2015年12月28日(月)

微積分第24回講義ノート

留数定理

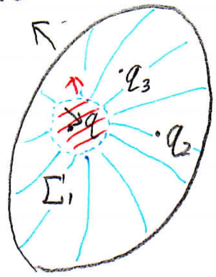
Coulombの法則 ⇒ Gaussの法則

電場(ベクトル場)

q 　$\mathrm{div}\,\mathbb{f}=0$

Gaussの発散定理　　　　重ね合せの原理

閉曲面Σで囲まれた領域Ω

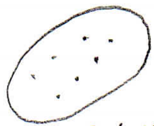


$\int_\Sigma \mathbb{f}\cdot d\mathbb{S} = \int_\Omega (\underbrace{\mathrm{div}\,\mathbb{f}}_{0})dV = 0$

面積分 $=0$　　体積分　$\Sigma\cup\Sigma_1$

$\int_{\Sigma\cup\Sigma_1} \mathbb{f}\cdot d\mathbb{S}=0 \rightarrow \int_\Sigma \mathbb{f}\cdot d\mathbb{S} = \int_{\Sigma_1} \mathbb{f}\cdot d\mathbb{S}$

$4\pi kq$

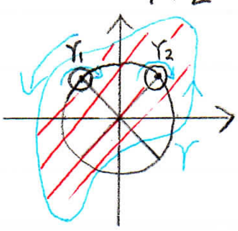


$\int_\Sigma f\cdot d\mathbb{S} = 4\pi k$ (内部にある電場の和)

$\int_\gamma \mathbb{f}\, d\mathbb{r}=0$

$\mathbb{f}$:解析的

$f(z) = \dfrac{z^2}{1+z^4}$　　$\int_\gamma f dz$



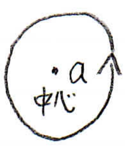
$\int_{\gamma\cup\gamma_1\cup\gamma_2} f dz = 0$

$\qquad\|$

$\int_\gamma f dz - \int_{\gamma_1} f dz - \int_{\gamma_2} f dz = 0$

$\int_\gamma f dz = \int_{\gamma_1} f dz + \int_{\gamma_2} f dz$

特異点　　Laurent展開　　級数



$f(z) = \sum_{-\infty}^{\infty} a_i (z-a)^i$

$\gamma: [a,b] \to \mathbb{C}$

基本定理　$F'=f$　原始関数

$\boxed{\int_\gamma f dz = F(\gamma(b)) - F(\gamma(a)) \quad (=0)}$

γが閉曲線

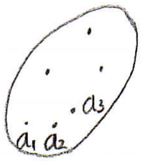
$\int_\gamma f dz = \sum_{-\infty}^{\infty} a_i \int_\gamma (z-a)^i dz$　　$z-a$,　$\dfrac{(z-a)^2}{2}$

$\int_\gamma (z-a)^{-1}dz = \int_\gamma \frac{dz}{z-a} = 2\pi i$

$\int_\gamma f(z)dz = 2\pi i\, a_{-1}$ Residue

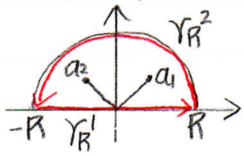
$\mathrm{Res}(f;a)$

fのaにおける留数



$\int_\gamma f(z)dz = 2\pi i(\mathrm{Res}(f;a_1)+\mathrm{Res}(f;a_2)+\mathrm{Res}(f;a_3))$

$\int_{-\infty}^{\infty}\frac{x^2}{1+x^4}dx$

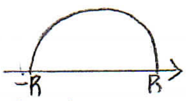


$\gamma_R = \gamma_R'$

$\int_{\gamma_R} f(z)dz = 2\pi i(\underline{\mathrm{Res}(f;a_1)}+\underline{\mathrm{Res}(f;a_2)}) = \frac{\pi}{\sqrt{2}}$ Rが十分大きい $R>1$

$\int_{-R}^{R}\frac{x^2}{1+x^4}dx = \underline{\int_{\gamma_R^1} f(z)dz} + \boxed{\int_{\gamma_R^2} f(z)dz}$

$\left|\int_a^b g(x)dx\right| \leq M(b-a)$



$|g(x)| \leq M$

$\left|\int_\gamma g(z)dz\right| \leq M$ (γの長さ)

$|g(z)| \leq M$ $\gamma_R^2: \theta\in[0,\pi] \to Re^{i\theta}$

$\left(\int_{\gamma_R^2} f(z)dz\right)$

$f(z) = \frac{z^2}{1+z^4}$

$R\to\infty$

$|f(Re^{i\theta})| = \left|\frac{R^2e^{2i\theta}}{1+R^4e^{4i\theta}}\right| = \left|\frac{R^2}{R^4}\frac{e^{2i\theta}}{\frac{1}{R^4}+e^{4i\theta}}\right| \pi R$ $\frac{1}{R}$

$\frac{R^2}{R^4}$

Report 次を示せ.

(1) $\int_0^\infty \frac{x^2dx}{x^4+x^2+1} = \frac{1}{2}\int_{-\infty}^{\infty}\frac{x^2dx}{x^4+x^2+1}$

(2) $\int_0^\infty \frac{dx}{(x^2+a^2)^2} = \frac{1}{2}\int_{-\infty}^{\infty}\frac{dx}{(x^2+a^2)^2}$

$(a>0)$ 重解

$g(z)$

$g'(-ai)$ α_1

$\mathrm{Res}(f;a)$ $w_1\, w_2$ $w_3\, w_4$ $\frac{z^2}{z^4+1} = \frac{z^2}{(z-w_1)(z-w_2)(z-w_3)(z-w_4)}$

$x^2+a^2 = (x+ai)(x-ai)$

$f(z) = \frac{1}{(z^2+a^2)^2} = \frac{1}{(z+ai)^2(z-ai)^2} = \mathrm{Res}(f;w_1)$ $g(z)$

$\mathrm{Res}(f;-ai)$ $\frac{g(z)=\sum_{i=0}^{\infty}\alpha_i(z+ai)^i}{(z+ai)^2}$

$g(z)$

$\alpha_0 = g(w_1)$

$\frac{\alpha_0}{z-w_1}$

$\frac{\sum_{i=0}\alpha_i(z-w_1)^i}{z-w}$