No. 1
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微積分第6回講義ノート

線形代数		
(linear algebra)	$f: \mathbb{R} \to \mathbb{R} \quad x, y, \alpha \in \mathbb{R}$	$f(x) = ⓐx$
真っ直ぐ	(1) $f(x+y) = f(x) + f(y)$	比例定数
原点	(2) $f(\alpha x) = \alpha f(x)$	
小学校		比例関数　比例定数
比例関数	$f(x) = f(x1) = x\underline{f(1)}$	$f \longleftrightarrow a$
1変数の線形関数	$\qquad \overset{\shortparallel}{a}$	

多変数の微分

多変数の線形代数

$f: \mathbb{R}^n \to \mathbb{R}^m$ (nとmは自然数)

線形関数 (1) $f(x+y) = f(x) + f(y)$　$(x, y \in \mathbb{R}^n, \alpha \in \mathbb{R})$

(写像) (2) $f(\alpha x) = \alpha f(x)$

$$e_1 = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \quad e_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \quad \cdots \quad e_n = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}$$

$$x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = x_1 e_1 + \cdots + x_n e_n$$

$$\begin{aligned} f(x) &= f(x_1 e_1 + \cdots + x_n e_n) \\ &= f(x_1 e_1) + \cdots + f(x_n e_n) \\ &= x_1 f(e_1) + \cdots + x_n f(e_n) \end{aligned}$$

$$\mathbb{R}^m \ni f(e_1) = a_1 = \begin{pmatrix} a_{11} \\ \vdots \\ a_{m1} \end{pmatrix}$$

$$f(e_2) = a_2 = \begin{pmatrix} a_{12} \\ \vdots \\ a_{m2} \end{pmatrix}$$

$$\vdots$$

$$f(e_n) = a_n = \begin{pmatrix} a_{1n} \\ \vdots \\ a_{mn} \end{pmatrix}$$

$$\overset{m \times n}{[a_1, \cdots, a_n]} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{array}{c|ccc} & 1列目 & 2列目 & n列目 \\ \hline 1行目 & a_{11} & a_{12} \cdots & a_{1n} \\ 2行目 & a_{21} & a_{22} \cdots & a_{2n} \\ & \vdots & & \vdots \\ m行目 & a_{m1} & a_{m2} \cdots & a_{mn} \end{array} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$$

$m \times n$の行列 (matrix)
行: row　列: column

$$f(x) = x_1 a_1 + \cdots + x_n a_n$$

1×1なら[a]

$\mathbb{R}^n \to \mathbb{R}^m$の線形関数　$m \times n$の行列

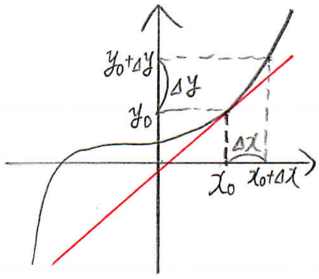
$$f \longleftrightarrow A$$

$\underline{m=n=1}$ 比例関数

$f:\mathbb{R}\to\mathbb{R}$

$f'(x_0)$ 微分係数

線形関数



$$\Delta y = f(x_0+\Delta x) - f(x_0)$$

$f:\mathbb{R}^n \to \mathbb{R}^m$ $m\times n$の行列

$x_0 \in \mathbb{R}^n$ で微分

$f'(x_0) = \mathbb{R}^n \to \mathbb{R}^m$ 線形関数

$$f'(x_0) = m\left\{\underbrace{\begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & & \vdots \\ \frac{\partial f_m}{\partial x_1} & \frac{\partial f_m}{\partial x_2} & \cdots & \frac{\partial f_m}{\partial x_n} \end{pmatrix}}_{n}\right.$$

$\boxed{\text{偏微分}}$

$$f(x) = \begin{pmatrix} f_1(x) \\ \vdots \\ f_m(x) \end{pmatrix}$$

$$f_i : \mathbb{R}^n \to \mathbb{R} \ (1 \le i \le m)$$

$n=2,\ m=1$

$z = f(x,y)$

$$\left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]\begin{pmatrix} x \\ y \end{pmatrix}$$
$$= \frac{\partial f}{\partial x}x + \frac{\partial f}{\partial y}y$$

合成関数の微分

$f:\mathbb{R}\to\mathbb{R}$ $f'(x)$ 比例関数

$g:\mathbb{R}\to\mathbb{R}$ $g'(f(x))$ 比例関数

$(g\circ f)'(x)=g'(f(x))\cdot f'(x)$

$f:\mathbb{R}^n\to\mathbb{R}^m$

$g:\mathbb{R}^m\to\mathbb{R}^l$

合成関数

$$(g\circ f)'(x)=\underbrace{g'(f(x))}_{B}\circ\underbrace{f'(x)}_{A}$$

$f:\mathbb{R}^2\to\mathbb{R}^2$, $g:\mathbb{R}^2\to\mathbb{R}$

$g\circ f$ $z=g(y_1,y_2)$

$$f(x_1,x_2)=\begin{pmatrix} f_1(x_1,x_2) \\ f_2(x_1,x_2)\end{pmatrix}$$

$$f'(x_1,x_2)=\begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2}\end{pmatrix}$$

$$g'(y_1,y_2)=\begin{pmatrix} \frac{\partial g}{\partial y_1} & \frac{\partial g}{\partial y_2}\end{pmatrix}$$

$$(g\circ f)'(x_1,x_2)=\begin{pmatrix} \frac{\partial g}{\partial y_1} & \frac{\partial g}{\partial y_2}\end{pmatrix}\begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2}\end{pmatrix}=\begin{pmatrix} \frac{\partial g}{\partial y_1}\frac{\partial f_1}{\partial x_1}+\frac{\partial g}{\partial y_2}\frac{\partial f_2}{\partial x_1} & \frac{\partial g}{\partial y_1}\frac{\partial f_1}{\partial x_2}+\frac{\partial g}{\partial y_2}\frac{\partial f_2}{\partial x_2}\end{pmatrix}$$

$g\circ f$

$$\begin{pmatrix} \frac{\partial(g\circ f)}{\partial x_1} & \frac{\partial(g\circ f)}{\partial x_2}\end{pmatrix}=\begin{pmatrix} \frac{\partial g}{\partial y_1}\frac{\partial f_1}{\partial x_1}+\frac{\partial g}{\partial y_2}\frac{\partial f_2}{\partial x_1} & \frac{\partial g}{\partial y_1}\frac{\partial f_1}{\partial x_2}+\frac{\partial g}{\partial y_2}\frac{\partial f_2}{\partial x_2}\end{pmatrix}$$

イコール

f, g がいろいろな場合が考えられる

ex) $f:\mathbb{R}^3\to\mathbb{R}^3$, $g:\mathbb{R}^3\to\mathbb{R}$

report

I $f(x,y)=\sin(x^2+5xy+y^2)$ の $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$ を求めなさい

II $\begin{pmatrix}1 & 5\\ 7 & 4\end{pmatrix}\begin{pmatrix}2 & 1\\ 8 & -2\end{pmatrix}$ を求めなさい (積)