No. 1
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微積分第9回講義ノート

多変数の微分

$f:\mathbb{R}^n\to\mathbb{R}^m$ $\quad x, a\in\mathbb{R}^n$

$f(x+ad)=f(x)+?\,d$ ∃!(唯一存在する)

$? \in \mathbb{R}^m$

$f'(x)(a)$

$t\in\mathbb{R}\mapsto f(x+at)\in\mathbb{R}^m$ 方向微分

$t\in\mathbb{R}\mapsto x+at\in\mathbb{R}^n$ 合成関数

$y\in\mathbb{R}^n\mapsto f(y)\in\mathbb{R}^m$

$\boxed{(g\circ f)'(x)=g'(f(x))f'(x)}$ $\quad f'(x):\mathbb{R}\to\mathbb{R}$

$f'(x):\mathbb{R}^n\to\mathbb{R}^m$ の線型写像 $\quad$ 線型

$f:\mathbb{R}^n\to\mathbb{R}^m,\ \mathbb{R}^m\to\mathbb{R}^l$ $\quad$ 比例

$g'(f(x))\circ f'(x)$

$t=0$ で微分

$f'(x)(a)$ $\quad a=e_1=\begin{pmatrix}1\\0\\\vdots\\0\end{pmatrix}$ のとき

$f'(x)(e_1)$ $\quad \dfrac{\partial f}{\partial x_1}$

$t\in\mathbb{R}\mapsto f(x+e_1 t)$

$$f\left(\begin{pmatrix}x_1\\x_2\\\vdots\\x_n\end{pmatrix}+\begin{pmatrix}t\\0\\\vdots\\0\end{pmatrix}\right)=f\begin{pmatrix}x_1+t\\x_2\\\vdots\\x_n\end{pmatrix}$$

formal $\quad f:\mathbb{R}^n\to\mathbb{R}^m$

$f'(x)(a)$ $\quad d\in D=\{d\in\mathbb{R}\mid d^2=0\}$

$f'(x):\mathbb{R}^n\to\mathbb{R}^m$ $\quad \alpha d\in D$

線型 $\quad \because (\alpha d)^2=\alpha^2 d^2=0$

$f'(x)(a_1+a_2)=f'(x)(a_1)+f'(x)(a_2)$

$f'(x)(\alpha a)=\alpha f'(x)(a)$

論理記号

∃ Existence (存在)

∀ All (全て)

∫ Summation (総和)

$\underline{f(x+(\alpha a)d)} = f(x)+f'(x)(\alpha a)d$

$f(x+\overset{\shortparallel}{a}(\underset{\overset{\cap}{D}}{\alpha d})) = f(x)+f'(x)(a)\alpha d$

$f'(x)$

$a = \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix} = a_1 e_1 + \cdots + a_n e_n$

$f'(x)(a) = f'(x)(a_1 e_1 + \cdots + a_n e_n)$

$= a_1 f'(x)(e_1) + a_2 f'(x)(e_2) + \cdots + a_n f'(x)(e_n)$

$= a_1 \dfrac{\partial f}{\partial x_1}(x) + a_2 \dfrac{\partial f}{\partial x_2} + \cdots + a_n \dfrac{\partial f}{\partial x_n}(x)$

合成関数の微分

$f: \mathbb{R}^n \to \mathbb{R}^m,\ g: \mathbb{R}^m \to \mathbb{R}^l$

$x, a \in \mathbb{R}^n$

$g\circ f(x+ad) = g\circ f(x) + (g\circ f)'(a)d$

$g(f(\overset{\shortparallel}{x}+ad)) = g(\underline{f(x)} + \underline{f'(x)(a)}d)$

$= g(f(x)) + g'(f(x))f'(x)(a)d \qquad \overset{\cap}{\mathbb{R}^m}$

$(g\circ f)'(x) = g'(f(x)) \circ f'(x)$

<report 問題>

$\frac{\partial z}{\partial u}, \frac{\partial z}{\partial v}$ を求めよ。

(1) $z = x^2 y^3$	(2) $z = e^{\frac{y}{x}}$
$x = uv$	$x = u\cos v$
$y = u^2 - v^2$	$y = u\sin v$

極値

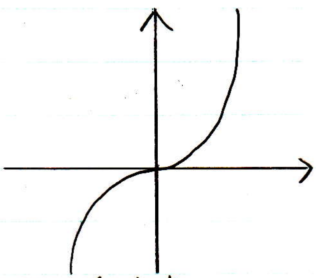
1変数

$\underline{f'(x) = 0}$

$f(x) = x^3$

$f'(x) = 3x^2 = 0$ 必要条件

$x = 0$ 十分条件



増減表

2次元

$\boxed{\begin{array}{ll} f''(x) > 0 & \text{極小値} \\ f''(x) < 0 & \text{極大値} \end{array}}$