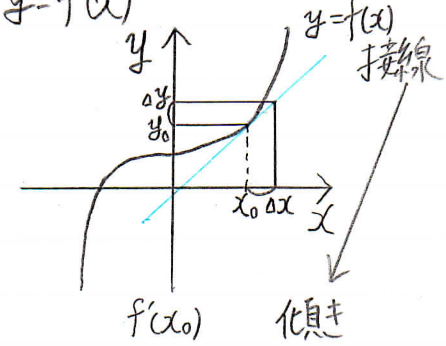
No. 1
Date 2015年5月11日(月)

微積分第4回講義ノート

多変数の微分

$y=f(x)$



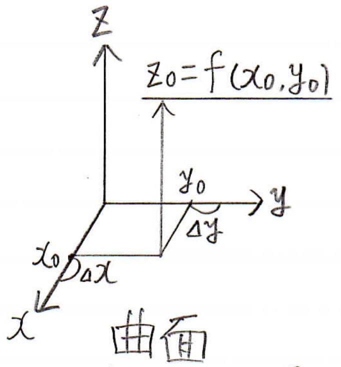
$\Delta y = y - y_0$

$\Delta x = x - x_0$

$\Delta y = f'(x_0)\Delta x$

曲がっているの嫌 ⇒ 真っ直ぐなものに置き換えましょう

$z=f(x,y)$



$\Delta x = x - x_0$

$\Delta y = y - y_0$

$\Delta z = z - z_0$

Pair

$$\left(\frac{\partial f}{\partial x}(x_0,y_0),\ \frac{\partial f}{\partial y}(x_0,y_0)\right)$$

接平面の方程式

$\Delta z = a\Delta x + b\Delta y$ (aとbは定数)

曲面

$y=y_0$ 平面 切る

$x=x_0$ 平面 切る

$\Delta y = 0$

$x \to f(x, y_0)$ 接線 $\boxed{a = \frac{\partial f}{\partial x}(x_0,y_0)}$

fのx方向の偏微分

$\Delta x = 0$

$y \to f(x_0, y)$ 接線 $\boxed{b = \frac{\partial f}{\partial y}(x_0,y_0)}$

fのy方向の偏微分

→ 他の変数を固定して1変数として微分する

3変数

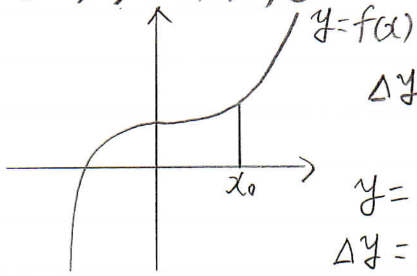
$u=f(x,y,z)$

グラフ　4次元

代数的に考える!

小学校

比例←最初に習う関数



$\Delta y=g(\Delta x)$

$y=\sin x$

$\Delta y=\sin(x_0+\Delta x)-\sin x_0$

$a=f'(x_0)$

$f(\Delta x)=a\Delta x$

$\Delta y=a\Delta x$

$f:\Delta x\mapsto a\Delta x$

$(\Delta x,\Delta y)\mapsto a\Delta x+b\Delta y$

$\Delta x=x-x_0$

$u=f(x,y,z)$

$u=f(x,y_0,z_0)$ $\begin{cases}\Delta y=0\\ \Delta z=0\end{cases}$

$(\Delta x,\Delta y,\Delta z)\mapsto a\Delta x+b\Delta y+c\Delta z$

aとbとcを決める

$x\mapsto f(x,y_0,z_0)$

$$a=\frac{\partial f}{\partial x}(x_0,y_0,z_0)$$

$$b=\frac{\partial f}{\partial y}(x_0,y_0,z_0)$$

$$c=\frac{\partial f}{\partial z}(x_0,y_0,z_0)$$

微積分
線型代数 (linear algebra)
(形)

$y = f(x)$, $x, y, \alpha \in \mathbb{R}$

$$\left.\begin{cases} f(x+y) = f(x) + f(y) \\ f(\alpha x) = \alpha f(x) \end{cases}\right\}$$ 線型関数

※ $f(x) = x^2$ は成り立たない
$f(\alpha x) = \alpha^2 x^2 = \alpha^2 f(x)$

比例関数 $f(x) = ax$
$\left(f(x) = f(x \cdot 1) = x \underset{\overset{\shortparallel}{a}}{f(1)} \right)$

ベクトルで考える ($\mathbb{x} = (x, y)$)
$\mathbb{x}, \mathbb{y} \in \mathbb{R}^2$, $\alpha \in \mathbb{R}$, $f : \mathbb{R}^2 \to \mathbb{R}$

$$\left.\boxed{\begin{array}{l} f(\mathbb{x}+\mathbb{y}) = f(\mathbb{x}) + f(\mathbb{y}) \\ f(\alpha \mathbb{x}) = \alpha f(\mathbb{x}) \end{array}}\right\}$$ 線型関数

$\mathbb{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = x_1 \mathbb{e}_1 + x_2 \mathbb{e}_2 \qquad \mathbb{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \mathbb{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$\begin{aligned} f(\mathbb{x}) &= f(x_1 \mathbb{e}_1 + x_2 \mathbb{e}_2) = f(x_1 \mathbb{e}_1) + f(x_2 \mathbb{e}_2) \\ &= x_1 \underbrace{f(\mathbb{e}_1)}_{a} + x_2 \underbrace{f(\mathbb{e}_2)}_{b} = a x_1 + b x_2 \end{aligned}$$

$\mathbb{R}^n = \mathbb{R} \times \cdots \times \mathbb{R}$
$f : \mathbb{R}^n \to \mathbb{R}$

$$\left.\begin{array}{l} f(\mathbb{x}+\mathbb{y}) = f(\mathbb{x}) + f(\mathbb{y}) \\ f(\alpha \mathbb{x}) = \alpha f(\mathbb{x}) \end{array}\right\}$$ 線型関数

$\mathbb{e}_1 = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \mathbb{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \mathbb{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \cdots, \mathbb{e}_n = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}$

$\mathbb{x} = x_1 \mathbb{e}_1 + \cdots + x_n \mathbb{e}_n$

$$\begin{aligned} f(\mathbb{x}) &= f(x_1 \mathbb{e}_1 + \cdots + x_n \mathbb{e}_n) \\ &= f(x_1 \mathbb{e}_1) + \cdots + f(x_n \mathbb{e}_n) \\ &= x_1 \underbrace{f(\mathbb{e}_1)}_{a_1} + \cdots + x_n \underbrace{f(\mathbb{e}_n)}_{a_n} \end{aligned}$$

$\mathbb{x} \mapsto a_1 x_1 + \cdots + a_n x_n$