$, or $I_{CAMP}(i) = 0$, otherwise.
$\boldsymbol{N}$	The number of sales campaign days organized over a certain period
$\boldsymbol{M}$	The total number of days in a certain period
$\boldsymbol{S_0}$	The numerical threshold level, or the decile cut-off point, in total sales $s(i)$
$\boldsymbol{T_0}$	The numerical threshold level, or the decile cut-off point, in the number of purchase transactions $t(i)$
$\boldsymbol{GSD}$	Good-Sales-Day
$\boldsymbol{\hat{I}_{GOOD:S_0T_0}(i)}$	The indicator function for the GSD of $i \in D_{TD}$ based on S_0 and T_0, where $\hat{I}_{GOOD:S_0T_0}(i) = 1$ when $s(i) \geq S_0$ and $t(i) \geq T_0$, and $\hat{I}_{GOOD:S_0T_0}(i) = 0$, otherwise
$\boldsymbol{\hat{I}_{GOOD:S_0T_0:TD}(j)}$	The indicator function for the GSD of $j \in D_{TD}$ based on S_0 and T_0 obtained from LD, where $\hat{I}_{GOOD:S_0T_0:TD}(j) = 1$ when $s(j) \geq S_0$ and $t(j) \geq T_0$, and $\hat{I}_{GOOD:S_0T_0:TD}(j) = 0$, otherwise
$\boldsymbol{\hat{I}_{GOOD}(j)}$	The indicator function for GSD, estimated by the logistic regression model and the confusion matrix for day $j \in D_{TD}$, where $\hat{I}_{GOOD}(j) = 1$ when day $\hat{I}_{GOOD:S_0T_0:TD}(j) = \hat{I}_{GOOD}(j) = 1$ and $\hat{I}_{GOOD}(j) = 0$, otherwise
$\boldsymbol{B}$	The sales campaign budget per day
$\boldsymbol{B_0}$	The standard sales campaign budget per day as provided from the management of the SC
$\Delta_{\boldsymbol{B}}$	The campaign budget increase per day, where $\Delta_B = B - B_0$
$\underline{\boldsymbol{d}}$	The campaign day assignment vector, under $\Delta_B = 0$, specified by $\underline{d} = [d(1), \cdots, d(j), \cdots, d(M)] \in \{0,1\}^M$ for $j \in D_{TD}$, where $d(j) = 1$ if day j is selected to be a sales campaign day, and $d(j) = 0$, otherwise

$\underline{\boldsymbol{d}}_{\Delta_B}(\boldsymbol{j})$	The campaign day assignment vector, under $\Delta_\mathrm{B} > 0$, specified by $\underline{d}_{\Delta_B} = \left[d_{\Delta_B}(1), \cdots, d_{\Delta_B}(j), \cdots, d_{\Delta_B}(M)\right] \in \{0,1\}^M$ for $j \in D_{TD}$, where $d_{\Delta_B}(j) = 1$ if day j is selected to be a sales campaign day, and $d_{\Delta_B}(j) = 0$, otherwise
$\hat{\boldsymbol{s}}_{(\boldsymbol{m},\boldsymbol{n})}$	The average total sales subject to $m = I_{CAMP}(i)$ and $n = \hat{I}_{GOOD:S_0T_0}(i), m, n \in \{0,1\}$ obtained from LD
$\hat{\boldsymbol{r}}_{(\boldsymbol{m},\boldsymbol{n})}$	The expected total sales per day of the future period under $\Delta_B = 0$, subject to $m = d(j)$ and $n = \hat{I}_{GOOD}(j), m, n \in \{0,1\}$, estimated based on $\hat{s}_{(m,n)}$ obtained from LD
$\hat{\boldsymbol{r}}_{(\boldsymbol{k},\boldsymbol{l})\to(\boldsymbol{m},\boldsymbol{n})}$	The expected total sales per day under $\Delta_B > 0$, subject to $k = d^*(j)$, $l = \hat{I}_{GOOD}(j)$, $m = d_{\Delta_B}(j)$, and $n = \hat{I}_{GOOD:\Delta_B}(j)$, $k, l, m, n \in \{0,1\}$
$\widehat{\boldsymbol{R}}(\underline{\boldsymbol{d}})$	The total expected sales (aggregate total sales per day) over a certain future period, subject to the campaign day assignment vector $\underline{d}$ when $\Delta_B = 0$
$\widehat{\boldsymbol{R}}(\underline{\boldsymbol{d}}_{\Delta_B}, \Delta_{\boldsymbol{B}})$	The total expected sales (aggregate total sales per day) over a certain future period, subject to the campaign day assignment vector $\underline{d}_{\Delta_B}$ and the campaign budget increase Δ_B
$\widehat{\boldsymbol{P}}(\underline{\boldsymbol{d}})$	The expected profit over a certain future period, subject to the campaign day assignment vector $\underline{d}$ when $\Delta_B = 0$
$\widehat{\boldsymbol{P}}(\underline{\boldsymbol{d}}_{\Delta_B}, \Delta_{\boldsymbol{B}})$	The expected profit over a certain future period, subject to the campaign day assignment vector $\underline{d}_{\Delta_B}$ and the campaign budget increase Δ_B

1 Introduction and Literature Review

1.1 Purpose of the Thesis

In examining the performance of a sales campaign for a Shopping Center (denoted by SC, hereafter), the literature guides one to consider two main elements: the total sales and the number of the purchase transactions for the entire SC for each season, as in Oliver and Swan (1989), Noordewier et al. (1990), and Parsons (2003). A sales campaign is typically organized in segments of consecutive days over a certain period where a sales campaign is organized in such a way that, good-sales-days (denoted by GSD, hereafter) of the previous year would be designated as sales campaign days in the future period, with the expectation that the campaign effect could enhance the potential of good-sales-days further. However, there is no theoretical foundation to support such business practices. Real data obtained from an SC in Tokyo revealed that such common business practices do not necessarily yield better performance for the period of the subsequent year. Tables 1.1.1 and 1.1.2 below exhibit the data of fall 2008 and 2009, and winter 2009 and 2010. One sees that scheduling sales campaign days in the same manner as the previous year did not yield improvement in the total sales or the number of the purchase transactions for all the periods across the three years. The purpose of this thesis is to challenge this common practice of scheduling sales campaign days in segments of consecutive days. It will be shown that mere reorganization of campaign days with flexibility could increase the profitability of the SC significantly.

Table 1.1.1 Comparison of Sales Campaign Performance in Winter 2009 and 2010

	Winter 2009			**Winter 2010**		
	Entire Period	**Win_1**	**Win_2**	**Entire Period**	**Win_1**	**Win_2**
Start Date	12/01/2009	12/01/2009	01/04/2010	12/01/2010	12/01/2010	01/04/2011
End Date	28/02/2010	12/25/2009	01/12/2010	02/28/2011	12/28/2010	01/11/2011
Total Number of Days	88	28	7	88	28	8
Average Total Sales (¥ Million)	4.34	4.59	4.59	4.29	4.41	3.99
Average of Purchase Transactions	3,043	3,141	3,114	2,971	3,055	2,967

Table 1.1.2 Comparison of Sales Campaign Performance in Fall 2008 and 2009

	Fall 2008			**Fall 2009**		
	Entire Period	**Fall_1**	**Fall_2**	**Entire Period**	**Fall_1**	**Fall_2**
Start Date	09/01/2008	10/23/2008	11/21/2008	09/01/2009	10/23/2009	11/21/2009
End Date	11/30/2008	11/03/2008	11/30/2008	11/30/2009	11/03/2009	11/30/2009
Total Number of Days	90	7	11	90	7	12
Average Total Sales (¥ Million)	4.35	4.48	4.48	4.01	4.14	4.12
Average Purchase Transactions	3,120	3,140	3,139	2,919	2,971	2,999

The key concept of this thesis is Marketing Flexibility, which enables one to alter the business process for allocating sales campaign days so as to achieve improvement. In the SC example introduced in this thesis, sales campaign days are optimally allocated freely based on marketing flexibility in order to optimize returns (aggregate total sales and profit) in the SC. More specifically, given a set of data over the past periods and a future period for which campaign days should be scheduled, the main steps toward this goal are summarized below.

1. We first estimate whether or not a day in the future period is a GSD, denoted by GSD, based on the logistic regression and the confusion matrix for a tentatively given campaign assignment vector.
2. Depending on whether or not a future day is chosen as a campaign day and whether or not it is a GSD, we next estimate the expected total sales for that future day based on the past data.
3. Finally, an optimization problem is formulated where the optimal campaign assignment vector is determined so as to maximize the expected total sales.
4. In order to maximize the expected total profit rather than the expected total sales, a new model is developed where the expected total sales of a day under sales campaign can be increased as a concave function of the campaign budget. Here, the optimal campaign assignment vector and the optimal campaign budget would be determined simultaneously.

The remainder of this chapter is devoted to the literature review. Section 1.2 provides a succinct summary of the entire literature. In Section 1.3, a general history of the shopping centers in the U.S. and Japan is discussed. Section 1.4 summarizes the evolution of the research on shopping centers. In

Section 1.5, different concepts of flexibility are introduced, enabling one to position Marketing Flexibility in an appropriate perspective.

1.2 Summary of the Literature Review

To summarize the literature review, we focus on three different perspectives: the evolution of SCs, the evolution of research on SC, and the flexibility concept. In the evolution of SCs, the history of the development of SCs, and the context of their advancements were described. The history of the birth of the western-style SC in Japan was also discussed following the line of research in Tsutsui (2009). In the evolution of research on SC, the common business practices prevalent in the management of sales campaigns in SCs were discussed. One of the most crucial points noted in this connection was the use of data accumulated through the POS system for analysis to develop marketing strategies and to achieve business excellence. In this regard, and because of the complexities involved in the management of the SC business in comparison to that of a single store, much more flexibility would be needed to enhance the profitability of an SC. The three main concepts of flexibility which were discussed here are: economic, organizational and business process flexibility. Understanding these different types of flexibility can facilitate the achievement of flexibility in the context of the management of SCs.

1.3 Evolution of the Shopping Centers

In the first half of this section, a succinct summary of the evolution of SCs in the U.S. is provided based on Gruen and Smith (1967). A similar summary is given in the second half regarding SCs in Japan based on Tsutsui (2009).

The 1888 electric street car, made possible to establish “street car suburbs” and decentralized commercial centers. In 1891 Edward Bouton built *Roland Park* near Baltimore that included a “store block” arranged in a linear pattern along a street to serve the commercial needs of a planned residential community. Similar store blocks were built in Los Angeles in 1908 for the College Tract on West 48th street in New York City (Howard and Spencer, 1953 p. 113). The industrial revolution of the nineteenth century produced the department store but made cities crowded and dirty, and the desire to improve life by moving away from the city gave birth to the suburb shopping centers (Macfadyen, 1970).

The early history of shopping places dates back to the city square. In ancient Greece, the “Agora”, which is a Latin word meaning “assembly” or “gathering point”, was built for people to gather and shop. This concept inspired a famous architect, Victor Gruen, to adopt its model. The first SC designed by Victor Gruen was built in Kansas City, Kansas, U.S. in 1922 and was named *Country Club Plaza*. Victor Gruen built another shopping center, the *Northland Shopping Center*, in the U.S. which became the largest in the world in 1954. Later in 1956, Victor Gruen designed *Southdale Center Mall* located in Edina, MN, near Minneapolis in the U.S. It was the first fully enclosed shopping center with a constant climate-controlled temperature.

According to Tsutsui (2009), in early 1920s to 1930s, Japan had witnessed urbanization of SCs. The destruction caused by the Kantō earthquake in 1923 made businesses and their employees move away from the central downtown toward the southern suburbs in Tokyo. Even after the reconstruction of downtown Tokyo, companies kept operating in their bases near Marunouchi area away from the center. This was made possible by the new private railway lines constructed by private railway companies, which also constructed department stores and shopping centers near terminals, stations, and transfer points, such as Shinjuku. As a response to this competition, department stores in Ginza, which had previously specialized in imported expensive goods and specialty items, started to display more everyday goods for consumption. Most people, especially those who belonged to middle-class, could not afford to buy many of the fashion and goods displayed in department stores in Ginza, but browsing and window shopping became a popular leisure pastime in Tokyo. *Tamagawa Takashimaya Shopping Center* in Tokyo, opened in 1969, is considered to be the first fully established SC in Japan.

In North America, the largest SC registered in Guinness Book is *West Edmonton Mall* in Edmonton, Alberta, Canada, founded by Ghermezian brothers who immigrated to Canada from Iran. It was opened in 1981 and completed in 1998 over 4 different development stages. This SC contains more than 800 stores with an amusement park, hotels and even an aquarium, attracting more than 20 million people per year (Emporis, 2012). Currently, the world’s largest shopping mall is *The Dubai Mall*, located in Dubai, United Arab Emirates U.A.E. It is part of the 20-billion-dollar downtown Dubai complex, and includes 1,200 shops. Dubai Mall was opened in November 2008, with about 635 retailers, marking the world’s largest-ever mall opening in retail history. In 2012, Dubai Mall continued to hold the title of the world’s most-visited shopping and leisure destination, and attracted more than 65 million visitors in that year. It has an aquarium and under water zoo, ice rink, and a theme park.

1.4 Evolution of the Research on Shopping Centers

The competitive edge of SCs over individual independent retail stores may be found in that, they have a variety of stores and services in one place for the convenience of consumers. Furthermore, they can provide the cost-performance efficiency for their business partners by allowing them to share parking lots, loading and unloading depots, and other related facilities.

Research on the retail industry has evolved over the years. One of the earliest publications of literature addressing marketing issues related to SCs is (Christaller, 1966), which focused on Central Place Theory. Walter Christaller originally proposed the Central Place Theory (CPT) in 1933, explained using geometric shapes, such as hexagons and triangles. Similar to other location theories propounded by (Weber and Von Thunen, 1969), the locations are assumed to be located in a Euclidean, isotropic plain with similar purchasing power in all locations. A Central Place is a settlement, or a hub, that serves the area around it with goods and services. Christaller's model was based on three assumptions: first, that all goods and services were purchased by consumers from the nearest possible central place; second, the demands placed on all central places in the plain were similar, and thus could be compared; and third, none of the central places made any excessive profit.

Eppli and Benjamin (1994) summarized the array of critical opinion on SC, and they discussed the benefits of locating anchor and non-anchor shops in the same location in order to create positive externalities. The authors analyzed Christaller's initial economic modeling of Central Place Theory, which he created before the first enclosed SC. The theory posits that shoppers will travel the minimum distance possible to purchase a good, and this was deemed reasonable by Eppli and Benjamin due to the high cost of transportation. They describe the evolution of the theory as different variables and assumptions are added, for instance, the assumption that people rarely went to the shops for just one item. This led to the research of multipurpose shopping behavior and to the realization that people often travelled further than the closest shopping center. In summary, Eppli and Benjamin found that shopping center research methods evolved with people's shopping patterns.

Although central place theory was appropriate in the 1930s, the subsequent popularization of the motor vehicle and the increasing ease of transportation meant that central place theory had to evolve. For example, similar shops in the same location was once deemed not to work, but it was later found to be the ideal setting for comparative shopping. Since then, this line of research had been expanded to different directions, including complex consumer shopping patterns and retailer behavior in agglomerated SCs. For example, Kumar, Shah and Venkatesan (2006) addressed themselves to issues surrounding how to evaluate customer lifetime value at individual customer level so as to maximize profitability. In addition, the analytic network process approach was employed in (Cheng, Li and Yu, 2005) in order to find the best location of an SC from a set of alternative locations.

The understanding of the spatial configuration of a shopping center, and the gradual commodification of the space, in itself, has also received critical attention. (Goss, 1993) examined the SC strategies in building and designing the space and of a symbolic landscape; in order to understand how the retail built environment would work (Goss, 1993). He examined the physical space of the retail environment as an object of value; that is, a private space designed for efficient circulation of commodities which itself is a commodity produced for profit. This presents an interesting dilemma; that is to say, even though the SCs are profit-oriented private properties, it would be possible for a potential consumer to spend an entire day in it without engaging in any shopping.

Accordingly, recent studies have shifted focus to assessing promotional techniques and loyalty programs as tools to optimize profits. The main goal of such tools is to stimulate higher sales by providing rewards, or incentives, to customers (Kivetz and Simonson 2002) and (Sharp and Sharp, 1997). In a traditional approach, a sales campaign is typically organized over segments of consecutive days, and two or three campaigns are organized in each season. Total sales is normally used as a key-performance-indicator for the effectiveness of sales campaigns. This is because it is a high-priority objective and because of its high impact (Parsons, 2003; Noordewier et al., 1990). The number of purchase transactions is also used in performance metrics because of its high control on inventory (Noordewier et al., 1990; Oliver and Swan, 1989). Accordingly, in examining the performance of sales campaigns in an SC, the literature guides one to consider both: the total sales and the number of the purchase transactions. This thesis follows this general framework.

A study of an SC in Iran, discussed in Balaghar, Majidazar and Niromand (2012), examined and assessed the effectiveness of promotional tools, such as advertisement, sales promotion, public relations and direct selling. Kahn and McAlister found that the reliance on sales promotions, especially monetary promotions, were often a short run driver of sales and profits, and that, in their argument, explained why so many were unprofitable (Kahn and McAlister, 1997), as the effects of monetary promotions eroded their capacity over time (Lal and Rao, 1997).

Perhaps the most singular finding from the many instances discussed in the literature is that sales campaigns that provide rewards to consumers tend to be successful in motivating behaviors of repeat purchases and customer loyalty (Hilgard and Bower, 1997), (Latham and Locke, 1991). These rewards vary in their nature; they may be stores-wide low prices and discounts, or they may assume the form of one of the more common promotional tools (Sharp and Sharp, 1997). Parsons (2003) examined the effects of common promotional activities measured by sales and visits based on a survey and actual data of an SC for three months. He suggested that wide sales strategies such as sales campaigns, is the preferred technique that encourages visits and spending over traditional promotional tools of individual stores.

It is now possible to collect and accumulate massive data from the market via a point of sale system (POS) and to utilize it so as to develop effective marketing strategies aim at enhancing sales. The data correlate information on actual consumer purchases (available from universal-product-code scanners used in shops) with information on the frequency and type of sales campaigns. An extensive literature exists, for analyzing consumer purchasing behaviors based on POS data, represented by (Ishigaki et al., 2011; Taguchi, 2010 ; Yada et al., 2006; Eugene, 1997) to name only a few. However, little research has been done concerning how to utilize POS data solely for management of the SC business.

1.5 Flexibility

The term flexibility could be loosely defined as the capacity to quickly and cost-effectively respond to a changing environment within a limited range and timeframe (Upton, 1994). Dwivedi and Momaya (2003) defined flexibility as, "having more options, an increased freedom of choice, and change mechanism." Johns and Ostroy (1984) similarly argued that the analysis of choices rely on the manner in which flexibility is used to exploit expected information. Substantial literature exists dealing with the concept of flexibility from various perspectives, such as economic flexibility, organizational flexibility, and business process flexibility, to name only a few. For any organization to succeed there is an essential necessity to acknowledge the notion of flexibility to some degree (Birkinshaw, 2004). Therefore, understanding what flexibility is, and the types of flexibility, can facilitate the path towards achieving it in a specific setting.

When studying flexibility from an economic point of view; core concepts commonly discussed, include: cost, pricing, demand, product, and supply. For example, Stigler (1939) developed his own theory on cost analysis, which differed from classical cost analysis in that, he characterized the flexibility of two alternative manufacturing plants using the second derivative of their total cost curves. Stigler's theory was later extended by Marschak and Nelson (1962), by recognizing flexibility as good current actions that would permit good later responses to later observations. As reported by Sethi and Sethi (1990), one of the earliest discussions revolving around economic flexibility was featured in Lavington's book, "The English Capital Market," (1921) which discussed the importance of considering the risk arising from the immobility of invested resources.

From the organizational point of view, March and Simon (1958) argued that, the resources of an organization would be necessary to cope with internal as well as environmental uncertainties. According to Harrington (1991), flexibility in a business process is necessary to increase the organization's ability to adapt to changing circumstances and to compete effectively. Feibleman and Friend (1945) defined organizational flexibility as the ability of an organization to suffer a limited change without severe disorganization. Amram and Kulatilaka (1999) compared flexibility to owning an option, but not the obligation to take an action in the future. According to the real-options paradigm, uncertainty can increase the value of a project, as long as flexibility is preserved and resources are not irreversibly committed. Recent research has argued that organizational flexibility is not only dynamic; it is also inherently paradoxical by nature. Flexibility is said to require managerial action, balancing dialectical forces of control and autonomy (Bahrami, 1992), juxtapositioning capabilities (Evans, 1991), and ultimately building a constructive friction between change and preservation. In recent literature, flexible organizational forms are those that are simultaneously able to explore new possibilities and exploit old certainties (March, 1991).

From the perspective of business process flexibility, Nelson and Nelson (1997) considered two fundamental aspects in defining flexibility, emphasizing structural and process flexibility. They characterized the contemporary business environment as one that requires dynamic, flexible business processes. Davenport (1992) defined a business process as: "A structured set of activities designed to produce a specified output for a particular customer or market." (Davenport, 1992 p.5). The definition of a business process ranges from Harrington's (1991) version of being a set of logically coherent and connected tasks that use the resources of the organization with the goal of producing results, to the version of Nelson and Nelson (1997) which described the tasks involved in the business process as interdependent, and that a process would be orientated towards a specified output to achieve optimization. Nelson and Nelson (1997) stated that: "Adaptability characterizes revolutionary changes in the business process environment, adaptation, as defined by Huber (1984), is the optimization of a particular niche or business process." (Nelson and Nelson, 1997).

Revolutionary changes, therefore, go hand in hand with the desire for optimization. Kusiak (1986) argued that system flexibility is measured by its adaptability to changing its functions or business processes. Sorescu et al. (2011) also argued that researchers could achieve desirable outcomes by examining different managerial common practices. However, as constraints and specifications inevitably influence the process design, a critical challenge, according to Halemane and Janszen (2004) would be how to restructure the constraints and specifications of a business process so as to achieve optimization. They defined specifications as the description of the requirements of a business process, whereas the constraints are the restrictions and limitations of the business process design.

Shankar and Yadav (2011) argued that, in retail businesses, modifications in process design could spur innovations. A similar argument was put forth by Sorescu et al. (2011), who clearly stated that altering the constraints and specifications of a business process would influence the process design and these changes could yield improvements and innovation.

To the best knowledge of the author, marketing flexibility was not clearly defined in the literature. In order to fill this absence; this thesis proposes a definition of marketing flexibility that overlaps with the main points describing economic, organizational and process flexibility. In the context of retail business, marketing flexibility could be defined as "a management approach that aims at optimizing the outcome of a business process by exploring possible options for reconfiguring the specifications and or the constraints of the business process that controls them."

2 Optimization Problem –I: Total Expected Sales

2.1 Introduction

This Chapter is devoted to the mathematical model for optimizing total expected sales by optimally reallocating sales campaign days freely rather than in segments of consecutive days. In Section 2.2 the data of the winter season is described. Section 2.3 is the model specification and the numerical results of implementing the model on the winter season.

2.2 Data Description of the Winter Period

We work on a set of real data obtained from an SC operating in Tokyo, Japan, for the winter period of 2009 and that of 2010, that is, December 2009, January 2010, and February 2010 for the winter period 2009, and December 2010, January 2011 and February 2011 for the winter period 2011. For the $i-th$ day of a winter period, the dataset comprises the following main elements

$I_{CAMP}(i)$: The campaign flag indicating if the $i-th$ day was under the sales campaign [$I_{CAMP}(i)$=1] or not [$I_{CAMP}(i)$=0]

$s(i)$: The total sales of the $i-th$ day in ¥ in the entire SC (2.2.1)

$t(i)$: The number of purchase transactions of the $i-th$ day in the entire SC

Two sales campaigns are organized in each winter period, that is Win_1 and Win_2 where $I_{CAMP}(i) = \text{Win\_1} + \text{Win\_2}$. Table 2.2.1 shows the organization of the sales campaigns days over the winter periods 2009 and 2010.

Table 2.2.1 The Organization of Sales Campaign days over the Winter Periods 2009 and 2010 as Obtained from the SC

Start Date	End Date	Campaign	# of Days
Winter 2009			
01/12/2009	27/12/2009	Win_1	27
28/12/2009	01/03/2010	No campaign	6
01/04/2010	01/11/2010	Win_2	8
01/12/2010	02/28/2010	No campaign	47
Winter 2010			
01/12/2010	28/12/2010	Win_1	28
29/12/2010	01/03/2011	No campaign	5
01/04/2011	01/11/2011	Win_2	8
01/12/2010	02/28/2010	No campaign	47

Figure 2.2.1 displays $s(i)$ and $t(i)$ as obtained from the SC for the winter periods 2009 and 2010 in a histogram format. The size and number of bins of the histogram are selected based on the Freedman Diaconis method (Freedman and Diaconis, 1981), the general equation for the rule is

$$\text{Bin size} = 2\ \text{IQR(x)}\ \text{n}^{-1/3} \quad ,$$

where IQR(x) is the interquartile range of the data and n is the number of observations in the sample x .

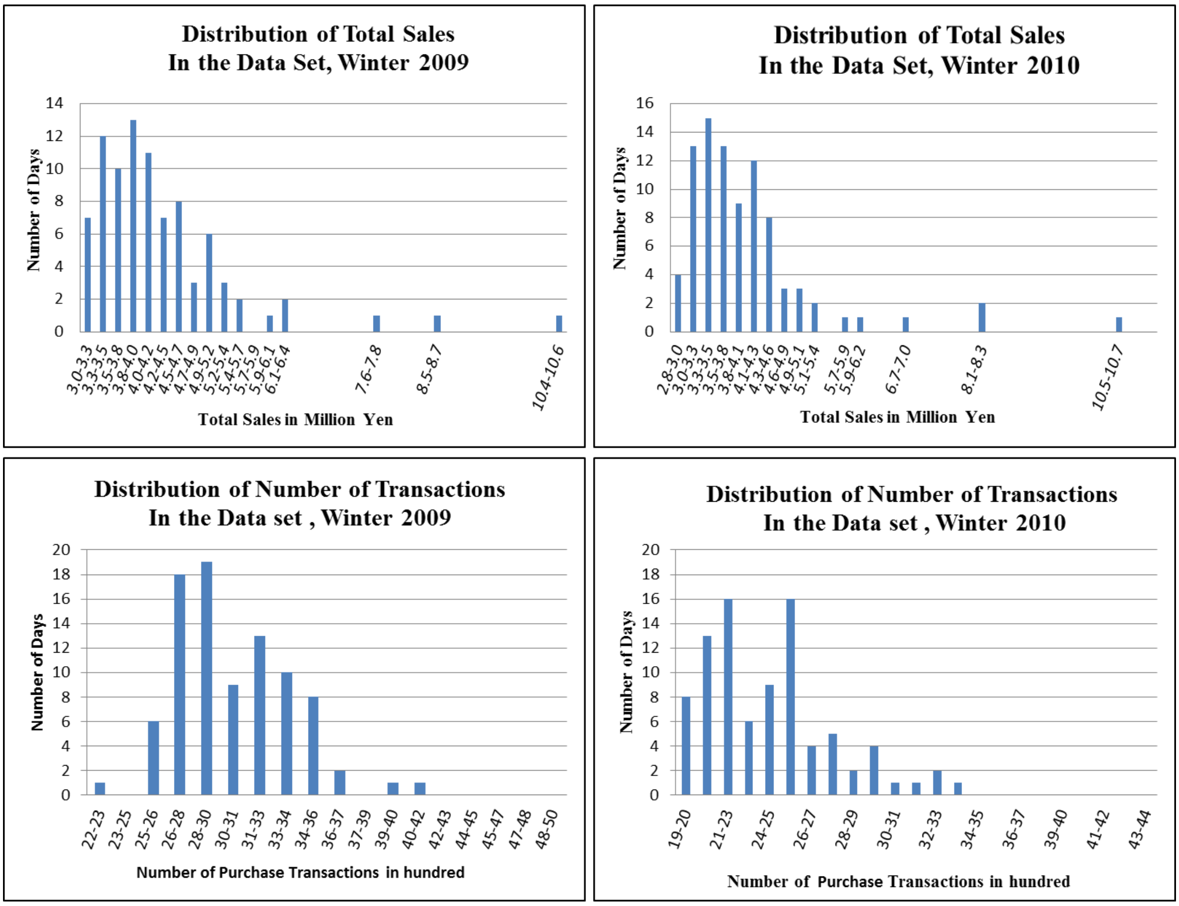

Figure 2.2.1 Total Sales and Number of Purchase Transactions for the Winter Periods 2009 and 2010 Before Cleaning Outliers

Throughout the year, the administration of the SC organizes some activities or special events that attract more visitors, consequently, these activities cause upsurges in $s(i)$ and $t(i)$, which we call outliers, hereafter. In order to achieve a better analysis quality, such outliers need to be normalized. More specifically, let μ_S and σ_S be the mean and the standard deviation of total sales over the period under consideration, and μ_T and σ_T similarly defined for the number of purchase transactions over that period. Then we define

$$s(i) \text{ is an outlier} \quad \Leftrightarrow \quad s(i) \geq \mu_S + 2\sigma_S \quad ,$$

$$t(i) \text{ is an outlier} \quad \Leftrightarrow \quad t(i) \geq \mu_T + 2\sigma_T \quad . \qquad (2.2.2)$$

If the normal distribution is assumed, this boundary value would represent the 95% level. In order to investigate the assumption of normality, we rely on the Quantile−Quantile plot or Q−Q plot. According to Neil Salkind (2007), the normal Q−Q plot is used to visually see the deviation from normality in a dataset. Based on the Q−Q plots, shown in Figure 2.2.2 below, by comparing the distributions against the diagonal line representing the expected normal one; it can be said that, despite the presence of some outliers, the distributions are sufficient to assume normality.

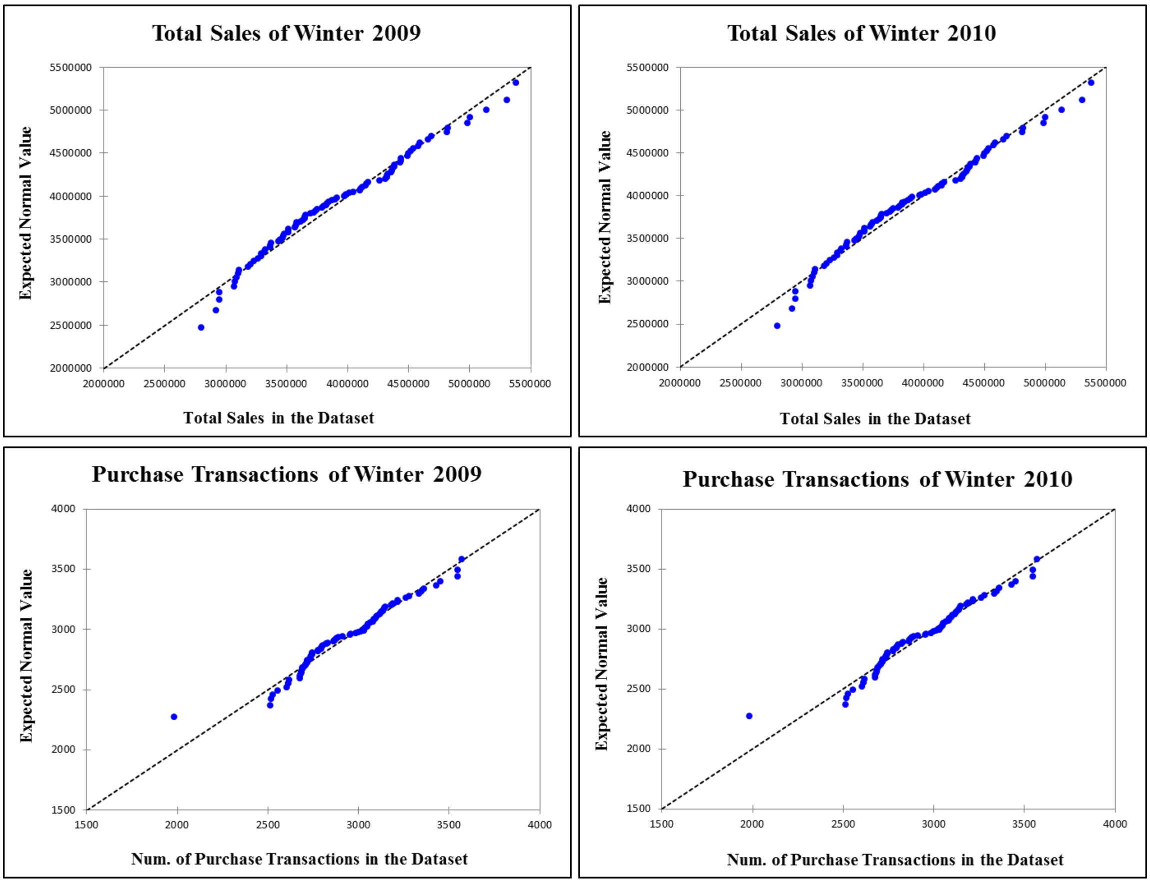

Figure 2.2.2 Q−Q Plots of the Total Sales and the Number of Purchase Transactions for the Winter Periods 2009 and 2010

Let $\mu_{S:\neg outlier}$ and $\mu_{S:outlier}$ be the average total sales of non-outlier days and that of outlier days, respectively. $\mu_{T:\neg outlier}$ and $\mu_{T:outlier}$ are defined similarly for the number of purchase transactions. Then $s(i)$ and $t(i)$, judged to be outliers, are adjusted based on the following formula

$$s(i) \leftarrow s(i) \times \mu_{S:\neg outlier} / \mu_{S:outlier} \quad ; \quad t(i) \leftarrow t(i) \times \mu_{T:\neg outlier} / \mu_{T:outlier} \ . \qquad (2.2.3)$$

Outliers may result from other reasons; for example, in this case, the SC under the study provides facilities for cultural classes, e.g. flower arrangement and piano. Monthly fees of such classes may be paid on a fixed date of the month which result on outliers in $s(i)$ and $t(i)$. In the winter season, only one store referred to as the *Music Store*, hereafter, caused such outliers. Outliers of this sort are adjusted by eliminating the total sales and the number of purchase transactions of such classes rather than using (2.2.3). Table 2.2.2 shows the adjusted outliers of the *Music Store*, whereas adjusted outliers, detected by the standard deviation as in (2.2.2), are shown in Table 2.2.3.

Table 2.2.2 Adjusted Outliers for the Winter Periods 2009 and 2010 of the *Music Store*

Winter 2009							
Total Sales				**Num. of Purchase Transactions**			
Date	**Entire SC**	**Music Store**	**Adjusted**	**Date**	**Entire SC**	**Music Store**	**Adjusted**
12/24/2008	¥ 9,667,621	¥ 4,112,600	¥ 5,555,021	12/24/2008	4,239	439	3,800
01/24/2009	¥ 9,794,751	¥ 4,053,000	¥ 5,741,751	01/24/2009	4,070	432	3,638
02/25/2009	¥ 7,901,727	¥ 4,044,500	¥ 3,857,227	02/25/2009	3,275	432	2,843
Winter 2010							
Total Sales				**Num. of Purchase Transactions**			
Date	**Entire SC**	**Music Store**	**Adjusted**	**Date**	**Entire SC**	**Music Store**	**Adjusted**
12/24/2009	¥ 8,737,528	¥ 3,907,800	¥ 4,829,728	12/24/2009	4,099	409	3,690
01/25/2010	¥ 7,808,626	¥ 3,853,300	¥ 3,955,326,	01/25/2010	3,222	405	2,817
02/25/2010	¥ 10,523,905	¥ 3,841,000	¥ 6,682,905	02/25/2010	3,481	406	3,075

Table 2.2.3 Adjusted Outliers for the Winter Periods 2009 and 2010, Detected by the Standard Deviation Method

Winter 2009			**Winter 2010**		
Date	**Purchase Transactions**	**Adjusted Transactions**	**Date**	**Purchase Transactions**	**Adjusted Transactions**
12/23/2008	4,177	3,221	12/23/2009	3, 360	3,140
12/25/2008	3,863	2,979	12/20/2009	3,210	2,934
12/24/2008	3,800	2,931			
Date	**Total Sales**	**Adjusted Total Sales**	**Date**	**Total Sales**	**Adjusted Total Sales**
12/6/2008	¥ 6,842,642	¥ 4,526,458	12/20/2009	¥ 6,289,894	¥ 4,115,768
12/25/2008	¥ 6,434,730	¥ 4,256,621	12/23/2009	¥ 6,682,905	¥ 4,372,933
02/26/2009	¥ 6,175,360	¥ 4,085,046	12/24/2009	¥ 6,235,075	¥ 4,079,898
02/28/2009	¥ 6,847,974	¥ 4,529,985	12/26/2009	¥ 6,052,768	¥ 3,960,606

The number of outliers in the dataset ranges from $5-7$ outliers in a sample size of 88, corresponding to around $6\ -8$ %, respectively. From Hampel et al.'s (1986) classical book on robust statistics, it is claimed that a routine dataset typically contains about $1-10$% outliers. One also notes that, no minimum extremes were detected in the winter periods 2009 and 2010. Accordingly, the datasets are ready for analysis and no further cleaning would be needed. The mean, variance, kurtosis, and skewness for the winter periods 2009 and 2010 before and after cleaning are summarized in Table 2.2.4 below. One sees that, upon adjusting the outliers of $s(i)$ and $t(i)$, the variance drops significantly.

Table 2.2.4 The Effect of Data Cleaning on Total Sales and Number of Purchase Transactions for the Winter Periods 2009 and 2010

		Winter 2009		**Winter 2010**	
		Before cleaning	**After Cleaning**	**Before cleaning**	**After Cleaning**
Total Sales	**Mean**	4,589,445	4,349,528	4,363,364	4,231,522
	Variance	1,147,588	613,325	1,156,849	770,962
	Skewness	2.75	0.95	2.91	0.38
	Kurtosis	10.74	0.73	11.23	-0.48
Purchase Transactions	**Mean**	3,089	3,044	3,069	3,037
	Variance	357	288	336	297
	Skewness	0.52	0.20	0.62	0.05
	Kurtosis	0.34	-0.41	1.03	0.81

Figure 2.2.2 below shows the effect of cleaning all outliers of $s(i)$ and $t(i)$ for the winter periods 2009 and 2010, one notes that, all spikes are smoothed.

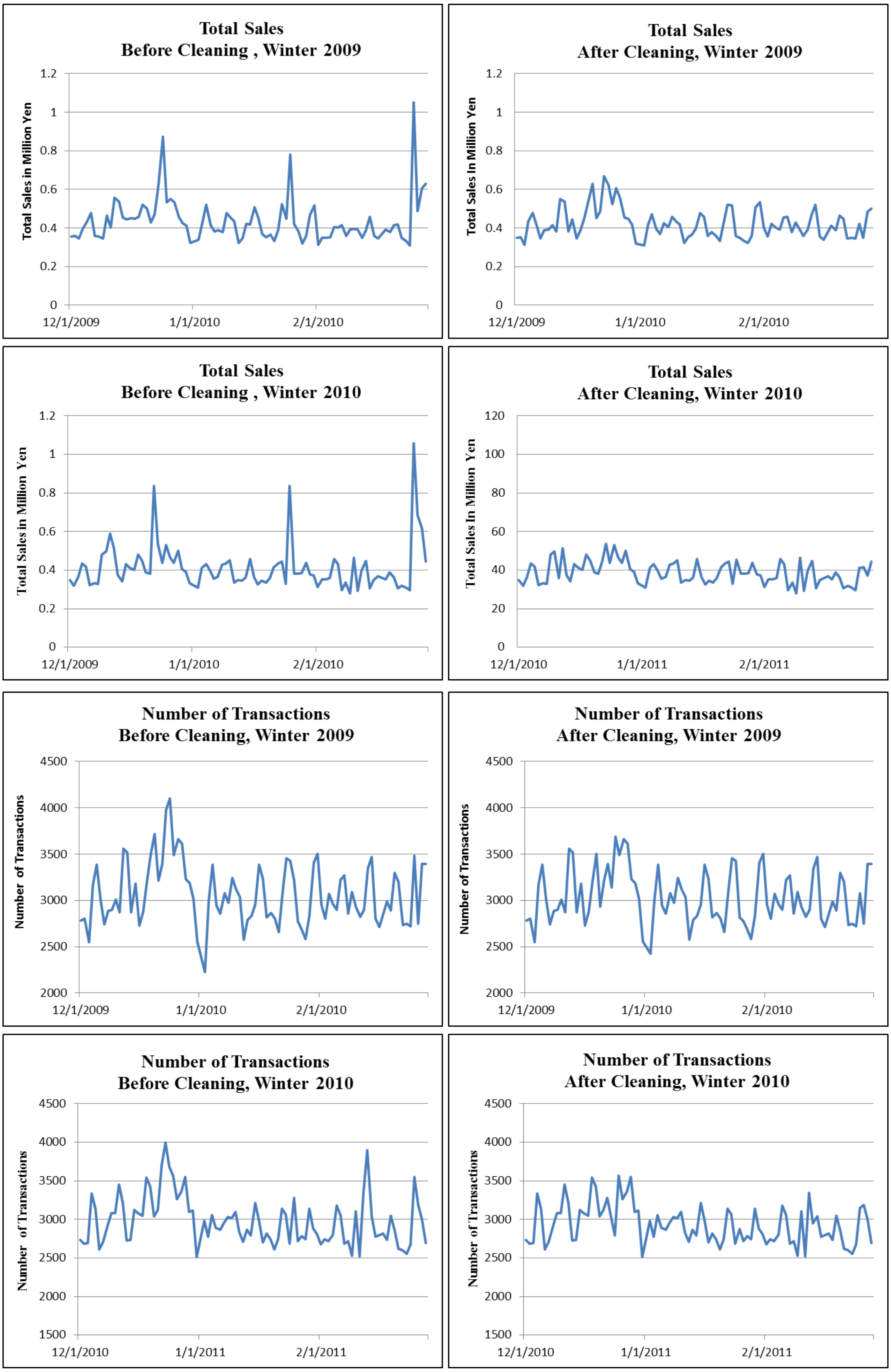

Figure 2.2.3 Before and After Cleaning of Total Sales and Number of Purchase Transactions for the Winter Periods 2009 and 2010

2.3 Model Specification: Optimizing Total Expected Sales

We consider a sales campaign to be organized so as to maximize the total expected sales over a given period of M days subject to the number of campaign days being N days, where $N < M$. A machine learning technique is employed where two datasets are considered: winter 2009 for Learning Data (LD) and winter 2010 for Testing Data (TD). For notational convenience, the set of days involved in LD is denoted by D_{LD} , and D_{TD} are defined similarly.

The model consists of four stages. In Stage I, we specify the two indicator functions; $I_{CAMP}(i)$ for a sales campaign day, and $I_{GOOD:S_0T_0}(i)$ for a GSD, where S_0 and T_0 are numerical threshold levels of $s(i)$ and $t(i)$ to be defined through the following procedure. All days in D_{LD} are first ordered in descending order by $s(i)$ and $t(i)$, separately. The decile points are then marked, yielding a two-dimensional matrix as shown in Figure 2.3.1. The decile points are summarized in Table 2.3.1.

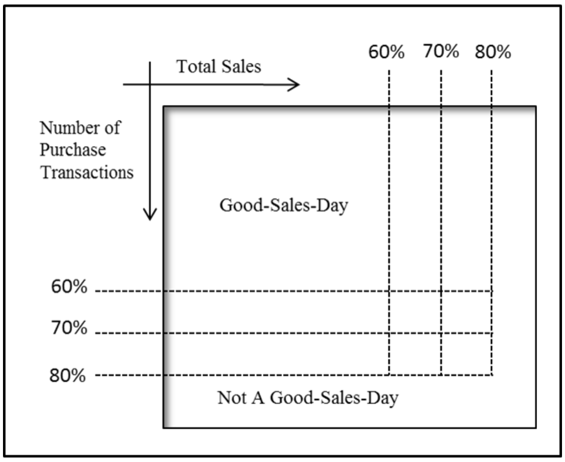

Figure 2.3.1 Two-dimensional Matrix for the Decile Points of Total Sales and Number of Purchase Transactions

Table 2.3.1 Decile Points in Total Sales and Number of Purchase Transactions of the Winter Period 2009 (*LD*)

Deciles	Total Sales	Number of Purchase Transactions
10%	¥ 5,517,359	3,502
20%	¥ 5,187,521	3,391
30%	¥ 4,671,986	3,233
40%	¥ 4,511,894	3,140
50%	¥ 4,226,882	3,014
60%	¥ 4,021,595	2,946
70%	¥ 3,876,067	2,884
80%	¥ 3,591,585	2,803
90%	¥ 3,459,930	2,733
100%	¥ 3,093,096	2,227

Given two threshold levels S_0 for $s(i)$ and T_0 for $t(i)$, the indicator function $I_{GOOD:S_0T_0}(i)$ for $i \in D_{LD}$ is defined as

$$I_{GOOD:S_0T_0}(i) = \begin{cases} 1 , & if \quad s(i) \geq S_0 \text{ and } t\,(i) \geq T_0 \\ 0 , & else \end{cases} . \tag{2.3.1}$$

The numerical threshold levels S_0 and T_0 obtained from LD are similarly used to determine $\hat{I}_{GOOD:S_0T_0:TD}(j) = 1$, for $j \in D_{TD}$ and $\hat{I}_{GOOD:S_0T_0:TD}(j) = 0$, otherwise. For a sales campaign of *N* days organized over a future winter period of *M* days, the key decision variable of the optimization problem is represented by the campaign day assignment vector, denoted by $\underline{d} = [d(1), \cdots, d(j), \cdots, d(M)] \in \{0,1\}^M$, where

$$d(j) = \begin{cases} 1 , & if \ day \ j \ is \ selected \ to \ be \ a \ campaign \ day \\ 0 , & else \end{cases} , \tag{2.3.2}$$

subject to the constraint that $\sum_{j=1}^{M} d(j) \leq N = \sum_{i=1}^{M} I_{CAMP}(i)$ where $N < M$. In order to establish a mathematical model to assign *N* campaign days over a future period of *M* days for maximizing the total expected sales, one has to deal with two issues: 1) how to determine whether or not a day is a GSD, this is addressed in Stage II; and 2) how to estimate the expected total sales for that day, given an allocation of *N* campaign days over *M* days is decided, addressed in Stage III.

In Stage II, a logistic regression model is developed for estimating whether or not a day is a GSD in the future winter period. For this purpose, we consider a set of explanatory variables given in Table 2.3.2. Following the standard procedure for eliminating multi-collinearity, the correlation structure of these variables is given in Table 2.3.3. In this case, it happened that the correlation of every pair is less than 0.5 and no variables are eliminated because of multi-collinearity.

Table 2.3.2 Variables Considered for Logistic Regression for the Winter Period 2010

Labels	Description
Week_k $(\boldsymbol{i})$, ***k = 1, 2, 3, 4 .***	Each month has four weeks, labeled as: *Week_1*, *Week_2*, *Week_3*, and *Week_4*. Any week consists of seven days, except that *Week_4* may include extra days until the end of the month. *Week_k*(i) =1 if day i belongs to week *k*, and *0* otherwise.
Weekday_k (i) , ***k = 1 , ⋯ , 5 .***	This binary variable takes the value of 1 when WeekDay_*k* (i) is a weekday and 0 otherwise. Each week has five weekdays, Mon, Tue, Wed, Thu, and Fri, labeled as Weekday_1, Weekday_2, Weekday_3, Weekday_4, and Weekday_5, respectively.
Weekend $(\boldsymbol{i})$	This binary variable takes the value of 1 when day i is Saturday or Sunday, and 0 otherwise.
National Holiday (i)	This binary flag indicates that day i is an official national holiday in Japan.
Non-national Holiday$(\boldsymbol{i})$	This binary flag indicates that day i is not an official national holiday but is likely to be very passive in business in Japan, e.g. *Dec 28, 29, 30, 31* during which offices are typically closed.
Win_1 $(\boldsymbol{i})$	This binary variable takes the value of 1 only if day i is in *December,* $I_{CAMP}(i)$=1, and *0* otherwise.
Win_2 $(\boldsymbol{i})$	This binary variable takes the value of 1 only if day i is in *January* or *February,* $I_{CAMP}(i)$=1, and 0 otherwise.
LY_Transactions$(\boldsymbol{i})$	This integer variable describes the number of purchase transactions of the same day of the month of the last year.

Table 2.3.3 The Correlation Matrix of Variables Tested for Multi-collinearity for Winter Period 2010

	Week _1	**Week _2**	**Week _3**	**Week _4**	**Mon**	**Tue**	**Wed**	**Thu**	**Fri**	**Weekend**	**National Holiday**	**Non-National Holiday**	**Win_1**	**Win_2**	**LY_ Trans**
Week_1	1														
Week_2	-0.304	1													
Week_3	-0.294	-0.304	1												
Week_4	-0.361	-0.372	-0.361	1											
Mon	0.022	0.011	0.022	-0.049	1										
Tue	0.003	-0.008	0.003	0.001	-0.165	1									
Wed	0.022	0.011	-0.057	0.023	-0.158	-0.165	1								
Thu	0.003	-0.008	0.003	0.001	-0.165	-0.173	-0.165	1							
Fri	-0.057	0.011	0.022	0.023	-0.158	-0.165	-0.158	-0.165	1						
Weekend	0.005	-0.012	0.005	0.001	-0.257	-0.270	-0.257	-0.270	-0.257	1					
National Holiday	-0.102	0.189	-0.102	0.011	0.108	-0.078	0.108	0.098	-0.075	-0.122	1				
Non-national Holiday	-0.118	-0.122	-0.118	0.328	0.072	0.063	0.072	0.063	-0.087	-0.141	-0.04	1			
Win_1	0.051	0.032	0.051	-0.122	-0.049	0.001	0.023	0.001	0.023	0.001	0.01	-0.145	1		
Win_2	0.206	0.194	-0.171	-0.210	0.105	-0.020	-0.010	-0.020	-0.010	-0.032	0.16	-0.069	-0.210	1	
LY_Trans	-0.093	0.078	0.049	-0.032	-0.118	-0.078	-0.155	0.372	0.216	-0.186	0.02	-0.198	0.423	-0.001	1

A logistic regression model is developed for estimating the likelihood, $\rho_{GOOD}(j)$, of whether or not day j in the future winter period is a GSD based on *LD*. Namely, from a set of the explanatory variables $x_k\,(i)$ for $i \in D_{LD}$ and $k = 1, \cdots, K$, let $\underline{x} = [x_1(i), \cdots, x_k(i)]$, and $\underline{\beta} = [\beta_0, \beta_1, \cdots, \beta_K]$. We define $r\left(\underline{x}, \underline{\beta}\right)$ by

$$r\left(\underline{x}\,, \underline{\beta}\right) = \beta_0 + \sum_{k=1}^{K} \beta_k \,.\, x_k\,(i) \quad . \tag{2.3.3}$$

The corresponding logistic regression model then yields the optimal coefficient vector $\underline{\beta}^*$, given by

$$\underline{\beta}^* = \min_{\underline{\beta}} \sum_{i \in D_{LD}} \left\{ I_{GOOD:S_0T_0}(i) - \frac{e^{r(x(i),\underline{\beta})}}{1 + e^{r(x(i),\underline{\beta})}} \right\}^2 \quad . \tag{2.3.4}$$

If $\underline{x}$ of day j in the future winter period can be known, (3.2.4) enables one to assess the likelihood of day j being a GSD. This measure, denoted by $\rho_{GOOD}(j)$, can be computed as

$$\rho_{GOOD}(j) = \frac{e^{r(x(j),\,\underline{\beta}^*)}}{1 + e^{r(x(j),\,\underline{\beta}^*)}} \quad . \tag{2.3.5}$$

In turn, (2.3.5) enables one to determine whether or not day j is judged to be a GSD by specifying a threshold level ρ_{GOOD}, where day j is judged to be a GSD, denoted by $\hat{I}_{GOOD}(j) = 1$, if $\rho_{GOOD}(j) \geq \rho_{GOOD}$, and day j is not a GSD, denoted by $\hat{I}_{GOOD}(j) = 0$, otherwise. In order to determine the threshold level ρ_{GOOD}, we employ the confusion matrix obtained from *TD* and given in Table 2.3.4 below. This approach is widely used in the area of machine learning. Since $\underline{x}(j)$ is known for $j \in D_{TD}$, and $\rho_{GOOD}(j)$ can be computed from (2.3.5) above, furthermore, it is known that day j is a GSD when ($\hat{I}_{GOOD:S_0T_0:TD}(j) = 1$) or not ($\hat{I}_{GOOD:S_0T_0:TD}(j) = 0$), consequently, we are in a position to see whether or not $\hat{I}_{GOOD}(j) = \hat{I}_{GOOD:S_0T_0:TD}(j)$, yielding the confusion matrix.

Table 2.3.4 General Confusion Matrix

		Actual			
		¬ GSD	**GSD**	**Total**	
Judgment	**¬ GSD**	x_{00}	x_{01}	X_0	
	GSD	x_{10}	x_{11}	X_1	$\boldsymbol{x_{11}/X_1}$
	Total	Y_0	Y_1	X	
			$\boldsymbol{x_{11}/Y_1}$		$\boldsymbol{(x_{00}+x_{11})/X}$

The common measures for assessing the appropriateness of the choice of ρ_{GOOD} is given by Recall = x_{11}/Y_1, Precision = x_{11}/X_1 and Accuracy = $(x_{00}+x_{11})/X$. We note that Recall describes the portion of actual GSDs which were judged to be a GSD, whereas Precision gives the portion of judged GSDs which were actually a GSD, and Accuracy represents the overall correctness of the judgment. It is clear that Recall decreases while Precision increases as ρ_{GOOD} increases. In order to balance the two conflicting measures, we consider the optimization problem of maximizing Precision subject to Recall $\geq$ 0.75. This optimization problem is solved by varying ρ_{GOOD} with a stepwise of 0.01. This process is repeated for every combination of $(S_0,\ T_0)$ obtained from the decile cut-off points of $s(i)$ and $t(i)$, yielding the best model with ${\rho^*}_{GOOD} = 0.64$, $S_0^* =$ 3,591,585 and T_0^* =2,870 representing the 80% and 70% levels of total sales and purchase transactions in LD, respectively. The estimated regression coefficients and other statistical measures are summarized in Table 2.3.5. The corresponding confusion matrix of the best model is shown in Table 2.3.6, yielding Precision = 0.81, Recall = 0.76 and Accuracy = 0.82.

Table 2.3.5 Estimated Coefficients of the Logistic Regression Model for Winter 2010

	Estimate	**Std. Error**	**z value**	**Pr(>|z|)**	**Sig**
(Intercept)	-11.8941	5.175097	-2.298	0.02154	*
Weekend	3.588255	1.148712	3.124	0.00179	**
Week1	-2.10841	1.006596	-2.095	0.03621	*
Week2	-2.05983	0.932031	-2.21	0.0271	*
LY_Transactions	0.003582	0.001751	2.046	0.04078	*
Non-national Holiday	3.326175	1.598921	2.08	0.0375	*
Win_1	1.879	0.87049	2.159	0.03088	*
Win_2	3.13367	1.26512	2.477	0.01325	*

Signif. codes: 0 '*' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1**

Table 2.3.6 The Confusion Matrix of the Logistic Regression Model for Winter 2010

		Actual			
		$\neg$ ***GSD***	***GSD***	**Total**	
Judgment	$\neg$ ***GSD***	43	9	52	**Precision**
	GSD	7	29	36	**80.6%**
	Total	50	38	88	
		Recall	**76.3%**	**Accuracy**	**81.8%**

In Stage III, we turn our attention to the second issue of how to estimate the expected total sales for day $j \in D_{TD}$, given a decision vector $\underline{d}$ specifying campaign days for the future winter period as well as the estimated coefficients for the explanatory variables of the logistic regression, one can compute $\rho_{GOOD}(j)$ from (2.3.5) which in turn enables one to determine $\hat{I}_{GOOD}(j)$ under $\rho^*{}_{GOOD}$. The matrix of the average total sales, denoted by $\hat{s}_{(m,n)}$, computed over days $i \in D_{LD}$ with $m = I_{CAMP}(i)$ and $n = I_{GOOD:S_0T_0}(i), m, n \in \{0,1\}$ is displayed in Table 2.3.7. The average total sales, obtained from *LD*, is then used to estimate the expected total sales of day $j \in D_{TD}$, denoted by $\hat{r}_{(m,n)}$ with $m = d(j)$ and $n = \hat{I}_{GOOD}(j), m, n \in \{0,1\}$.

Table 2.3.7 Average Total Sales (¥ million) Obtained from Winter 2009 (*LD*)

$\hat{s}_{(m,n)}$		$n = I_{GOOD:S_0T_0}(i)$	
		0	**1**
$m = I_{CAMP}(i)$	**0**	¥ 3.65	¥ 4.68
	1	¥ 3.89	¥ 4.82

The total expected sales over the future period of *M* days, denoted by $\hat{R}(\underline{d})$, can then be estimated as

$$\hat{R}(\underline{d}) = \sum_{j=1}^{M} \sum_{m,n \in \{0,1\}} \hat{r}_{(m,n)} \delta_{\{d(j)=m\}} \delta_{\{\hat{I}_{GOOD}(j)=n\}} \quad , \tag{2.3.6}$$

where $\delta_{\{STATEMENT\}} = 1$ if *STATEMENT* is true, and $\delta_{\{STATEMENT\}} = 0$, otherwise. In order to test the validity of this approach, the formula of total expected sales $\hat{R}(\underline{d})$ in (2.3.6) above, is used with actual campaign days in *TD*, and then compared with the actual total sales R of *TD*. More specifically, let $\underline{I}_{CAMP} = [I_{CAMP}(1), \cdots, I_{CAMP}(M)] \in \{0,1\}^M$ be the sales campaign days in the actual practice, then $\hat{R}(\underline{I}_{CAMP})$ based on (2.3.6) is compared against actual total sales R of *TD*, achieving a relative accuracy of less than 2 % as shown in Table 2.3.8 below.

Table 2.3.8 The Validity of the Systematic Approach of Estimating Total Sales for Winter 2009 (*TD*)

	Notation	**Value**
Total expected sales	$\hat{R}\left(\underline{I}_{CAMP}\right)$	¥ 366.48
Actual total sales	R	¥ 360.25
Relative accuracy	$\left\lvert\hat{R}\left(\underline{I}_{CAMP}\right) - R\right\rvert \times 100/R$	1.72 %

Finally, in Stage VI, the problem of optimally reallocating sales campaign days, specified by $\underline{d} = [d(1), \cdots, d(j), \cdots, d(M)] \in \{0,1\}^M$ subject to $\sum_{j=1}^{M} d(j) \leq N$ so as to maximize the total expected sales, can be formulated as

$$\max_{\underline{d} \in \{0,1\}^M} \hat{R}\left(\underline{d}\right) \quad subject\ to \quad \sum_{j=1}^{M} d(j) = N \quad . \tag{2.3.7}$$

To assess the impact of this flexible allocation of sales campaign days, we compare the optimal solution against the actual total sales, which was achieved through two separate sales campaigns, each consisting of certain segments of consecutive days. For this purpose, we set M = 88 and N = 36 as obtained from *TD* of the winter period with $\sum_{j=1}^{88} d(j) =$36. This optimization problem can be readily solved, yielding $\hat{R}\left(\underline{d}^*\right) =$ ¥ 385.78 million. We note that the difference between the optimal expected total sales and the actual total sales is given by $\hat{R}\left(\underline{d}^*\right) - R =$ ¥ 25.53 million, or about 7 % increase.

Table 2.3.9 demonstrates how the optimal allocation of sales campaign days, $\underline{d}^*$, differs from the actual sales campaign days, $\underline{I}_{CAMP}$, obtained from *TD*. We find that only 13 sales campaign days are in common out of 36 sales campaign days. There are 23 days for which sales campaign is assigned only in the actual practice, or only by the optimal decision.

Table 2.3.9 Sales Campaign Days (Actual vs. Optimal) of $\hat{R}\left(\underline{d}^*\right)$ for Winter 2010 (*TD*)

		Optimal		**Total**
		1	**0**	
Actual	**1**	13	23	36
	0	23	29	52
Total		36	52	88

In Table 2.3.10, the effect of the optimal allocation of sales campaign days on GSDs is summarized, where 31 GSDs and 24 ¬GSDs are common, amounting to 63% of 88 days in the winter period. It should be noted that the optimal decision approach converted 26 ¬GSDs into GSDs, while only 7 days were downgraded from GSD in the actual practice to ¬GSD. Consequently, the optimal decision approach yielded 57 GSDs or 65% of 88 days.

Table 2.3.10 The Effect of the Optimal Decision Approach on GSDs of $\hat{R}(\underline{d}^*)$ for Winter 2010 (*TD*)

# of Days		Optimal		Total
		GSD	¬GSD	
Actual	GSD	31	7	38
	¬GSD	26	24	50
Total		57	31	88

Table 2.3.11 below demonstrates how the above improvement by the optimal decision approach was achieved in further detail, where GSD vs. ¬GSD are classified according to sales campaign days only in the actual practice, those in common (actual and optimal), and those only by the optimal decision approach.

Table 2.3.11 GSD vs. ¬GSD Transitions by Optimal Decision Approach of $\hat{R}(\underline{d}^*)$ for Winter 2010 (*TD*)

Actual Only

Campaign Days		Optimal		Total
		GSD	¬GSD	
Actual	GSD	14	3	17
	¬GSD	0	6	6
Total		14	9	23

In Common

Campaign Days		Optimal		Total
		GSD	¬GSD	
Actual	GSD	3	3	6
	¬GSD	1	6	7
Total		4	9	13

Optimal Only

Campaign Days		Optimal		Total
		GSD	¬GSD	
Actual	GSD	2	0	2
	¬GSD	20	1	21
Total		22	1	23

In the actual practice, 17 + 6 = 23 sales campaign days (or 64%) are assigned to GSDs in the actual practice, and 6 + 7 = 13 days (or 36%) to ¬GSDs, whereas the optimal decision approach allocated only 6 + 2 = 8 days (or 22%) to GSDs in the actual practice, and 7 + 21 = 28 days (or 78%) to ¬GSDs. This result supports the original observation that the effect of a sales campaign for enhancing the total sales of ¬GSD may exceed that for strengthening the total sales of GSD further.

3 Optimization Problem –II: Expected Profit

3.1 Introduction

The mathematical model in *Chapter 2* described the optimization problem of how to reallocate sales campaign days specified by $\underline{d} = [d(1), \cdots, d(j), \cdots, d(M)]$ subject to $\sum_{j=1}^{M} d(j) \leq N$ so as to maximize the total expected sales, achieving 7 % increase from the actual total sales for the winter period. In this chapter, two further extensions of this optimization problem are considered. Firstly, by introducing the standard campaign budget of $B = B_0$ per day, the objective function of the optimization problem is modified to maximize the expected profit rather than the total expected sales. Secondly, we introduce a campaign budget increase $\Delta_B = B - B_0$ per day, and examine the optimal budget size B^* along with the optimal campaign day assignment vector so as to maximize the expected profit.

This chapter is organized in the following manner; the first extension of the optimization problem is introduced in Section 3.2. In Section 3.3, the second extension is described, and in Section 3.4, basic properties of the expected total sales under the campaign budget increase are examined. Finally, in Section 3.5, the results of the optimal solution are reported.

3.2 Model Specification: Optimizing Expected Profit Under the Standard Campaign Budget $B = B_0$

For the two winter campaigns in *LD* and those in *TD*, the campaign cost B_0 per day is estimated to be ¥ 0.4 million in the following manner; we were informed by the SC that the total cost per day in the winter period would be approximately 20 % of the total sales per day, which turned out to be: (¥ 4.36 million / day × 20 % = ¥ 0.872 million / day). We were also told by the SC that the campaign cost per day was 46 % of the total cost, yielding $B_0 =$ ¥ 0.4 million.

In this section, the objective function for optimally reallocating sales campaign days, specified by $\underline{d} = [d(1), \cdots, d(j), \cdots, d(M)] \in \{0,1\}^M$ subject to $\sum_{j=1}^{M} d(j) \leq N$, so as to maximize the expected profit under B_0, is formulated. As described in *Chapter 2*, the expected total sales per day, $\hat{r}_{(m,n)}$ with $m = d(j)$ and $n = \hat{I}_{GOOD}(j), m, n \in \{0, 1\}$ is estimated based on $\hat{s}_{(m,n)}$ obtained from *LD*, with $m = I_{CAMP}(i)$ and $n = I_{GOOD:S_0T_0}(i)$, $m, n \in \{0, 1\}$. Accordingly, the total expected sales, $\hat{R}(\underline{d})$ for the future period, is computed as in (2.3.6). The optimization problem of expected profit, denoted by $\hat{P}(\underline{d}^*)$, can then be written as

$$\max_{\underline{d}\in\{0,1\}^M}\left[\hat{R}(\underline{d})-\ B_0\times\ \sum_{j=1}^{M} d(j)\right]\ ,\qquad subject\ to\ \ \sum_{j=1}^{M} d(j)\leq N\quad . \tag{3.2.1}$$

This optimization problem can be readily solved, yielding an optimal expected profit of $\hat{P}(\underline{d}^*) =$ ¥ 372.46 by reallocating 26 sales campaign days with total cost of ¥ 10.40 million and $\hat{R}(\underline{d}) =$ ¥ 383.66 million. This optimal solution amounts to 7.69 % increase rate from actual profit, $P(\underline{I}_{CAMP}) =$ ¥ 345.85 million, where $\sum_{i=1}^{M} I_{CAMP}(i) = 36$.

3.3 Model Specification: Optimizing Expected Profit Under the Enhanced Campaign Budget $\boldsymbol{B = B_0 + \Delta_B}$

In the second extension, the campaign budget is enhanced and the campaign budget increase is incorporated as part of the decision variables. More specifically, let $B = B_0 + \Delta_B$ be the new enhanced campaign budget per day, where the campaign budget increase Δ_B is considered as a decision variable along with the campaign day assignment vector, denoted here by $\underline{d}_{\Delta_B} = [d_{\Delta_B}(1),\cdots,d_{\Delta_B}(j),\cdots,d_{\Delta_B}(M)] \in \{0,1\}^M$, and let $\hat{I}_{GOOD:\Delta_B}(j) = 1$ or 0, if day j is estimated to be a GSD or not under $\Delta_B > 0$ by following the procedure described in *Chapter 2* to determine $\hat{I}_{GOOD}(j)$.

Now, we are in a position to estimate the expected total sales per day under $\Delta_B > 0$. If day j in the future winter period under consideration is not chosen for campaign, that is, if $d_{\Delta_B}(j) = 0$, the campaign budget increase Δ_B would not affect the expected total sales of day j. On the other hand, if $d_{\Delta_B}(j) = 1$, it is natural to assume that the expected total sales would be increased with the effect of diminishing return. Namely, let $g(x)$ be a strictly increasing concave function of x with $g(0) = 1$, and $lim_{\Delta_B\to\infty}\, g(x) = 1 + \frac{a}{b}$. If $d^*(j) = 0$ and $d_{\Delta_B}(j) = 1$, that is, if the optimal decision for day j with $\Delta_B = 0$ is not to campaign and day j is chosen for campaign with $\Delta_B > 0$, the expected total sales per day would be increased from $\hat{s}_{(0,l)}$ to $\hat{s}_{(0,l)} + (\hat{s}_{(1,n)} - \hat{s}_{(0,l)}) \times g_s(\Delta_B)$, where $\hat{s}_{(0,l)}$ is the expected total sales under $\Delta_B = 0$, $l = \hat{I}_{GOOD}(j)$ and $n = \hat{I}_{GOOD:\Delta_B}(j)$. This can be reasoned in the following manner; if day j switched from $d^*(j) = 0$ under $\Delta_B = 0$ to campaign $d_{\Delta_B}(j) = 1$ under $\Delta_B > 0$, the expected total sales would be increased by $(\hat{s}_{(1,n)} - \hat{s}_{(0,l)})$, this increase is strengthened by a factor of $g_s(\Delta_B)$ as a result of increasing the campaign budget from B_0 to $B_0 + \Delta_B$. Whereas, if day $d^*(j) = 1$ does not switch to a non-campaign day, that is $d^*(j) = d_{\Delta_B}(j) = 1$, the expected total sales of day j under $\Delta_B = 0$ would be increased by $g_{\neg s}(\Delta_B)$. In order to describe these assumptions succinctly, we define $\hat{r}_{(k,l)\to(m,n)}(j)$ to be the expected total sales of day j with $\Delta_B > 0$, where $k = d^*(j)$, $l = \hat{I}_{GOOD}(j)$, $m = d_{\Delta_B}(j)$, and $n = \hat{I}_{GOOD:\Delta_B}(j)$. One then has

$$\begin{array}{lcll} \hat{r}_{(k,l)\to(0,n)}(j) & = & \hat{s}_{(0,n)} & k,l,n \in \{0,1\} \\ \hat{r}_{(0,l)\to(1,n)}(j) & = & \hat{s}_{(0,l)} + \left(\hat{s}_{(1,n)} - \hat{s}_{(0,l)}\right) \times g_s(\Delta_B) & l,n \in \{0,1\}, n \geq l \\ \hat{r}_{(1,l)\to(1,n)}(j) & = & \hat{s}_{(1,n)} \times g_{\neg s}(\Delta_B) & l,n \in \{0,1\}, n = l \end{array} \quad (3.3.1)$$

The total expected sales, denoted by $\hat{R}\left(\underline{d}_{\Delta_B}, \Delta_B\right)$, can now be computed as

$$\hat{R}\left(\underline{d}_{\Delta_B}, \Delta_B\right)$$

$$= \sum_{j=1}^{M} \sum_{k,l\in\{0,1\}} \sum_{m,n\in\{0,1\}} \hat{r}_{(k,l)\to(m,n)}(j) \,.\, \delta_{\{k=d^*(j)\}}\, \delta_{\{l=\hat{I}_{GOOD}(j)\}} \delta_{\{m=d_{\Delta_B}(j)\}} \delta_{\{n=\hat{I}_{GOOD:\Delta_B}(j)\}} \;, \quad (3.3.2)$$

Accordingly, for this extension, the optimization problem of expected profit, denoted by $\hat{P}\left(\underline{d}_{\Delta_B}, \Delta_B\right)$, can then be written as

$$\max_{\underline{d}_{\Delta_B}, \Delta_B} \left[\hat{R}\left(\underline{d}_{\Delta_B}, \Delta_B\right) - (B_0 + \Delta_B) \times \sum_{j=1}^{M} d_{\Delta_B}(j)\right] \; subject\ to \; \sum_{j=1}^{M} d_{\Delta_B}(j) \leq N \;. \quad (3.3.3)$$

3.4 Basic Properties of the Total Expected Sales Under $B = B_0 + \Delta_B$

Let $g(\Delta_B)$ be a strictly increasing concave function defined by two parameters, a and b as

$$g(\Delta_B) = 1 + \frac{a \times \Delta_B}{1 + b \times \Delta_B} \quad , \quad (3.4.1)$$

where $lim_{\Delta_B\to\infty}\, g(\Delta_B) = 1 + \frac{a}{b}$, and $g(0) = 1$. By differentiating $g(\Delta_B)$ twice with respect to Δ_B, one finds

$$\frac{\partial}{\partial \Delta_B}\, g(\Delta_B) = \frac{a(1 + b \times \Delta_B) - a \times b \times \Delta_B}{(1 + b \times \Delta_B)^2} = \frac{a}{(1 + b \times \Delta_B)^2} > 0 \quad , \quad (3.4.2)$$

$$\left(\frac{\partial}{\partial \Delta_B}\right)^2 g(\Delta_B) = -2 \times b\, \frac{a}{(1 + b \times \Delta_B)^2} < 0 \quad , \quad (3.4.3)$$

hence, $g(\Delta_B)$ is concave over Δ_B. Similarly, we define the functions; $g_s(\Delta_B)$ and $g_{\neg s}(\Delta_B)$, as

$$g_s(\Delta_B) = \ 1 \ + \ \frac{a_s \times \Delta_B}{1 + b_s \times \Delta_B} \quad ; \quad g_{\neg s}(\Delta_B) = \ 1 \ + \ \frac{a_{\neg s} \times \Delta_B}{1 + b_{\neg s} \times \Delta_B} \ . \tag{3.4.4}$$

One also differentiate $\hat{R}\left(\underline{d}_{\Delta_B}, \Delta_B\right)$, defined in (3.3.2), twice with respect to Δ_B. In order to describe the number of days concisely, we define $D_{(k,l)\to(m,n)}$ to be the sum of days corresponding to $\hat{r}_{(k,l)\to(m,n)}(j)$, as follows

$$D_{(k,l)\to(m,n)} = D_{(k,l)\to(0,n)} + D_{(0,l)\to(1,n)} + D_{(1,l)\to(1,n)} \quad , \tag{3.4.5}$$

where $D_{(k,l)\to(m,n)} = M$ and $\left\{D_{(0,l)\to(1,n)} + D_{(1,l)\to(1,n)}\right\} = N = \sum_{j=1}^{M} d_{\Delta_B}(j)$. Accordingly, one has

$$\frac{\partial}{\partial \Delta_B} \hat{R}\left(\underline{d}_{\Delta_B}, \Delta_B\right) = \frac{\partial}{\partial \Delta_B} \begin{Bmatrix} \hat{r}_{(k,l)\to(0,n)}(j) \times D_{(k,l)\to(0,n)} + \\ \hat{r}_{(0,l)\to(1,n)}(j) \times D_{(0,l)\to(1,n)} + \\ \hat{r}_{(1,l)\to(1,n)}(j) \times D_{(1,l)\to(1,n)} \end{Bmatrix} \quad , \tag{3.4.6}$$

Substituting for $\hat{r}_{(k,l)\to(m,n)}(j)$ as defined in (3.3.1), one sees that

$$\frac{\partial}{\partial \Delta_B} \hat{R}\left(\underline{d}_{\Delta_B}, \Delta_B\right) = \frac{\partial}{\partial \Delta_B} \left\{ \begin{matrix} \hat{s}_{(0,n)} \times D_{(k,l)\to(0,n)} \ + \\ \left(\hat{s}_{(0,l)} + \left(\left(\hat{s}_{(1,n)} - \hat{s}_{(0,l)}\right) \times \ g_s(\Delta_B)\right)\right) \times D_{(0,l)\to(1,n)} \ + \\ \hat{s}_{(1,n)} \ \times \ g_{\neg s}(\Delta_B) \times \ D_{(1,l)\to(1,n)} \end{matrix} \right\} \quad , \tag{3.4.7}$$

yielding

$$\frac{\partial}{\partial \Delta_B} \hat{R}\left(\underline{d}_{\Delta_B}, \Delta_B\right) = \left(\hat{s}_{(1,n)} - \hat{s}_{(0,l)}\right) \times \frac{\partial}{\partial \Delta_B} g_s(\Delta_B) \ \times D_{(0,l)\to(1,n)} + \\ \hat{s}_{(1,n)} \times \ \frac{\partial}{\partial \Delta_B} g_{\neg s}(\Delta_B) \times D_{(1,l)\to(1,n)} \qquad > 0 \quad , \tag{3.4.8}$$

$$\left(\frac{\partial}{\partial \Delta_B}\right)^2 \hat{R}\left(\underline{d}_{\Delta_B}, \Delta_B\right) = \left(\hat{s}_{(1,n)} - \hat{s}_{(0,l)}\right) \times \ \left(\frac{\partial}{\partial \Delta_B}\right)^2 g_s(\Delta_B) \ \times D_{(0,l)\to(1,n)} + \\ \hat{s}_{(1,n)} \ \times \left(\frac{\partial}{\partial \Delta_B}\right)^2 g_{\neg s}(\Delta_B) \ \times D_{(1,l)\to(1,n)} \quad < 0 \quad . \tag{3.4.9}$$

Hence, $\hat{R}\left(\underline{d}_{\Delta_B}, \Delta_B\right)$ is concave over Δ_B. As for expected profit, one has

$$\hat{P}\left(\underline{d}_{\Delta_B}, \Delta_B\right) = \left[\begin{matrix} \hat{s}_{(0,n)} \times D_{(k,l)\to(0,n)} \ + \\ \left(\ \hat{s}_{(0,l)} + \left(\left(\hat{s}_{(1,n)} - \hat{s}_{(0,l)}\right) \times g_s(\Delta_B)\right)\right) \times D_{(0,l)\to(1,n)} \ + \\ \hat{s}_{(1,n)} \times g_{\neg s}(\Delta_B) \times \ D_{(1,l)\to(1,n)} \end{matrix} \right] - (B_0 + \Delta_B) \times \sum_{j=1}^{M} d_{\Delta_B}(j) \tag{3.4.10}$$

In order to determine the optimal campaign budget increase ${\Delta_B}^*$ that yields the optimal expected profit $\hat{P}\left({\underline{d}_{\Delta_B}}^*, {\Delta_B}^*\right)$, one differentiate $\hat{P}\left(\underline{d}_{\Delta_B}, \Delta_B\right)$ with respect to Δ_B as

$$\frac{\partial}{\partial \Delta_B}\hat{P}(\underline{d}_{\Delta_B}, \Delta_B) = \frac{\partial}{\partial \Delta_B}\begin{bmatrix}(\hat{s}_{(1,n)} - \hat{s}_{(0,l)}) \times \ g_s(\Delta_B) \times D_{(0,l)\to(1,n)} + \\ \hat{s}_{(1,n)} \times g_{\neg s}(\Delta_B) \times \ D_{(1,l)\to(1,n)}\end{bmatrix} - \sum_{j=1}^{M} d_{\Delta_B}(j) \qquad , \qquad (3.4.11)$$

The first derivative of the optimal expected profit diminishes in the optimal budget increase $\Delta_B{}^*$, this yield

$$\sum_{j=1}^{M} d_{\Delta_B}(j) = \frac{\partial}{\partial \Delta_B} g_s(\Delta_B^*) \times (\hat{s}_{(1,n)} - \hat{s}_{(0,l)}) \times D_{(0,l)\to(1,n)} + \frac{\partial}{\partial \Delta_B} g_{\neg s}(\Delta_B^*) \times \hat{s}_{(1,n)} \times D_{(1,l)\to(1,n)},$$

$$(3.4.12)$$

Similarly written as

$$\frac{\sum_{j=1}^{M} d_{\Delta_B}(j)}{(\hat{s}_{(1,n)} - \hat{s}_{(0,l)}) \times D_{(0,l)\to(1,n)}} = \frac{\partial}{\partial \Delta_B} g_s(\Delta_B^*) + \frac{\partial}{\partial \Delta_B} g_{\neg s}(\Delta_B^*) \times \frac{\hat{s}_{(1,n)} \times D_{(1,l)\to(1,n)}}{(\hat{s}_{(1,n)} - \hat{s}_{(0,l)}) \times D_{(0,l)\to(1,n)}} \quad ,$$

$$(3.4.13)$$

For convenience, the expression $\frac{\hat{s}_{(1,n)} \times D_{(1,l)\to(1,n)}}{(\hat{s}_{(1,n)} - \hat{s}_{(0,l)}) \times D_{(0,l)\to(1,n)}}$ will be referred to as C

$$\frac{\partial}{\partial \Delta_B} g_s(\Delta_B^*) + \frac{\partial}{\partial \Delta_B} g_{\neg s}(\Delta_B^*) \times \ C = \frac{\sum_{j=1}^{M} d_{\Delta_B}(j)}{(\hat{s}_{(1,n)} - \hat{s}_{(0,l)}) \times D_{(0,l)\to(1,n)}} \qquad , \qquad (3.4.14)$$

Substituting for $\frac{\partial}{\partial \Delta_B} g_s(\Delta_B^*)$ and $\frac{\partial}{\partial \Delta_B} g_{\neg s}(\Delta_B^*)$, and by taking the square root of both sides, one finds

$$\frac{\sqrt{a_s}}{(1 + \ b_s \ \Delta_B^*)} + \frac{\sqrt{a_{\neg s}}}{(1 + \ b_{\neg s} \ \Delta_B^*)} \times \sqrt{C} = \sqrt{\frac{\sum_{j=1}^{M} d_{\Delta_B}(j)}{(\hat{s}_{(1,n)} - \hat{s}_{(0,l)}) \times D_{(0,l)\to(1,n)}}} \qquad , \qquad (3.4.15)$$

$$\frac{\sqrt{a_s}(1 + \ b_{\neg s} \ \Delta_B^*) + \sqrt{C}\sqrt{a_{\neg s}} \times (1 + b_s \ \Delta_B^*)}{(1 + b_s \ \Delta_B^*)(1 + b_{\neg s} \ \Delta_B^*)} = \sqrt{\frac{\sum_{j=1}^{M} d_{\Delta_B}(j)}{(\hat{s}_{(1,n)} - \hat{s}_{(0,l)}) \times D_{(0,l)\to(1,n)}}} \quad , \qquad (3.4.16)$$

as $b_s = b_{\neg s}$, one has

$$\frac{(1 + \ b_s \ \Delta_B^*)(\sqrt{a_s} + \sqrt{C}\sqrt{a_{\neg s}})}{(1 + b_s \ \Delta_B^*)(1 + b_s \ \Delta_B{}^*)} = \sqrt{\frac{\sum_{j=1}^{M} d_{\Delta_B}(j)}{(\hat{s}_{(1,n)} - \hat{s}_{(0,l)}) \times D_{(0,l)\to(1,n)}}} \quad , \qquad (3.4.17)$$

$$\Delta_B{}^* = \left(\sqrt{\frac{(a_s + C a_{\neg s}) \times (\hat{s}_{(1,n)} - \hat{s}_{(0,l)}) \times D_{(0,l)\to(1,n)}}{\sum_{j=1}^{M} d_{\Delta_B}(j)}} - 1\right) \ b_s{}^{-1} \qquad , \qquad (3.4.18)$$

Finally, substituting for C , the optimal campaign budget increase can be expressed by

$$\Delta_B{}^{*}=\left(\sqrt{\frac{\left(a_s\times\left(\hat{s}_{(1,n)}-\hat{s}_{(0,l)}\right)\times D_{(0,l)\to(1,n)}\right)+\left(a_{\neg s}\times\hat{s}_{(1,n)}\times D_{(1,l)\to(1,n)}\right)}{\sum_{j=1}^{M} d_{\Delta_B}(j)}}-1\right) b_s{}^{-1} \ . \quad (3.4.19)$$

One can easily see that, the value of the optimal campaign budget increase $\Delta_B{}^{*}$ can be determined only through numerical exploration. This is because the number of the sales campaign days is part of the formula 3.4.19 above.

3.5 The Optimal Solution of Expected Profit $\widehat{\boldsymbol{P}}\left(\underline{\boldsymbol{d}}_{\Delta_B}^{*}, \Delta_B{}^{*}\right)$

In order to solve the optimization problem of expected profit, $\hat{P}\left(\underline{d}_{\Delta_B}^{*}, \Delta_B{}^{*}\right)$, the best-bet values of the parameters of functions $g_s(\Delta_B)$ and $g_{\neg s}(\Delta_B)$ should be estimated. For this purpose, sensitivity analysis is conducted. Based on the partial approach of sensitivity analysis, the sensitivity of the function $g(\Delta_B)$ with respect to the parameters a , b is equal to the partial derivative of Δ_B with respect to the parameters a , b. Assuming that the campaign budget increase would be an increment of 10 % of the standard campaign budget per day $B = B_0 = ¥\,0.4$ million, accordingly, the campaign budget increment amount is estimated to be ¥ 0.04 million.

Sensitivity analysis of $g(\Delta_B)$ is conducted by holding parameter a fixed while varying parameter b with respect to $\Delta_B = 0.04$. Namely, we define $a \in \{1, \cdots, 10\}$ and $b \in \{1, \cdots, 10\}$ with 0.1 and 1.0 stepwise increments to investigate the sensitivity of the system under such conditions. Figure 3.5.1 below, shows the first derivative of $g(\Delta_B)$ with parameter $a = 5$ and $a = 0.5$ and varying parameter b in such a way that $a > b$ and $a < b$ within the range $\Delta_B = \{\, 0.04\,, \cdots, 0.8\,\}$. One finds that, the slope is flat when $b = 0$, and decreases as b increases. Although the curves do not look exactly the same, the general mode of the behavior of the system does not change. When parameter b varies, the first derivative decreases until the system stabilizes. It is clear that the stability of the system occurs at a later point in case of $a > b$ than that of $a < b$. Moreover, with 0.1 stepwise increments as $a > b$, the slop tends to be linear within the range $\Delta_B = \{0.04\,, \cdots, 0.8\,\}$. Accordingly, within the range of $\Delta_B = \{\, 0.04\,, \cdots, 0.8\,\}$, it is feasible to consider a 1.0 stepwise increment of the parameters a , b for $g_s(\Delta_B)$ and $g_{\neg s}(\Delta_B)$.

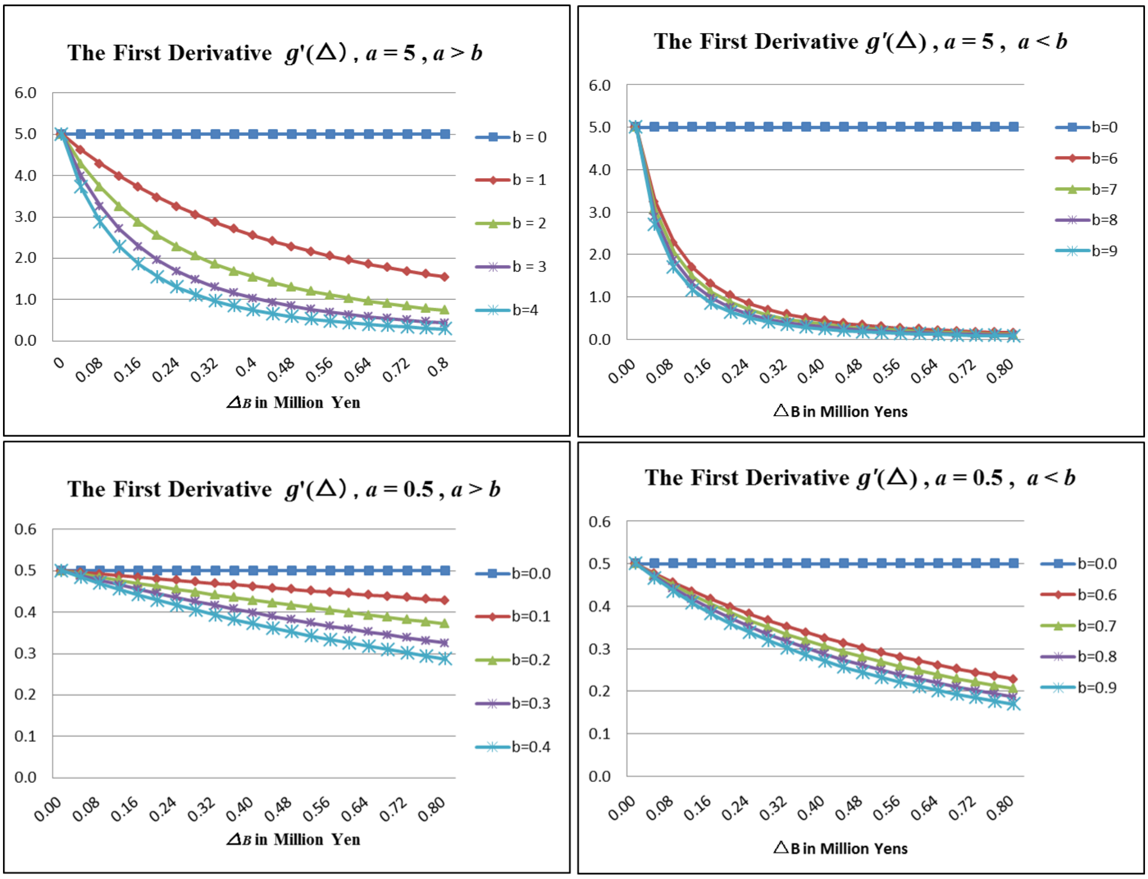

Figure 3.5.1 The First Derivative of the Function $g(\Delta_B)$ with $\Delta_B = 0.04$

Now, one can estimate the values of the parameters of $g_s(\Delta_B)$ and $g_{\neg s}(\Delta_B)$. In order to do so, the effect of the sales campaign to enhance total sales from $d(j) = 0$ to $d(j) = 1$ is first considered under $\Delta_B = 0$. Based on this, the strengthening effect of the sales campaign under $\Delta_B > 0$, expressed by $\left(\hat{s}_{(1,n)} - \hat{s}_{(0,l)}\right) \times g_s(\Delta_B), n \geq l,$ is estimated. This can be reasoned in the following manner; the strengthening factor of $\Delta_B > 0$ expressed by $g_s(\Delta_B)$ on $\left(\hat{s}_{(1,n)} - \hat{s}_{(0,l)}\right)$ where $n \geq l$, results when $\underline{d}^*(j) = 0$ switches to $\underline{d}^*_{\Delta_B}(j) = 1$. When $n \geq l$, three possibilities are conceived, that is $\left(\hat{s}_{(1,0)} - \hat{s}_{(0,0)}\right)$, $\left(\hat{s}_{(1,1)} - \hat{s}_{(0,1)}\right)$, and $\left(\hat{s}_{(1,1)} - \hat{s}_{(0,0)}\right)$. Accordingly, as shown in Table 3.5.1 below, the average increase rate of such effect is estimated to be 0.14 as shown below

$$\frac{\frac{\left(\hat{s}_{(1,0)} - \hat{s}_{(0,0)}\right)}{\hat{s}_{(0,0)}} + \frac{\left(\hat{s}_{(1,1)} - \hat{s}_{(0,0)}\right)}{\hat{s}_{(0,0)}} + \frac{\left(\hat{s}_{(1,1)} - \hat{s}_{(0,1)}\right)}{\hat{s}_{(0,1)}}}{3} = \frac{(0.066 + 0.03 + 0.321)}{3} = 0.139 \cong 0.14\,. \qquad (3.5.1)$$

By definition, the two functions $g_s(\Delta_B)$ and $g_{\neg s}(\Delta_B)$ express the same thing, that is, the effect of the campaign budget increase Δ_B on the expected total sales of day $d_{\Delta_B}(j) = 1$. One computes the effect of the sales campaign to improve $d(j) = 1$ from ¬ GSD to GSD as

$$\frac{\left(\hat{s}_{(1,1)} - \hat{s}_{(1,0)}\right)}{\hat{s}_{(1,0)}} = \frac{0.931}{3.89} = 0.24 \quad , \qquad (3.5.2)$$

therefore, one may assume that the strengthening effect of $g_{\neg s}(\Delta_B)$ to be 24 % of that of $g_s(\Delta_B)$.

Table 3.5.1 Basic Computations on Average Total Sales (¥ million) of Winter 2009 (LD) for Determining the Best-bet Values of the parameters of $g_s(\Delta_B)$ and $g_{\neg s}(\Delta_B)$

$\hat{s}_{(m,n)}$		$n = I_{GOOD:S_0T_0}(i)$		
		0	**1**	
$m = I_{CAMP}(i)$	**0**	¥ 3.65	¥ 4.68	
	1	¥ 3.89	¥ 4.82	
		$\hat{s}_{(1,0)} - \hat{s}_{(0,0)}$	$\hat{s}_{(1,1)} - \hat{s}_{(0,1)}$	$\hat{s}_{(1,1)} - \hat{s}_{(0,0)}$
		0.240	0.138	1.17
$\left(\hat{s}_{(1,n)} - \hat{s}_{(0,l)}\right) / \hat{s}_{(0,l)}$		6.6 %	3.0 %	32.1 %

From Table 3.5.2 below, one finds that parameters $(a_s, b_s) = (4, 3)$ correspond to $g_s(\Delta_B) = 1.14$, based on (3.5.1) and $(a_{\neg s}, b_{\neg s}) = (1, 4)$ correspond to $g_{\neg s}(\Delta_B) = 1.034$, based on (0.24 × 0.14 = 0.0336).

Table 3.5.2 The Output of The Function $g(\Delta_B)$ with 1.0 Stepwise Increments of Parameters a , b , $a \neq b$

b	a									
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1		1.0769	1.1154	1.1538	1.1923	1.2308	1.2692	1.3077	1.3462	1.3846
2	1.0370		1.1111	1.1481	1.1852	1.2222	1.2593	1.2963	1.3333	1.3704
3	1.0357	1.0714		1.1401	1.1786	1.2143	1.2500	1.2857	1.3214	1.3571
4	1.0340	1.0690	1.1034		1.1724	1.2069	1.2414	1.2759	1.3103	1.3448
5	1.0333	1.0667	1.1000	1.1333		1.2000	1.2333	1.2667	1.3000	1.3333
6	1.0323	1.0645	1.0968	1.1290	1.1613		1.22581	1.2581	1.2903	1.3226
7	1.0313	1.0625	1.0938	1.1250	1.1563	1.1875		1.2500	1.2813	1.3125
8	1.0303	1.0606	1.0909	1.1212	1.1515	1.1818	1.2121		1.2727	1.3030
9	1.0294	1.0588	1.0882	1.1176	1.1471	1.1765	1.2059	1.2353		1.2941
10	1.0290	1.0571	1.0857	1.1143	1.1429	1.1714	1.2000	1.2286	1.2571	

The optimization problem can now be readily solved yielding optimal expected profit $\hat{P}\left(\underline{d}^*_{\Delta_B}, \Delta_B{}^*\right) =$ ¥ 380.28 million, amounting to 9.95 % increase from actual profit $P\left(\underline{I}_{CAMP}\right) =$ ¥ 345.85 million, one notes that, the difference between the optimal and actual profit is found to be ¥ 34.43 million. This optimal solution is achieved by reallocating 28 campaign days with an optimal campaign budget $B^* =$ ¥ 0.68 million per day, amounting to 70 % budget increase ($\Delta_B = 0.28$) from the standard campaign budget $B_0 =$ ¥ 0.4 million. Figure 3.5.2 shows the curve of the optimal expected profit and Figure 3.5.3, displays the curves of the functions $g_s(\Delta_B)$ and $g_{\neg s}(\Delta_B)$ achieving the optimal solution. We note that, only 2 days switched from $d^*(j) = 0$ to $d^*_{\Delta_B}(j) = 1$, corresponding to expected total sales per day, $\hat{r}_{(0,1)\rightarrow(1,1)}(j)$, and accumulating 6.5 % of expected total sales of ¥ 150.55 million specific only to sales campaign days .

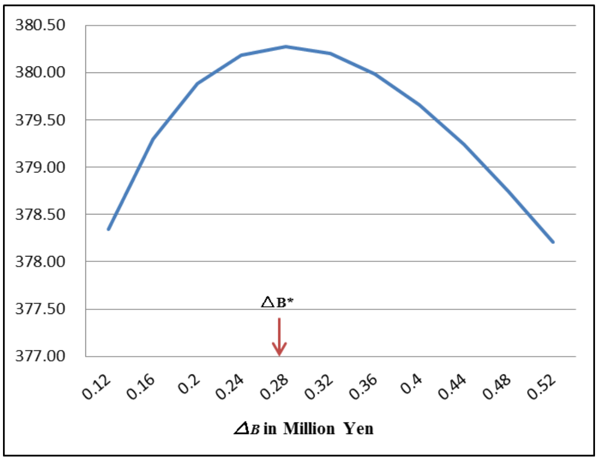

Figure 3.5.2 The Optimal Expected Profit (¥ million) in the Winter Period achieved by $\Delta_B{}^* =$ ¥ 0.28 million and $\sum_{j=1}^{M} d^*_{\Delta_B}(j) = 28$

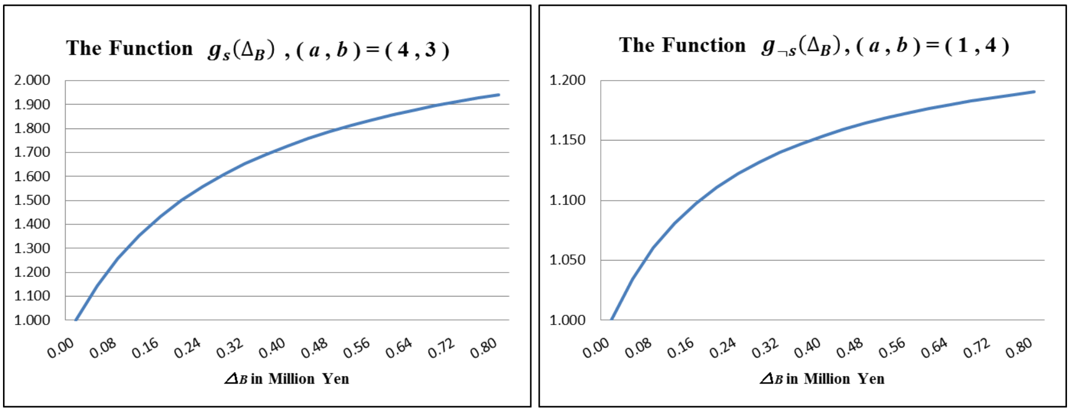

Figure 3.5.3 The Functions $g_s(\Delta_B)$ and $g_{\neg s}(\Delta_B)$ Achieved by $(a, b) = (4, 3)$ and (1 , 4), Respectively, with Respect to $\Delta_B = 0.04$

In order to investigate the robustness of the solution of $\hat{P}\left(\underline{d}^*_{\Delta_B}, \Delta_B{}^*\right)$ in face of varying parameter b of the functions $g_s(\Delta_B)$ and $g_{\neg s}(\Delta_B)$, we vary parameter b by 1.0 stepwise increase and decrease for each function separately as shown in Table 3.5.3.

Table 3.5.3 Optimal Expected Profit of Winter 2010 with Varying Parameter b by 1.0 Stepwise, $a \neq b$

Varying $b_{\neg s}$ of the function $g_{\neg s}(\Delta_B)$ with (a_s , b_s) = (4 , 3)				
	(*a* , *b*)	$\hat{P}(\underline{d}^*_{\Delta_B}, \Delta_B{}^*)$	$\Delta_B{}^*$	$\underline{d}^*_{\Delta_B}$
1.0 Stepwise Decrease of Parameter *b*	(1 , 2)	¥ 388.92	¥ 0.56	28
	(1 , 3)	¥ 383.16	¥ 0.36	28
The Best-bet Value	(1 , 4)	¥ 380.28	¥ 0.28	28
1.0 Stepwise Increase of Parameter *b*	(1 , 5)	¥ 378.53	¥ 0.24	28
	(1 , 6)	¥ 376.88	¥ 0.28	27
	…			
	(1 , 10)	¥ 375.12	¥ 0.12	27
Varying b_s of the function $g_s(\Delta_B)$ with ($a_{\neg s}$, $b_{\neg s}$) = (1 , 4)				
	(*a* , *b*)	$\hat{P}(\underline{d}^*_{\Delta_B}, \Delta_B{}^*)$	$\Delta_B{}^*$	$\underline{d}^*_{\Delta_B}$
1.0 Stepwise Decrease of Parameter *b*	(4 , 1)	¥ 380.35	¥ 0.28	28
	(4 , 2)	¥ 380.31	¥ 0.28	28
The Best-bet Value	(4 , 3)	¥ 380.28	¥ 0.28	28
1.0 Stepwise Increase of Parameter *b*	(4 , 5)	¥ 380.24	¥ 0.28	28
	(4 , 6)	¥ 380.22	¥ 0.28	28
	…			
	(4 , 10)	¥ 380.19	¥ 0.28	28

The results indicate that, when varying parameter $b_{\neg s}$ of the function $g_{\neg s}(\Delta_B)$, the best-case and worst-case scenarios yielded $\hat{P}(\underline{d}^*_{\Delta_B}, \Delta_B{}^*) =$ ¥ 388.92 and ¥ 375.12 million, respectively, achieving a sensitivity index (*SI*) of less than 4 % as computed below

$$SI = \frac{\hat{P}(\underline{d}^*_{\Delta_B}, \Delta_B{}^*)_{MAX} - \hat{P}(\underline{d}^*_{\Delta_B}, \Delta_B{}^*)_{MIN}}{\hat{P}(\underline{d}^*_{\Delta_B}, \Delta_B{}^*)_{MAX}} = \frac{13.8}{388.92} = 0.035 \quad , \qquad (3.5.2)$$

whereas the best-case and worst-case scenarios of varying parameter b_s of the function $g_s(\Delta_B)$ yielded $\hat{P}(\underline{d}^*_{\Delta_B}, \Delta_B{}^*) =$ ¥ 380.35 and ¥ 380.19 million, respectively, yielding a *SI* of less than 1 % as shown below

$$SI = \frac{\hat{P}(\underline{d}^*_{\Delta_B}, \Delta_B{}^*)_{MAX} - \hat{P}(\underline{d}^*_{\Delta_B}, \Delta_B{}^*)_{MIN}}{\hat{P}(\underline{d}^*_{\Delta_B}, \Delta_B{}^*)_{MAX}} = \frac{0.16}{380.35} = 0.00042 \quad , \qquad (3.5.3)$$

In order to assess the impact of the flexible approach for optimally reallocating sales campaign days with varying campaign budget, the optimal solutions of the expected profit $\hat{P}\left(\underline{d}^*_{\Delta_B}, \Delta_B{}^*\right)$ and $\hat{P}\left(\underline{d}^*\right)$ are compared against the actual profit, $P\left(\underline{I}_{CAMP}\right) =$ ¥ 345.85 million obtained from traditionally organizing sales campaigns in segments of consecutive days. The expected profit of optimal total expected sales, denoted by $\hat{P}(\ \hat{R}(\underline{d}^*)\ )$ is also compared against actual profit. Table 3.5.4 below shows the results of the optimal solutions and their increase rate from the actual profit. One finds that, there is an increase rate of only 0.46 % of $\hat{P}\left(\underline{d}^*\right)$ from $\hat{P}(\hat{R}(\underline{d}^*))$. Moreover, when the campaign budget is increased by 70 % from the standard campaign budget B_0, the increase rate of the optimal solution $\hat{P}\left(\underline{d}^*_{\Delta_B}, \Delta_B{}^*\right) =$ ¥ 380.28 from actual profit is 2.26 % higher than the optimal solution of $\hat{P}\left(\underline{d}^*\right) =$ ¥ 372.46.

Table 3.5.4 The Optimal Solutions of Expected Profit Compared Against the Actual Profit (¥ million) for the Winter Period 2010

Actual Profit $\boldsymbol{P\left(\underline{I}_{CAMP}\right)}$	$\boldsymbol{B_0}$	$\boldsymbol{\sum_{j=1}^{M} \underline{I}_{CAMP}}$	$\boldsymbol{B_0 \times \sum_{j=1}^{M} \underline{I}_{CAMP}}$	$\boldsymbol{R\left(\underline{I}_{CAMP}\right)}$	$\boldsymbol{P\left(\underline{I}_{CAMP}\right)}$	**Increase Rate From Actual Profit** $\boldsymbol{P\left(\underline{I}_{CAMP}\right)}$
	¥ 0. 40	36	¥ 14.4	¥ 360.25	¥ 345.85	
The Objective Function	**Results**					
Optimal Expected Profit $\boldsymbol{\hat{P}\left(\underline{d}^*_{\Delta_B}, \Delta_B{}^*\right)}$ $\boldsymbol{\Delta_B}^*=$ ¥ 0.28	$\boldsymbol{B^*}$	$\boldsymbol{\sum_{j=1}^{M} \underline{d}^*_{\Delta_B}}$	$\boldsymbol{(B_0 + \Delta_B^*) \times \sum_{j=1}^{M} \underline{d}^*_{\Delta_B}}$	$\boldsymbol{\hat{R}\left(\underline{d}_{\Delta_B}, \Delta_B\right.}$	$\boldsymbol{\hat{P}\left(\underline{d}^*_{\Delta_B}, \Delta_B{}^*\right)}$	**9.95%**
	¥ 0.68	28	¥ 19.04	¥ 399.32	¥ 380. 28	
Optimal Expected Profit $\boldsymbol{\hat{P}\left(\underline{d}^*\right)}$	$\boldsymbol{B_0}$	$\boldsymbol{\sum_{j=1}^{M} \underline{d}^*}$	$\boldsymbol{B_0 \times \sum_{j=1}^{M} \underline{d}^*}$	$\boldsymbol{\hat{R}\left(\underline{d}\right)}$	$\boldsymbol{\hat{P}\left(\underline{d}^*\right)}$	**7.84 %**
	¥ 0. 40	26	¥ 10.40	¥ 383.38	¥ 372.98	
Optimal Total Expected Sales $\boldsymbol{\hat{R}\left(\underline{d}^*\right)}$	$\boldsymbol{B_0}$	$\boldsymbol{\sum_{j=1}^{M} \underline{d}^*}$	$\boldsymbol{B_0 \times \sum_{j=1}^{M} \underline{d}^*}$	$\boldsymbol{\hat{R}\left(\underline{d}^*\right)}$	$\boldsymbol{\hat{P}\left(\hat{R}\left(\underline{d}^*\right)\right)}$	**7.38 %**
	¥ 0.40	36	¥ 14.40	¥ 385.78	¥ 371.38	

Table 3.5.5 below, demonstrates how the optimal allocation of sales campaign days, $\underline{d}^* = 26$ under $\Delta_B = 0$ and $\underline{d}_{\Delta_B}{}^* = 28$ with $\Delta_B{}^* = 0.28$ differ from the actual sales campaign days $\underline{I}_{CAMP} = 36$. In the winter period, we set $M = 88$ and $N = 36$, where the sales campaign Win_1 and Win_2 are each organized in a segment of consecutive days of 28 and 8, respectively. One find that, for $\hat{P}\left(\underline{d}^*\right)$, only 4 campaign days are in common out of 26 optimal and 36 actual. There are 32 days for which sales campaign is assigned only in the actual practice, and 22 by the optimal decision.

For $\hat{P}(\underline{d}^*)$, only 5 campaign days are in common out of 28 optimal and 36 actual days. There are 31 days for which sales campaign is assigned only in the actual practice, and 23 by the optimal decision.

Table 3.5.5 The Effect of the Optimal Decision Approach on GSDs Under Budget B_0 and $B_0 + \Delta_B$ for the Winter Period 2010

$\hat{P}(\underline{d}^*)$					$\hat{P}(\underline{d}^*_{\Delta_B}, \Delta_B{}^*)$				
# of GSD Days		**Optimal GSD**		**Total**	**# of GSD Days**		**Optimal GSD**		**Total**
		1	**0**				**1**	**0**	
Actual GSD	**1**	4	32	36	**Actual**	**1**	5	31	36
	0	22	30	52	**GSD**	**0**	23	29	52
Total		26	62	88	**Total**		28	60	88

In Table 3.5.6 below, the effect of the optimal allocation of sales campaign days (SCD) on GSDs is summarized for the optimal solutions of expected profit $\hat{P}(\underline{d}^*)$ and $\hat{P}(\underline{d}^*_{\Delta_B}, \Delta_B{}^*)$. In case of $\hat{P}(\underline{d}^*)$, 31 GSDs and 24 ¬GSDs are in common, amounting to 62.5 % of 88 days in winter 2010. It should be noted that the optimal decision of $\hat{P}(\underline{d}^*)$ approach converted 26 actual ¬GSDs into GSDs in the optimal decision, while only 7 days were downgraded from GSD in the actual practice to ¬GSD. Consequently, the optimal decision approach of $\hat{P}(\underline{d}^*)$, yielded 57 GSDs or 65% of 88 days. In case of $\hat{P}(\underline{d}^*_{\Delta_B}, \Delta_B{}^*)$, 30 GSDs and 25 ¬GSDs are common, amounting to 62.5 % of 88 days in winter 2010. The optimal decision approach of $\hat{P}(\underline{d}^*_{\Delta_B}, \Delta_B{}^*)$, converted 25 actual ¬GSDs into GSDs in the optimal decision, while 8 days were downgraded from GSD in the actual practice to ¬GSD in the optimal decision, consequently, the optimal decision approach of $\hat{P}(\underline{d}^*_{\Delta_B}, \Delta_B{}^*)$ yielded 55 GSDs or 62.5% of 88 days.

Table 3.5.6 The Allocation of Sales Campaign Days Across GSD for $\hat{P}(\underline{d}^*)$ and $\hat{P}(\underline{d}^*_{\Delta_B}, \Delta_B{}^*)$ for the Winter Period 2010

$\hat{P}(\underline{d}^*)$					$\hat{P}(\underline{d}^*_{\Delta_B}, \Delta_B{}^*)$				
# of SCD Days		**Optimal SCD**		**Total**	**# of SCD Days**		**Optimal SCD**		**Total**
		GSD	**¬GSD**				**GSD**	**¬GSD**	
Actual SCD	**GSD**	31	7	38	**Actual SCD**	**GSD**	30	8	38
	¬GSD	26	24	50		**¬GSD**	25	25	50
Total		57	31	88	**Total**		55	33	88

Table 3.5.7 and 3.5.8 demonstrate how the improvement by the optimal decision approach of $\hat{P}(\underline{d}^*)$ and $\hat{P}(\underline{d}^*_{\Delta_B}, \Delta_B{}^*)$ was achieved in further detail, where transitions of GSD vs.¬GSD are classified according to sales campaign days in the following manner: in the actual practice only, in

common only (actual and optimal), and by the optimal decision approach only. The optimal solution of $\hat{P}(\underline{d}^*)$ yielded 20 + 3 = 23 sales campaign days (or 64%) are assigned to GSDs in the actual practice, and 12 + 1 = 13 days (or 36%) to ¬GSDs, whereas the optimal decision approach allocated only 3 + 2 = 6 days (or 19%) to GSDs in the actual practice, and 1 + 20 = 21 days (or 81%) to ¬GSDs. On the other hand, the optimal solution of $\hat{P}(\underline{d}^*_{\Delta_B}, \Delta_B{}^*)$ yielded 19 + 4 = 23 sales campaign days (or 64%) are assigned to GSDs in the actual practice, and 12 + 1 = 13 days (or 36%) to ¬GSDs, whereas the optimal decision approach allocated only 4 + 2 = 6 days (or 21%) to GSDs in the actual practice, and 1 + 21 = 22 days (or 79%) to ¬GSDs. This result supports the original observation that the effect of a sales campaign for enhancing the total sales of ¬GSD may exceed that for strengthening the total sales of GSD further.

Table 3.5.7 GSD vs. ¬GSD Transitions by the Optimal Decision Approach for $\hat{\boldsymbol{P}}(\underline{\boldsymbol{d}}^*)$, $\underline{\boldsymbol{d}}^*$ = 26 for the Winter Period 2010

Actual Only

SCD		Optimal GSD	Optimal ¬GSD	Total
Actual	GSD	14	6	20
	¬GSD	0	12	12
Total		14	18	32

In Common

SCD (Common)		Optimal GSD	Optimal ¬GSD	Total
Actual	GSD	3	0	3
	¬GSD	1	0	1
Total		4	0	4

Optimal Only

SCD		Optimal GSD	Optimal ¬GSD	Total
Actual	GSD	2	0	2
	¬GSD	20	0	20
Total		22	0	22

Table 3.5.8 GSD vs. ¬GSD Transitions by the Optimal Decision Approach for $\hat{\boldsymbol{P}}(\underline{\boldsymbol{d}}^*_{\Delta_B}, \Delta_B{}^*)$, $\underline{\boldsymbol{d}}^*_{\Delta_B}$ = 28 for the Winter Period 2010

Actual Only

SCD		Optimal GSD	Optimal ¬GSD	Total
Actual	GSD	13	6	19
	¬GSD	0	12	12
Total		13	18	31

In Common

SCD		Optimal GSD	Optimal ¬GSD	Total
Actual	GSD	3	1	4
	¬GSD	1	0	1
Total		4	1	5

Optimal Only

SCD		Optimal GSD	Optimal ¬GSD	Total
Actual	GSD	2	0	2
	¬GSD	20	1	21
Total		22	1	23

4 Numerical Examples

This chapter is devoted to numerical example of the fall season. The mathematical models developed in Chapters 2 and 3 are implemented on the fall datasets obtained from the same SC in Tokyo. Section 4.1 describes the datasets; and Section 4.2 reports the numerical results of the optimization problems.

4.1 Data Description of the Fall Period

A dataset from the same SC in Tokyo for the fall periods 2009 and 2010 are obtained, that is, September, October and November 2009 for fall 2009, and September, October and November 2010 for fall 2010. The dataset comprises the following main elements: total sales, number of purchase transactions, and the campaign flag, as defined previously in (2.2.1). Two sales campaigns are organized in the fall period, that is, Fall_1 and Fall_2. Unlike the winter period, Fall_1 is organized in two segments of consecutive days rather than one segment, whereas Fall_2 is organized in one segment only. The organization of sales campaign days of the fall periods 2009 and 2010 is given in Table 4.1.1 below.

Table 4.1.1 The Organization of Sales Campaign days over the Fall Periods 2009 and 2010 as Obtained from the SC

Start Date	End Date	Campaign	# of Days
Fall 2009			
09/01/2009	09/16/2009	no camp	16
09/17/2009	09/19/2009	Fall_1	3
09/20/2009	10/08/2009	no camp	19
10/09/2009	10/12/2009	Fall_1	4
10/13/2009	11/19/2009	no camp	37
11/20/2009	11/30/2009	Fall_2	11
Fall 2010			
09/01/2010	09/16/2010	no camp	16
09/17/2010	09/19/2010	Fall_1	3
09/20/2010	10/08/2010	no camp	19
10/09/2010	10/12/2010	Fall_1	4
10/13/2010	11/18/2010	no camp	36
11/19/2010	11/30/2010	Fall_2	12

Figure 4.1.1 shows $s(i)$ and $t(i)$ as obtained from the SC for fall periods 2009 and 2010 in a histogram format. One can easily detect some outliers in the datasets.

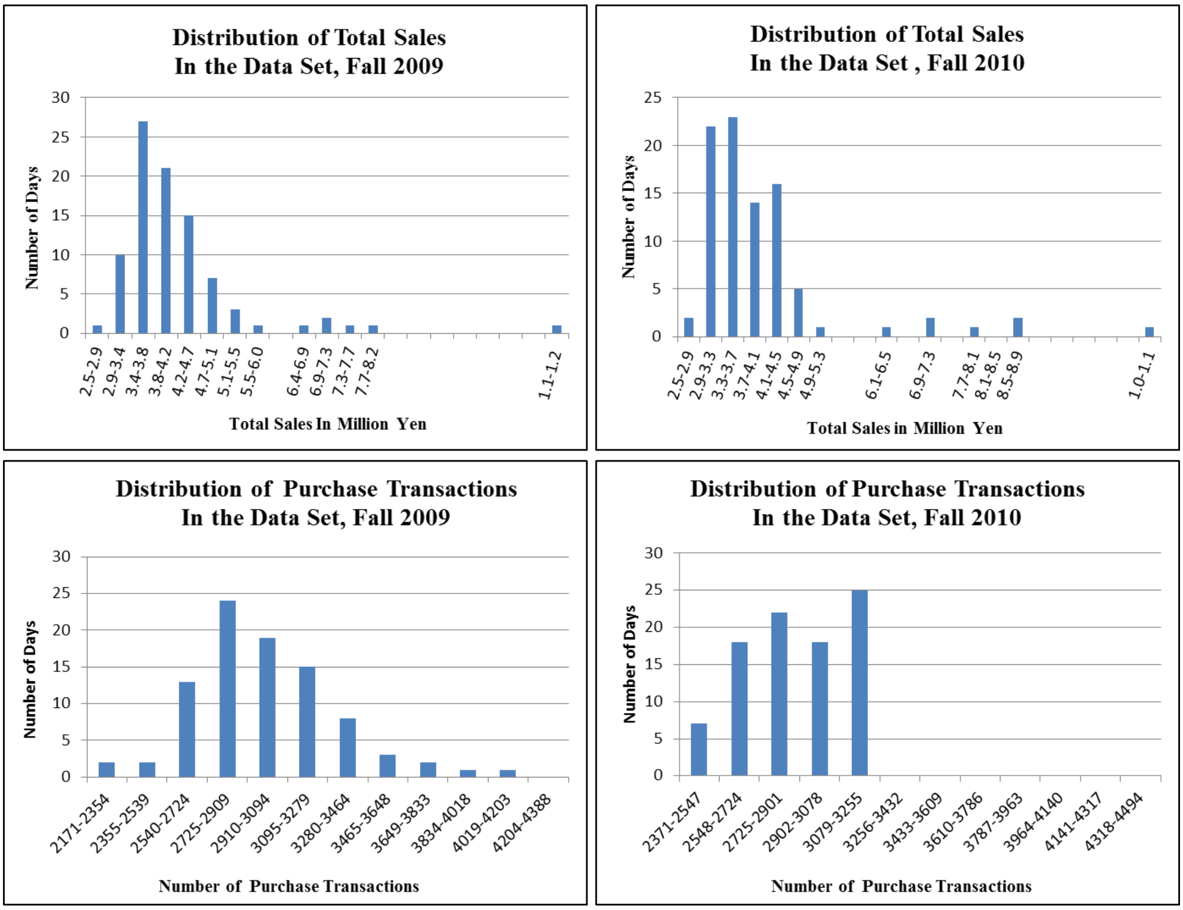

Figure 4.1.1 Total Sales and Number of Purchase Transactions for Fall 2009 and 2010 Before Cleaning

As in the winter season, outliers resulted from the Music Store are adjusted in Table 4.1.2., whereas outliers detected by the standard deviation method are adjusted by the formula given previously in (2.2.3) and shown in Table 4.1.3 below.

Table 4.1.2 Adjusted Outliers of the *Music Store* for the Fall Periods 2009 and 2010

Fall 2009							
Total Sales				Purchase Transactions			
Date	Entire SC	Store X	Adjusted Sales	Date	Entire SC	Store X	Adjusted Transactions
09/25/2009	¥ 7,985,535	¥ 4,041,400	¥ 3,944,135	09/25/2009	3,167	421	2,746
10/24/2009	¥ 11,375,837	¥ 4,574,900	¥ 6,800,937	10/24/2009	4,102	414	3,688
11/25/2009	¥ 7,565,088	¥ 3,908,000	¥ 3,657,088	11/25/2009	3,116	408	2,708

Fall 2010								
Total Sales					Purchase Transactions			
Date	Entire SC	Store X	Adjusted Sales		Date	Entire SC	Store X	Adjusted Transactions
09/25/2010	¥ 8,704,249	¥ 4,054,200	¥ 4,650,049		09/25/2009	3,687	433	3,254
10/25/2010	¥ 11,134,432	¥ 4,529,592	¥ 6,604,840		10/24/2009	3,397	432	2,965
11/25/2010	¥ 8,656,996	¥ 3,941,442	¥ 4,715,554		11/25/2009	3,191	420	2,771

Table 4.1.3 Adjusted Outliers for the Fall 2009 and 2010, Detected by the Standard Deviation Method

Fall 2009			Fall 2010		
Date	**Transactions**	**Adjusted Transactions**	**Date**	**Transactions**	**Adjusted Transactions**
09/19/2009	3,697	2,898	10/02/2010	3,670	2,992
10/10/2009	3,837	3,008	10/23/2010	3,541	2,887
10/11/2009	3,676	2,882	11/03/2010	3,442	2,806
10/24/2009	3,688	2,891			
Date	**Total Sales**	**Adjusted Total Sales**	**Date**	**Total Sales**	**Adjusted Total Sales**
10/22/2009	¥ 7,241,547	¥ 4,174,873	10/23/2010	¥ 8,050,988	¥ 4,221,772
10/23/2009	¥ 7,142,666	¥ 4,117,866	10/22/2010	¥ 7,300,568	¥ 3,828,268
10/24/2009	¥ 6,800,937	¥ 3,920,854	10/24/2010	¥ 7,172,099	¥ 3,760,901
10/25/2009	¥ 6,676,526	¥ 3,849,129	10/25/2010	¥ 6,604,840	¥ 3,463,442
			10/26/2010	¥ 6,481,638	¥ 3,398,838

Checking for minimum extremes in the dataset yielded two minimum extremes in the number of purchase transactions of fall 2009, both minimum extremes are present on a Wednesday. This sort of outliers is adjusted by the average purchase transactions of the weekdays of the same week. Table 4.1.4 below lists the minimum extremes and their adjusted values.

Table 4.1.4 Minimum Extremes in the Number of Purchase Transactions of Fall 2009

Fall 2009			
Date	**Day**	**Transactions**	**Adjusted Transactions**
10/07/2009	Wednesday	2,275	2,759
11/11/2009	Wednesday	2,171	2,731

Before the data cleaning, the Q−Q plots of the total sales and the number of purchase transactions in Figure 4.1.2 were examined to check for the normality assumption.

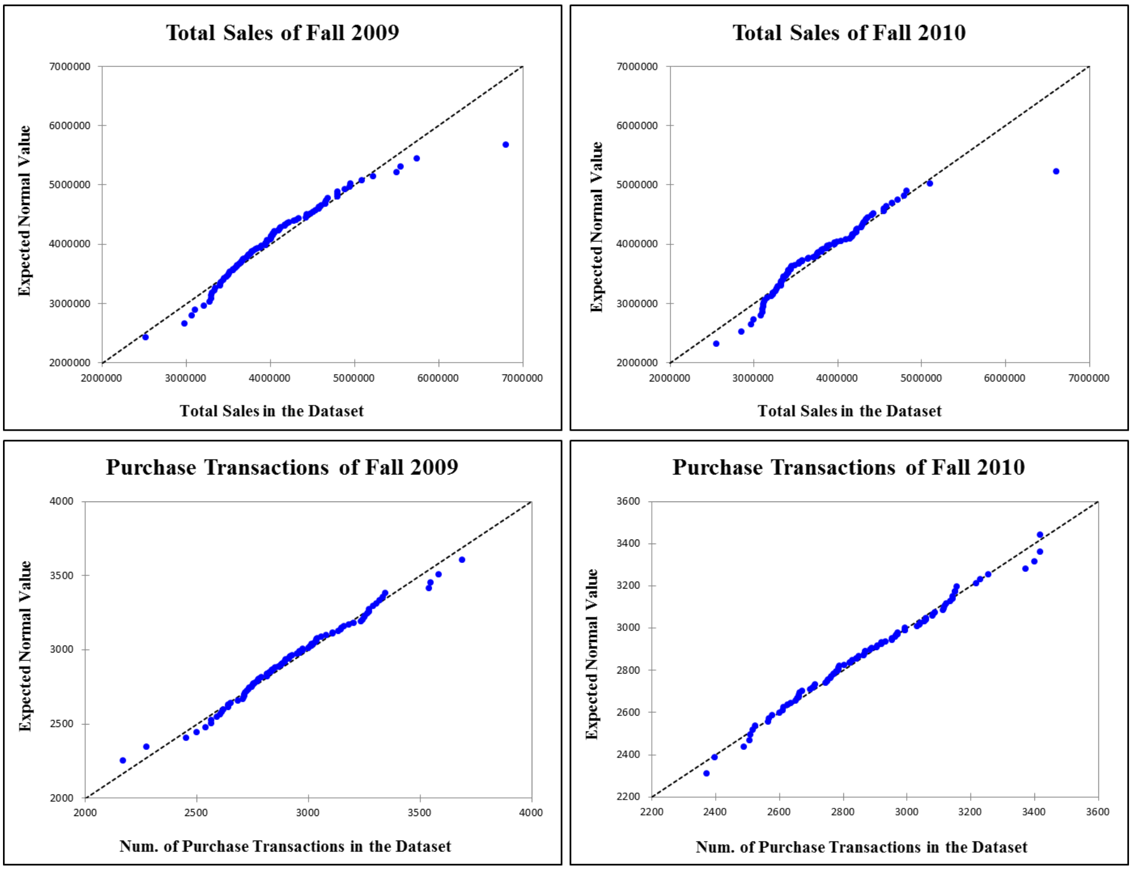

Figure 4.2.2 Q−Q Plots of Total Sales and Number of Purchase Transactions For Fall 2009 and 2010

4.2 Numerical Results

Numerical results of the fall season are reported in the following manner; first, the numerical thresholds S_0 and T_0 are summarized in Table 4.2.1 below. Second, the results of the logistic regression model and its associated confusion matrix are presented. Third, the average total sales matrix obtained from *LD* is reported. Fourth, the results of the optimization problem of total expected sales are shown and finally, the two extensions of the optimization problem of expected profit are reported.

First, the numerical threshold levels S_0 and T_0 of the fall period are summarized in Table 4.2.1 below. The numerical thresholds obtained from *LD* are used to mark the cut-off points in *TD* to define the variables $I_{GOOD:S_0T_0}(i)$ for $i \in D_{LD}$ and $\hat{I}_{GOOD:S_0T_0:TD}(j)$ for $j \in D_{TD}$, similarly.

Table 4.2.1 Numerical Thresholds of Total Sales and Number of Purchase Transactions Obtained from Fall 2009 (*LD*)

Deciles	Total Sales	Number of Purchase Transactions
10%	¥ 4,939,577	3,289
20%	¥ 4,658,289	3,204
30%	¥ 4,436,692	3,081
40%	¥ 4,174,873	3,007
50%	¥ 4,014,895	2,918
60%	¥ 3,920,854	2,865
70%	¥ 3,738,611	2,790
80%	¥ 3,585,832	2,725
90%	¥ 3,406,258	2,640
100%	¥ 2,521,625	2,171

Second, following the standard procedure for eliminating multi-collinearity of the explanatory variables in Table 4.2.2, the correlation structure for these variables is given in Table 4.2.3. It happened that the correlation of every pair of variables is less than 0.6 and no variables are eliminated because of multi-collinearity.

Table 4.2.2 Variables Considered for Logistic Regression for the Fall Period 2010

Label	Description
Week_k $(\boldsymbol{i})$, ***k = 1, 2, 3, 4 .***	Each month has four weeks, labeled as: *Week_1*, *Week_2*, *Week_3*, and *Week_4*. Any week consists of seven days, except that *Week_4* may include extra days until the end of the month. *Week_k* (i) =1 if day i belongs to week *k*, and *0*, otherwise.
Weekday_k (i), k = 1 , ⋯ , 5 .	This binary variable takes the value of 1 when WeekDay_k (i) is a weekday and 0 otherwise. Each week has five weekdays, Mon, Tue, Wed, Thu, and Fri, labeled as Weekday_1, Weekday_2, Weekday_3, Weekday_4, and Weekday_5, respectively.
Weekend $(\boldsymbol{i})$	This binary variable takes the value of 1 when day i is Saturday or Sunday, and 0, otherwise.
LY_Transactions $(\boldsymbol{i})$	This integer variable describes the number of purchase transactions of the same day of the month of the last year.
National Holiday (i)	This binary flag indicates that day i is an official national holiday in Japan.
Fall_1 $(\boldsymbol{i})$	This binary variable takes the value of 1 only if day i is in *September* or *October* and $I_{CAMP}(i)$=0, otherwise.
Fall_2 $(\boldsymbol{i})$	This binary variable takes the value of 1 only if day i is in *November* and $I_{CAMP}(i)$=0, otherwise.

Table 4.2.3 The Correlation Structure of Variables Tested for Multi-collinearity for the Fall Period 2010

	Week_1	Week_2	Week_3	Week_4	Mon	Tue	Wed	Thu	Fri	Weekend	LY_trans	Holiday_1	Fall_1	Fall_2
Week_1	1													
Week_2	−0.304	1												
Week_3	−0.295	−0.295	1											
Week_4	−0.371	−0.371	−0.359	1										
Mon	−0.002	−0.002	0.008	−0.003	1									
Tue	−0.002	−0.002	0.008	−0.003	−0.169	1								
Wed	0.015	0.015	−0.052	0.019	−0.161	−0.161	1							
Thu	−0.002	−0.002	0.008	−0.003	−0.169	−0.169	−0.161	1						
Fri	−0.002	−0.002	0.008	−0.003	−0.169	−0.169	−0.161	−0.169	1					
weekend	−0.004	−0.004	0.013	−0.005	−0.262	−0.262	−0.250	−0.262	−0.262	1				
LY_trans	0.000	0.126	−0.108	−0.018	−0.094	−0.288	−0.145	−0.299	−0.108	0.600	1			
Holiday_1	−0.019	−0.019	−0.013	0.047	0.314	0.038	0.048	−0.100	−0.100	−0.155	−0.020	1		
Fall_1	−0.160	0.232	0.144	−0.195	−0.001	−0.119	−0.114	−0.001	0.117	0.089	0.151	0.111	1	
Fall_2	−0.206	−0.206	−0.036	0.409	0.040	−0.057	−0.047	−0.057	0.040	0.062	−0.003	0.058	−0.108	1

The estimated regression coefficients and other statistical measures of the best logistic regression model are summarized in Table 4.2.4. The corresponding confusion matrix with maximum Precision subject to Recall $\geq$ 0.75 is shown in Table 4.2.5 below, yielding Precision $= 0.77$, Recall $=$ 0.77 and Accuracy $= 0.76$, with threshold value ${\rho^*}_{GOOD} =$ 0.06 and $S_0^* =$ 3,585,832 , $T_0^* =$ 2,725, representing the 80% threshold levels in the total sales and the number of purchase transactions in *LD*.

Table 4.2.4 Estimated Coefficients of the Logistic Regression for Fall 2010

	Estimate	Std. Error	z value	Pr(>|z|)	Sig
(Intercept)	-22.5623	7.629081	-2.95	0.00310	**
Weekend	5.01152	1.485726	3.37	0.00074	***
National Holiday	5.40635	1.755616	3.07	0.00207	**
Thursday	2.53315	1.281845	1.97	0.04813	*
Friday	2.49443	1.117791	2.23	0.02564	*
Week_1	1.92318	0.956927	2.01	0.04445	*
Fall_1	3.27627	1.612787	2.03	0.04221	*
Fall_2	2.96813	1.361948	2.17	0.02930	*
LY_Transactions	0.00601	0.002307	2.60	0.00915	**

Signif. codes: 0 '*' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1**

Given the decision vector $\underline{d}$ specifying campaign days for the future fall period, as well as the estimated coefficients of the explanatory variables in Table 4.2.4, one can compute $\rho_{GOOD}(j)$ as in (2.3.5) which in turn enables one to determine $\hat{I}_{GOOD}(j) = 1$ or 0.

Table 4.2.5 The Confusion Matrix of the Logistic Regression Model for Fall 2010

		Actual			
		$\neg\, GSD$	GSD	Total	
Judgment	$\neg\, GSD$	32	11	43	**Precision**
	GSD	11	36	47	**76.6%**
	Total	43	47	90	
		Recall	**76.60%**	**Accuracy**	**75.56%**

Third, the matrix of the average total sales, denoted by $\hat{s}_{(m,n)}$, computed over days $i \in D_{LD}$ in the fall period with $m = I_{CAMP}(i)$ and $n = I_{GOOD:S_0T_0}(i), m, n \in \{0,1\}$ is displayed in Table 4.2.6 below. The average total sales, obtained from *LD*, is then used to estimate the expected total sales of day $j \in D_{TD}$, denoted by $\hat{r}_{(m,n)}$ with $m = d(j)$ and $n = \hat{I}_{GOOD}(j), m, n \in \{0,1\}$.

Table 4.2.6 Average Total Sales (¥ million) Obtained from Fall 2009 (*LD*)

		$n = I_{GOOD:S_0T_0}(i)$	
		0	**1**
$m = I_{CAMP}(i)$	**0**	¥ 3.33	¥ 4.21
	1	¥ 3.60	¥ 4.35

In order to test the validity of this approach, the formula of total expected sales $\hat{R}(\underline{d})$ as in (2.3.6) is used with actual campaign days $\underline{I}_{CAMP}$ in *TD*, and then compared with the actual total sales R of *TD* achieving the relative accuracy of 1.40 % as shown in Table 4.2.7 below.

Table 4.2.7 The Validity of the Systematic Approach for Estimating Total Sales for Fall 2009 (*TD*)

(¥ Million)	Notation	Value
Expected total sales	$\hat{R}(\underline{I}_{CAMP})$	¥ 343.98
Actual total sales	R	¥ 339.26
Relative accuracy	$\lvert\hat{R}(\underline{I}_{CAMP}) - R\rvert \times 100/R$	1.40 %

Fourth, we report the results of the optimization problem of total expected sales. To assess the impact of this flexible allocation of sales campaign days, we compare the optimal solution of total expected sales against the actual total sales. For this purpose, we set M = 90 and N = 19 as obtained from *TD* of the fall period with $\sum_{j=1}^{90} d(j)$ =19. This optimization problem can now be solved, yielding $\hat{R}(\underline{d}^*) =$ ¥ 355.16 million. We note that the difference between the optimal total expected sales and the actual total sales, R = ¥ 339.26 is given by $\hat{R}(\underline{d}^*) - R =$ ¥ 15.9 million, or about 4.69 % increase.

Table 4.2.8 demonstrates how the optimal allocation of sales campaign days, $\underline{d}^*$, differs from the actual sales campaign days, $\underline{I}_{CAMP}$, obtained from *TD*. We find that only 2 sales campaign days are in common out of 19 sales campaign days. There are 17 days for which sales campaign is assigned only in the actual practice, or only by the optimal decision.

Table 4.2.8 Sales Campaign Days (Actual vs. Optimal) of $\hat{R}(\underline{d}^*)$ for Fall Period 2010 (*TD*)

		Optimal		Total
		1	**0**	
Actual	**1**	2	17	19
	0	17	54	71
Total		19	71	90

In Table 4.2.9, the effect of the optimal allocation of sales campaign days on GSDs is summarized, where 34 GSDs and 24 ¬GSDs are common, amounting to 64% of 90 days in the fall period. It should be noted that the optimal decision approach converted 26 ¬GSDs into GSDs, while only 6 days were downgraded from GSD to ¬GSD in the actual practice. Consequently, the optimal decision approach yielded 60 GSDs or 66.6% of 90 days.

Table 4.2.9 The Effect of the Optimal Decision Approach on GSDs of $\hat{R}(\underline{d}^*)$ for Fall 2010 (*TD*)

# of Days		Optimal		Total
		GSD	**¬GSD**	
Actual	**GSD**	34	6	40
	¬GSD	26	24	50
Total		60	30	90

Table 4.2.10 below demonstrates how the above improvement by the optimal decision approach was achieved in further detail, where GSD vs. ¬GSD are classified according to sales campaign days only in the actual practice, those in common (actual and optimal), and those only by the optimal decision approach.

Table 4.2.10 GSD vs. ¬GSD Transitions by Optimal Decision Approach of $\hat{R}(\underline{d}^*)$ for Fall 2010 (*TD*)

Actual Only				
Campaign Days		Optimal		Total
		GSD	¬GSD	
Actual	GSD	11	2	13
	¬GSD	0	4	4
Total		11	6	17

In Common				
Campaign Days		Optimal		Total
		GSD	¬GSD	
Actual	GSD	0	0	0
	¬GSD	2	0	2
Total		2	0	2

Optimal Only				
Campaign Days		Optimal		Total
		GSD	¬GSD	
Actual	GSD	7	0	7
	¬GSD	10	0	10
Total		17	0	17

In the actual practice, 13 + 0 = 13 sales campaign days (or 68%) are assigned to GSDs in the actual practice, and 4 + 2 = 6 days (or 32%) to ¬GSDs, whereas the optimal decision approach allocated only 7 + 0 = 7 days (or 37%) to GSDs in the actual practice, and 10 + 2 = 12 days (or 63%) to ¬GSDs. This result supports the original observation that the effect of a sales campaign for enhancing the total sales of ¬GSD may exceed that for strengthening the total sales of GSD further.

Finally, regarding the results of the two extensions of the optimization problem of expected profit, the campaign cost B_0 per day for the Fall campaigns in *LD* and those in *TD* is estimated to be ¥ 0.40 million in the following manner; the SC stated that the total cost per day would be approximately 20% of the total sales per day, which turned out to be: (¥ 3.98 million / day × 20% = ¥ 0.795 million / day). The SC also stated that the campaign cost per day was 50 % of the total cost for the fall period, yielding $\mathrm{B}_0 = ¥\,0.40$ million.

The strengthening effect of the sales campaign under $\Delta_B > 0$, expressed by $\left(\hat{s}_{(1,n)} - \hat{s}_{(0,l)}\right) \times g_s(\Delta_B), n \geq l$, is estimated based on the campaign effect under $\Delta_B = 0$ to enhance the total sales from $\hat{s}_{(0,l)}$ to $\hat{s}_{(1,n)}$ based on *LD*. When $n \geq l$, three possibilities are conceived, that is $\left(\hat{s}_{(1,0)} - \hat{s}_{(0,0)}\right)$, $\left(\hat{s}_{(1,1)} - \hat{s}_{(0,1)}\right)$, and $\left(\hat{s}_{(1,1)} - \hat{s}_{(0,0)}\right)$. Accordingly, as shown in Table 4.2.11 below, the average increase rate of such effect is estimated to be 0.14 as shown below

$$\frac{\frac{\left(\hat{s}_{(1,0)} - \hat{s}_{(0,0)}\right)}{\hat{s}_{(0,0)}} + \frac{\left(\hat{s}_{(1,1)} - \hat{s}_{(0,0)}\right)}{\hat{s}_{(0,0)}} + \frac{\left(\hat{s}_{(1,1)} - \hat{s}_{(0,1)}\right)}{\hat{s}_{(0,1)}}}{3} = \frac{(0.081 + 0.033 + .306\,)}{3} = \ 0.14. \tag{4.2.1}$$

One also computes the effect of the sales campaign to improve $d(j) = 1$ from ¬ GSD to GSD as

$$\frac{\left(\hat{s}_{(1,1)} - \hat{s}_{(1,0)}\right)}{\hat{s}_{(1,0)}} = \frac{0.75}{3.60} = \ 0.208 \qquad , \tag{4.2.2}$$

Assuming that the strengthening effect of $g_{\neg s}(\Delta_B)$ to be 21 % of that of $g_s(\Delta_B)$, the parameters $(a_s\,, b_s\,) \ = (\ 4\ ,\ 3\ )$, obtained from Table 3.5.2, correspond to $g_s(\Delta_B) = 1.14$, and $(a_{\neg s}\,, b_{\neg s}\,) =$

(1 , 10) correspond to $g_{\neg s}(\Delta_B) = 1.029$.

Table 4.2.11 Basic Computations on Average Total Sales (¥ million) of Fall 2009 (*LD*) for Determining the Best-bet Values of the parameters of $g_s(\Delta_B)$ and $g_{\neg s}(\Delta_B)$

$\hat{s}_{(m,n)}$		$n = I_{GOOD:S_0T_0}(i)$		
		0	1	
$m = I_{CAMP}(i)$	0	¥ 3.33	¥ 4.21	
	1	¥ 3.60	¥ 4.35	
		$\hat{s}_{(1,0)} - \hat{s}_{(0,0)}$	$\hat{s}_{(1,1)} - \hat{s}_{(0,1)}$	$\hat{s}_{(1,1)} - \hat{s}_{(0,0)}$
		0.27	0.14	1.02
$\left(\hat{s}_{(1,n)} - \hat{s}_{(0,l)}\right) / \hat{s}_{(0,l)}$		8.1 %	3.3%	30.6 %

Now, the optimization problem can be readily solved yielding optimal expected profit $\hat{P}\left(d^*_{\Delta_B}, \Delta_B{}^*\right) =$ ¥ 353.47 million, which amounts to 6.58 % increase from actual profit $P\left(\underline{I}_{CAMP}\right) =$ ¥ 331.66 million. This optimal solution is achieved by reallocating 19 campaign days with an optimal campaign budget $B^* =$ ¥ 0.72 million per day, amounting to 80 % ($\Delta_B{}^* = 0.32$) increase from the standard campaign budget $B_0 =$ ¥ 0.4 million. Figure 4.2.1 shows the curve of the optimal expected profit and Figure 4.2.2 displays the curves of the functions $g_s(\Delta_B)$ and $g_{\neg s}(\Delta_B)$ achieving the optimal solution. We note that, 17 days switched from $d^*(j) = 0$ to $d^*_{\Delta_B}(j) = 1$ generating $\hat{r}_{(0,0)\to(1,1)}(j)$ and accumulating 90.1 % of total expected sales of ¥ 94.63 million specific to sales campaign days only.

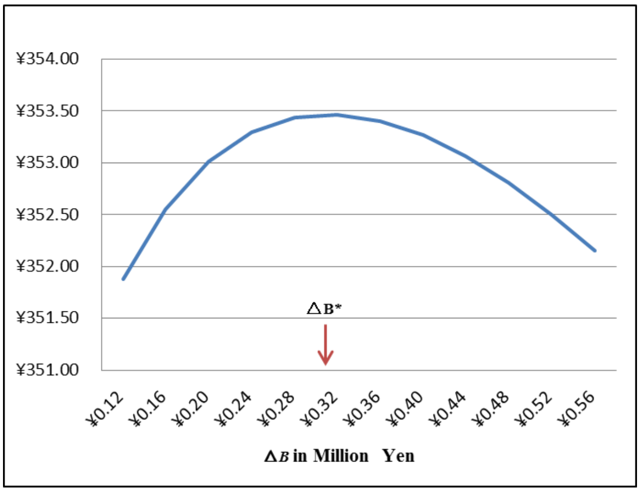

Figure 4.2.1 The Optimal Expected Profit (¥ million) in the Fall Period achieved by $\Delta_B{}^* =$ ¥ 0.32 million and $\sum_{j=1}^{M} d^*_{\Delta_B}(j) = 19$

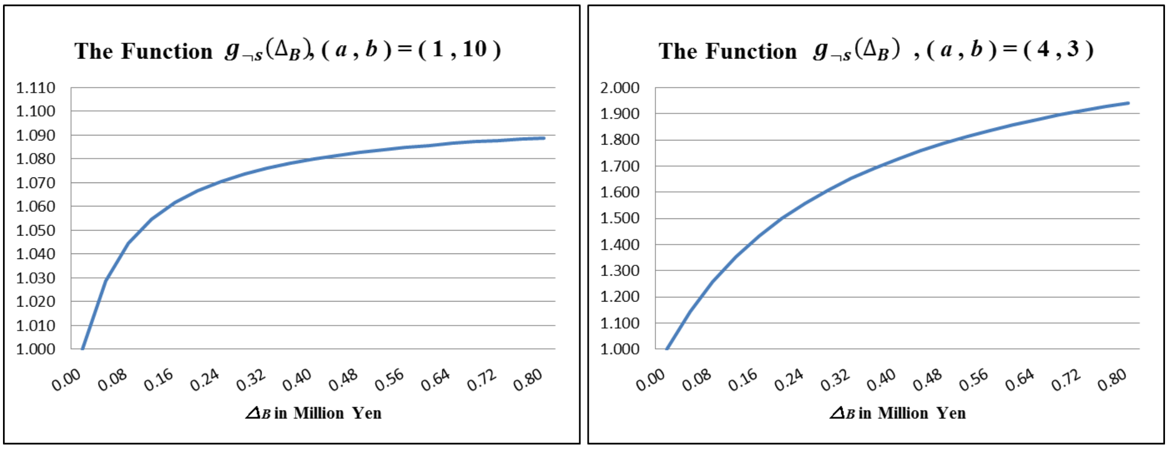

Figure 4.2.2 The Functions $g_s(\Delta_B)$ and $g_{\neg s}(\Delta_B)$ Achieved by (a , b) = (4 , 3) and (1 , 10), Respectively, with Respect to $\Delta_B = 0.04$

In order to investigate the robustness of the solution of $\hat{P}(\underline{d}^*_{\Delta_B}, {\Delta_B}^*)$ in face of varying parameter b of the functions $g_s(\Delta_B)$ and $g_{\neg s}(\Delta_B)$, we vary parameter b by 1.0 stepwise increase and decrease for each function separately, as shown in Table 4.2.12.

Table 4.2.12 Optimal Expected Profit of Fall 2010 with Varying Parameter b by 1.0 Stepwise, $a \neq b$

Varying $b_{\neg s}$ of the function $g_{\neg s}(\Delta_B)$ with (a_s , b_s) = (4 , 3)				
	(a , b)	$\hat{P}(\underline{d}^*_{\Delta_B}, {\Delta_B}^*)$	${\Delta_B}^*$	$\underline{d}^*_{\Delta_B}$
1.0 Stepwise Decrease of Parameter b	(1 , 2)	¥ 358.75	¥ 0.56	19
	…			
	(1 , 8)	¥ 353.55	¥ 0.28	19
	(1 , 9)	¥ 353.52	¥ 0.32	19
The Best-bet Value	(1 , 10)	¥ 353.47	¥ 0.32	19
Varying b_s of the function $g_s(\Delta_B)$ with ($a_{\neg s}$, $b_{\neg s}$) = (1 , 10)				
	(a , b)	$\hat{P}(\underline{d}^*_{\Delta_B}, {\Delta_B}^*)$	${\Delta_B}^*$	$\underline{d}^*_{\Delta_B}$
1.0 Stepwise Decrease of Parameter b	(4 , 1)	¥ 363.84	¥ 0.76	19
	(4 , 2)	¥ 356.15	¥ 0.48	19
The Best-bet Value	(4 , 3)	¥ 353.47	¥ 0.32	19
1.0 Stepwise Increase of Parameter b	(4 , 5)	¥ 350.97	¥ 0.28	19
	(4 , 6)	¥ 350.72	¥ 0.16	19
	…			
	(4 , 10)	¥ 349.70	¥ 0.12	19

The results indicate that, when holding parameter $a_{\neg s}$ fixed and varying parameter $b_{\neg s}$, the best-case and worst-case scenarios yielded $\hat{P}\left(\underline{d}^*_{\Delta_B}, {\Delta_B}^*\right) =$ ¥ 358.75 and ¥ 353.47 million, respectively, achieving a *SI* of less than 1.5 % as computed below

$$SI = \frac{\hat{P}\left(\underline{d}^*_{\Delta_B}, {\Delta_B}^*\right)_{MAX} - \hat{P}\left(\underline{d}^*_{\Delta_B}, {\Delta_B}^*\right)_{MIN}}{\hat{P}\left(\underline{d}^*_{\Delta_B}, {\Delta_B}^*\right)_{MAX}} = \frac{5.28}{358.75} = 0.015 \quad , \tag{3.5.2}$$

whereas the best-case and worst-case scenarios of varying parameter b_s of the function $g_s(\Delta_B)$ yielded $\hat{P}\left(\underline{d}^*_{\Delta_B}, {\Delta_B}^*\right) =$ ¥ 363.84 and ¥ 349.70 million, respectively, yielding a *SI* of 4.0 %.

$$SI = \frac{\hat{P}\left(\underline{d}^*_{\Delta_B}, {\Delta_B}^*\right)_{MAX} - \hat{P}\left(\underline{d}^*_{\Delta_B}, {\Delta_B}^*\right)_{MIN}}{\hat{P}\left(\underline{d}^*_{\Delta_B}, {\Delta_B}^*\right)_{MAX}} = \frac{14.4}{363.84} = 0.0395 \quad , \tag{3.5.3}$$

In order to assess the impact of the flexible approach for optimally reallocating sales campaign days with varying the campaign budget, the optimal solutions of the expected profit $\hat{P}\left(\underline{d}^*_{\Delta_B}, {\Delta_B}^*\right)$ and $\hat{P}\left(\underline{d}^*\right)$ are compared against the actual profit, $P\left(\underline{I}_{CAMP}\right) =$¥ 331.66 million. Table 4.2.13 below shows the results of the optimized solutions and their increase rate from the actual profit.

Table 4.2.13 The Optimal Solutions of Expected Profit Compared Against the Actual Profit (¥ million) for the Fall Period 2010

Actual Profit $P\left(\underline{I}_{CAMP}\right)$	$\boldsymbol{B_0}$	$\sum_{j=1}^{M} \underline{I}_{CAMP}$	$\boldsymbol{B_0} \times \sum_{j=1}^{M} \underline{I}_{CAMP}$	$R\left(\underline{I}_{CAMP}\right)$	$P\left(\underline{I}_{CAMP}\right)$	**Increase Rate From Actual Profit $P\left(\underline{I}_{CAMP}\right)$**
	¥ 0. 40	19	¥ 7.60	¥ 339.26	¥ 331.66	
The Objective Function	**Results**					
Optimal Expected Profit $\hat{P}\left(\underline{d}^*_{\Delta_B}, {\Delta_B}^*\right)$ ${\Delta_B}^* =$ ¥ 0.32	$\boldsymbol{B^*}$	$\sum_{j=1}^{M} \underline{d}^*_{\Delta_B}$	$(\boldsymbol{B_0} + \Delta^*_B) \times \sum_{j=1}^{M} \underline{d}^*_{\Delta_B}$	$\hat{R}\left(\underline{d}_{\Delta_B}, \Delta_B\right.$	$\hat{P}\left(\underline{d}^*_{\Delta_B}, {\Delta_B}^*\right)$	**6.58 %**
	¥ 0.72	19	¥ 13.68	¥ 367.15	¥ 353.47	
Optimal Expected Profit $\hat{P}\left(\underline{d}^*\right)$	$\boldsymbol{B_0}$	$\sum_{j=1}^{M} \underline{d}^*$	$\boldsymbol{B_0} \times \sum_{j=1}^{M} \underline{d}^*$	$\hat{R}\left(\underline{d}\right)$	$\hat{P}\left(\underline{d}^*\right)$	**4.79 %**
	¥ 0. 40	19	¥ 7.60	¥ 355.16	¥ 347.56	
Optimal Total Expected Sales $\hat{R}\left(\underline{d}^*\right)$	$\boldsymbol{B_0}$	$\sum_{j=1}^{M} \underline{d}^*$	$\boldsymbol{B_0} \times \sum_{j=1}^{M} \underline{d}^*$	$\hat{R}\left(\underline{d}^*\right)$	$P\left(\hat{R}\left(\underline{d}^*\right)\right)$	**4.79 %**
	¥ 0.40	19	¥ 7.60	¥ 355.16	¥ 347.56	

One finds that, there is no difference in the increase rate of $\hat{P}(\underline{d}^*)$ and $P(\hat{R}(\underline{d}^*))$ from actual profit. Moreover, when the campaign budget is increased by 80 % from the standard campaign budget B_0, the optimal solution $\hat{P}(\underline{d}^*_{\Delta_B}, \Delta_B{}^*) =$ ¥ 353.47 was 1.79 % higher than the optimal solution $\hat{P}(\underline{d}^*) =$ ¥ 347.56.

Table 4.2.14 below, demonstrates how the optimal allocation of sales campaign days, $\underline{d}^*$ = 19 under $\Delta_B = 0$ and $\underline{d}_{\Delta_B}{}^*$ = 19 with $\Delta_B{}^*$ = 0.32 differ from the actual sales campaign days $\underline{I}_{CAMP}$ = 19. We find that, for $\hat{P}(\underline{d}^*)$ only 5 campaign days are in common out of 19 optimal and actual. There are 14 days for which sales campaign is assigned only in the actual practice and by the optimal decision. For $\hat{P}(\underline{d}^*_{\Delta_B}, \Delta_B{}^*)$, only 3 campaign days are in common out of 19 optimal and actual. There are 16 days for which sales campaign is assigned only in the actual practice and by the optimal decision.

Table 4.2.14 The Effect of the Optimal Decision Approach on GSDs Under Budget $\boldsymbol{B_0}$ and $\boldsymbol{B_0 + \Delta_B}$ for the Fall Period 2010

$\hat{P}(\underline{d}^*)$					$\hat{P}(\underline{d}^*_{\Delta_B}, \Delta_B{}^*)$				
# of GSD Days		**Optimal GSD 1**	**Optimal GSD 0**	**Total**	**# of GSD Days**		**Optimal GSD 1**	**Optimal GSD 0**	**Total**
Actual GSD	**1**	5	14	19	**Actual GSD**	**1**	3	16	19
	0	14	57	71		**0**	16	55	71
Total		19	71	90	**Total**		19	71	90

In Table 4.2.15 below, the effect of the optimal allocation of sales campaign days (SCD) on GSDs is summarized for the optimal solutions of the expected profit $\hat{P}(\underline{d}^*)$ and $\hat{P}(\underline{d}^*_{\Delta_B}, \Delta_B{}^*)$. In case of $\hat{P}(\underline{d}^*)$, 36 GSDs and 26 ¬GSDs are in common, amounting to 69 % of 90 days in fall 2010. It should be noted that the optimal decision of $\hat{P}(\underline{d}^*)$ approach converted 24 actual ¬GSDs into GSDs in the optimal decision, while only 4 days were downgraded from GSD in the actual practice to ¬GSD. Consequently, the optimal decision approach of $\hat{P}(\underline{d}^*)$, yielded 60 GSDs or 67% of 90 days.

In case of $\hat{P}(\underline{d}^*_{\Delta_B}, \Delta_B{}^*)$, 31 GSDs and 21 ¬GSDs are common, amounting to 58 % of 90 days in fall 2010. The optimal decision approach of $\hat{P}(\underline{d}^*_{\Delta_B}, \Delta_B{}^*)$, converted 29 actual ¬GSDs into GSDs in the optimal decision, while 9 days were downgraded from GSD in the actual practice to ¬GSD in the optimal decision. Consequently, the optimal decision approach of $\hat{P}(\underline{d}^*_{\Delta_B}, \Delta_B{}^*)$ yielded 60 GSDs or 67% of 90 days.

Table 4.2.15 The Allocation of Sales Campaign Days Across GSD for $\hat{\boldsymbol{P}}(\underline{\boldsymbol{d}}^*)$ and $\hat{\boldsymbol{P}}(\underline{\boldsymbol{d}}^*_{\Delta_B}, \Delta_B{}^*)$ for the Fall Period 2010

$\hat{P}(\underline{d}^*)$					$\hat{P}(\underline{d}^*_{\Delta_B}, \Delta_B{}^*)$				
# of SCD Days		Optimal SCD		Total	# of SCD Days		Optimal SCD		Total
		GSD	¬GSD				GSD	¬GSD	
Actual SCD	GSD	36	4	40	Actual SCD	GSD	31	9	40
	¬GSD	24	26	50		¬GSD	29	21	50
Total		60	30	90	Total		60	30	90

Table 4.2.16 and 4.2.17 demonstrate how the improvement by the optimal decision approach of $\hat{P}(\underline{d}^*)$ and $\hat{P}(\underline{d}^*_{\Delta_B}, \Delta_B{}^*)$ was achieved in further detail, where transitions of GSD vs.¬GSD are classified in the following manner: in the actual practice only, in common only (actual and optimal), and by the optimal decision approach only. The optimal solution of $\hat{P}(\underline{d}^*)$ yielded 12 + 1 = 13 sales campaign days (or 68%) are assigned to GSDs in the actual practice, and 2 + 4 = 6 days (or 32%) to ¬GSDs, whereas the optimal decision approach allocated only 1 + 8 = 9 days (or 10%) to GSDs in the actual practice, and 4 + 6 = 10 days (or 11%) to ¬GSDs. On the other hand, the optimal solution of $\hat{P}(\underline{d}^*_{\Delta_B}, \Delta_B{}^*)$ yielded 12 + 1 = 13 sales campaign days (or 68%) are assigned to GSDs in the actual practice, and 4 + 2 = 6 days (or 32%) to ¬GSDs, whereas the optimal decision approach allocated only 1 + 3 = 4 days (or 21%) to GSDs in the actual practice, and 2 + 13 = 15 days (or 49%) to¬GSDs. This result supports the original observation that the effect of a sales campaign for enhancing the total sales of ¬GSD may exceed that for strengthening the total sales of GSD further.

Table 4.2.16 GSD vs. ¬GSD Transitions by the Optimal Decision Approach for $\hat{\boldsymbol{P}}(\underline{\boldsymbol{d}}^*)$, $\underline{\boldsymbol{d}}^*$ =19 for the Fall Period 2010

Actual Only				
SCD		Optimal		Total
		GSD	¬GSD	
Actual	GSD	11	1	12
	¬GSD	0	2	2
Total		11	3	14

In Common				
SCD		Optimal		Total
		GSD	¬GSD	
Actual	GSD	1	0	1
	¬GSD	4	0	4
Total		5	0	5

Optimal Only				
SCD		Optimal		Total
		GSD	¬GSD	
Actual	GSD	8	0	8
	¬GSD	6	0	6
Total		14	0	14

Table 4.2.17 GSD vs. ¬GSD Transitions by the Optimal Decision Approach for $\hat{\boldsymbol{P}}(\underline{\boldsymbol{d}}^*_{\Delta_B}, \Delta_B{}^*)$, $\underline{\boldsymbol{d}}^*_{\Delta_B} =$ 19 for the Fall Period 2010

Actual Only				
SCD		Optimal		Total
		GSD	¬GSD	
Actual	GSD	11	1	12
	¬GSD	0	4	4
Total		11	5	16

In Common				
SCD		Optimal		Total
		GSD	¬GSD	
Actual	GSD	0	1	1
	¬GSD	2	0	2
Total		3	0	3

Optimal Only				
SCD		Optimal		Total
		GSD	¬GSD	
Actual	GSD	3	0	3
	¬GSD	13	0	13
Total		16	0	16

5 Conclusion and Discussion

An extensive literature exists concerning SCs and sales optimization, where different approaches are taken; e.g. how to find the optimal location of SCs among available alternatives, and how to determine the configuration of space and design so as to achieve either cost-performance efficiency or profit generation. To the best knowledge of the researcher, the problem of optimally allocating campaign days over a certain period, e.g. the winter and fall seasons, has not been addressed in the literature. The purpose of this thesis is to fill this gap by developing a mathematical model to optimize returns in an SC by optimally reallocating sales campaign days, based on the marketing flexibility concept.

Through numerical examples, the proposed model for maximizing total expected sales demonstrated the power of marketing flexibility. By comparing the optimal total expected sales against the actual total sales of the winter season, the total expected sales increased by 7% by optimally reallocating sales campaign days with no additional cost. By implementing the same mathematical model on the fall season, the results similarly indicated an increase in optimal expected sales by 4.69% with no additional cost. This implies that, by mere reorganization of sales campaign days freely rather than in segments of consecutive days, the total expected sales would increase with no additional cost.

Furthermore, we compare the effect of the optimal allocation of sales campaign days only against that of reallocating both sales campaign days and the campaign budget on expected profit. The results of the winter season indicated that, optimal expected profit increased by 7.84% from actual profit by optimally reallocating sales campaign days only. However, by optimally reallocating both sales campaign days and the campaign budget, optimal expected profit increased by 9.95% from actual profit. This implies that, the optimal campaign budget is responsible for only (9.95 – 7.84 = 2.26%) of the improvement in optimal expected profit. The numerical example of the fall season provided similar evidence. By optimally reallocating both sales campaign days and the campaign budget, optimal expected profit increased by 6.58% from actual profit. Comparing this result with the 4.79% increase rate, achieved by optimally reallocating sales campaign days only, the optimal campaign budget would be responsible for only (6.58 – 4.79 = 1.79%).

In both numerical examples, the optimal campaign budget was responsible for about 2% only of the improvement in optimal expected profit, while the optimal allocation of sales campaign days only was responsible for about double this amount in the fall season (4.79%) and more than triple this amount in the winter season (7.84 %). This result is consistent with that reported by Fischer et al., (2011), they state that, profit improvement from better allocation across products or regions is much higher than that from improving the overall budget. Similarly, one can state that, optimal allocation of sales campaign days achieves better improvement in optimal expected profit than that achieved by only improving the overall budget.

One of the main assumptions of the procedure for estimating expected total sales per day under the influence of an enhanced campaign budget is that, non-sales-campaign-days switching to sales campaign days under the effect of the budget increase would experience better improvement in expected total sales than non-switching sales campaign days. In respect to the winter season, only 2 days switched from non-sales-campaign-days to campaign days corresponding to $\hat{r}_{(0,1)\to(1,1)}(j)$, and accumulating 6.5% of ¥150.55 million of expected total sales specific to sales campaign days only. On the other hand, 17 days switched from $d^*(j) = 0$ to $d^*_{\Delta_B}(j) = 1$ in the fall season, corresponding to $\hat{r}_{(0,0)\to(1,1)}(j)$ and accumulating 90.1 % of ¥ 94.63 million of total expected sales specific to sales campaign days. This can be interpreted in the following manner; regardless of the number of sales campaign days switching from non-sales-campaign-days, the impact of the optimal reallocation of sales campaign days would overwhelm that of the optimal campaign budget.

The proposed approach would be quite useful for the management of an SC, where different stores in one place can organize common sales campaigns to share the advantages of implementing a marketing flexibility-based strategy. To effectively allocate resources, optimal allocation of sales campaign days is recommended to maximize returns. For further improvement, the campaign budget could be optimally allocated along with the sales campaign days. These recommendations challenge the common business practices of improving the overall budget of a sales campaign to further boost its effectiveness. For this approach to be implemented efficiently, it is recommended for the management of the SC to share the timetable of scheduled campaign days with its customers. With the advent of smart phones, reaching out to customers has never been easier. Visitors of the SC can be kept informed through traditional channels of communication and advertising as well.

Future Work

This approach may be applicable in the telecommunication market in India, where Organized Trade (OT) and General Trade (GT) are cohabited together. The impact of the sales campaigns on the mobile device market might be analyzed from a similar perspective. To support this notion, a recent paper by Vidyarthi and Singh (2011), describing the relatively new Indian telecommunication market, gives insight into new directions of research that could be pursued in the future.

Limitations

One of the limitations of this study may be that, the available data was limited to two seasons only. One may expand the implementation of the proposed approach on a more extensive data from different industries. Furthermore, the size of the datasets, 88 and 90 days for winter and fall, respectively, may also be perceived as a limitation. However, one finds this to be inevitable in the context of SC retail business. Because over 50% of the SC stores are fashion stores, such stores highly rely on sales campaigns to lower their inventory before every new season in order to be able to introduce new lines of fashion on seasonal bases. Due to this practice, seasonal analysis deemed to be necessary, as in Pauwels (2007), Poel et al. (2004), and Arnold et al. (1983). Another limitation may be that, the effect of the campaign budget increase on expected total sales was estimated based on a previous dataset that was not treated by such effect.

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