微積分 第3回講義ノート

$d_1, d_2, d_3, d_4 \in D$

① $f(x+d_1) = f(x) + f'(x)\, d_1$

② $f(x+d_1+d_2) = \underbrace{f(x)}_{(1=\rho^2_0(d_1,d_2))} + f'(x)\underbrace{(d_1+d_2)}_{\rho^2_1(d_1,d_2)} + f''(x)\underbrace{d_1 d_2}_{\rho^2_2(d_1,d_2)}$

③ $f(x+d_1+d_2+d_3)$

$= \underbrace{f(x)}_{(1=\rho^3_0(d_1,d_2,d_3))} + f'(x)\underbrace{(d_1+d_2+d_3)}_{\rho^3_1(d_1,d_2,d_3)} + f''(x)\underbrace{(d_1d_2+d_1d_3+d_2d_3)}_{\rho^3_2(d_1,d_2,d_3)} + f'''(x)\underbrace{d_1d_2d_3}_{\rho^3_3(d_1,d_2,d_3)}$

④ $f(x+d_1+d_2+d_3+d_4) = f(x+d_1+d_2+d_3) + f'(x+d_1+d_2+d_3)d_4$

$= \boxed{f(x) + f'(x)(d_1+d_2+d_3) + f''(d_1d_2+d_1d_3+d_2d_3) + f'''(x)d_1d_2d_3}$

$+ \boxed{\{f'(x) + f''(x)(d_1+d_2+d_3) + f'''(d_1d_2+d_1d_3+d_2d_3) + f^{(4)}(x)\, d_1d_2d_3\}\, d_4}$

$= f(x) + f'(x)(d_1+d_2+d_3+d_4) + f''(x)(d_1d_2+d_1d_3+d_1d_4+d_2d_3+d_2d_4+d_3d_4)$

$+ f'''(x)(d_1d_2d_3+d_1d_2d_4+d_1d_3d_4+d_2d_3d_4) + f^{(4)}(x)\, d_1d_2d_3d_4$

基本対称式

$\rho^2_0(x_1,x_2) = 1$ 0次の基本対称式

$\rho^2_1(x_1,x_2) = x_1+x_2$ 1次 〃

$\rho^2_2(x_1,x_2) = x_1x_2$ 2次 〃

$\rho^3_0(x_1,x_2,x_3) = 1$ 0次

$\rho^3_1(x_1,x_2,x_3) = x_1+x_2+x_3$ 1次

$\rho^3_2(x_1,x_2,x_3) = x_1x_2+x_1x_3+x_2x_3$ 2次

$\rho^3_3(x_1,x_2,x_3) = x_1x_2x_3$ 3次

※ $\underbrace{x_1^2+x_2^2}_{\text{対称式}} = (x_1+x_2)^2 - 2x_1x_2$

補題 $\rho^{n+1}_{r+1}(x_1,\cdots,x_{n+1}) = \rho^n_{r+1}(x_1,\cdots,x_n) + x_{n+1}\rho^n_r(x_1,\cdots,x_n)$

組み合わせ論 $\boxed{{}_{n+1}C_{r+1} = {}_nC_{r+1} + {}_nC_r}$ ← 上手くやれば簡単にできる

∵ ${}_nC_{r+1} + {}_nC_r = \dfrac{n!}{(r+1)!(n-r-1)!} + \dfrac{n!}{r!+(n-r)!}$

$= \dfrac{n!\{(n-\cancel{r})+(\cancel{r}+1)\}}{(r+1)!\,(n-r)!}$

$= \dfrac{n!(n+1)}{(r+1)!\,(n-r)!}$

$= \dfrac{(n+1)!}{(r+1)!\,(n-r)!} = {}_{n+1}C_{r+1}$ □

ⓝ $d_1, \cdots, d_n \in D$

$$f(x+d_1+\cdots+d_n) = f(x) + f'(x)\rho_1^n(d_1,\cdots,d_n) + f''(x)\rho_2^n(d_1,\cdots,d_n) + \cdots + f^{(n)}(x)\rho_n^n(d_1,\cdots,d_n)$$

$$= f(x) + f'(x)(d_1+\cdots+d_n) + \frac{f''(x)}{2!}(d_1+\cdots+d_n)^2 + \cdots + \frac{f^{(n)}(x)}{n!}(d_1+\cdots+d_n)^n$$

$$\rho_1^n = d_1+\cdots+d_n$$

$$\rho_2^n = \frac{(d_1+\cdots+d_n)^2}{2!}$$

$$\rho_3^n = \frac{(d_1+\cdots+d_n)^3}{3!}$$

$\vdots$

$$\rho_n^n = \frac{(d_1+\cdots+d_n)^n}{n!}$$

$$f(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$$

Taylor展開

任意の関数は
無限小のLevelでは
いつでも多項式に書ける

巾零無限小

高校数学Ⅱ　多項式(次数)
高校数学Ⅲ　三角関数，指数関数 ← 無限次の多項式

$$f(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n + a_nx^{n+1} + \cdots$$

$f(x) = e^x$　$f(0) = 1 = a_0$
$f'(x)$　$f'(0) = 1 = a_1$
$f''(x)$　$\frac{f''(0)}{2!} = a_2$
$\frac{f'''(0)}{3!} = a_3$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

$g(x) = \sin x$, $g'(x) = \cos x$, $g''(x) = -\sin x$, $g'''(x) = -\cos x$, $g^{(4)}(x) = \sin x$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} \cdots$$

$h(x) = \cos x$, $h'(x) = -\sin x$, $h''(x) = -\cos x$, $h'''(x) = \sin x$, $h^{(4)}(x) = \cos x$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} \cdots$$

微分方程式とお友達になりましょう!

$y = f(x) = e^x$ — 指数関数

$y' = y$ ← 微分方程式

法則

仮定:解は無限次の多項式に書ける

$f(x) = a_0 + a_1 x + a_2 x^2 + \cdots$

$f'(x) = a_1 + 2a_2 x + 3a_3 x^2 + \cdots$

$f' = f$ より $a_0 = a_1,\ a_1 = 2a_2,\ a_2 = 3a_3,\ \cdots,\ a_{n-1} = n a_n$

$$a_1 = a_0$$
$$a_2 = \frac{a_1}{2} = \frac{a_0}{2}$$
$$a_3 = \frac{a_2}{3} = \frac{a_0}{3!}$$
$$\vdots$$
$$a_n = \frac{a_0}{n!}$$

$$f(x) = a_0\left\{1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots\right\}$$
$$= c\,e^x \quad (c \in \mathbb{C})$$