No. 1
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微積分第10回講義ノート

多変数の微分
合成関数の微分
2階の微分
$f: \mathbb{R}^2 \to \mathbb{R}$
$f(x,y)$

2重線型関数
$f'(x,y) = \mathbb{R}^2 \to \mathbb{R}$ の線型写像

$$\overset{1\times 2}{\left(\frac{\partial f}{\partial x}(x,y),\ \frac{\partial f}{\partial y}(x,y)\right)}$$

$f' : \mathbb{R}^2 \to \underbrace{L(\mathbb{R}^2;\mathbb{R})}_{\mathbb{R}^2 \to \mathbb{R}\text{の線型写像の全体}}$ ― 本質的には$\mathbb{R}^2$と同じ

※ 空間(数学用語):物の集まり

$f''(x,y)$ 1×2

$f''(x,y) : \overset{[x]}{\mathbb{R}^2} \to \underline{L(\mathbb{R}^2 \overset{y}{;} \mathbb{R})}$ の 線型写像

$f''(x,y)(x)(y) \in \mathbb{R}$

$L(\mathbb{R}^2; L(\mathbb{R}^2;\mathbb{R}))$
$= L(\mathbb{R}^2; \mathbb{R}^2; \mathbb{R})$
$\mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$ 2重線型写像

$$\left(p_{ij}\right)\left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}\right) \quad i=1,2 \quad j=1,2$$

$= p_{11}x_1y_1 + p_{12}x_1y_2 + p_{21}x_2y_1 + p_{22}x_2y_2$

$p_{11} = \frac{\partial}{\partial x}\frac{\partial f}{\partial x} = \frac{\partial^2 f}{\partial x^2}$ $\qquad p_{12} = \frac{\partial}{\partial y}\frac{\partial f}{\partial x} = \frac{\partial^2 f}{\partial y \partial x}$

$p_{22} = \frac{\partial}{\partial y}\frac{\partial f}{\partial y} = \frac{\partial^2 f}{\partial y^2}$ $\qquad p_{21} = \frac{\partial}{\partial x}\frac{\partial f}{\partial y} = \frac{\partial^2 f}{\partial x \partial y}$

$f(x,y) = x^3 + 2xy + y^3$

$\frac{\partial f}{\partial x} = 3x^2 + 2y \qquad \frac{\partial f}{\partial y} = 2x + 3y^2$

$$\boxed{\frac{\partial}{\partial y}\frac{\partial f}{\partial x} = 2} = \boxed{\frac{\partial}{\partial x}\frac{\partial f}{\partial y} = 2}$$

偏微分は順序によらない

$f: \mathbb{R}^2 \to \mathbb{R}$

$p_{12} = p_{21}$

$$\begin{array}{cc} p_{11} & p_{12} \\ \multicolumn{2}{c}{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}} \\ p_{21} & p_{22} \end{array}$$

2重線型写像は対称

$x \cdot y = y \cdot x$

$|x\,y| = -|y\,x|$

$f: \mathbb{R}^n \to \mathbb{R}^m$　$x, a \in \mathbb{R}^n$

$f'(x)(a)$　　　　　　　　　　$f': \mathbb{R}^n \to L(\mathbb{R}^n; \mathbb{R}^m)$

$f(x+ad_1) - f(x) = \underset{\in \mathbb{R}^m}{f'(x)(a)}\, d_1 \; (\forall d_1 \in D)$

$f'(x+bd_2) - f'(x) = f''(x)(b)\, d_2 \; (\forall d_2 \in D)$

$\boxed{f'(x+bd_2)(a)d_1} - \boxed{f'(x)(a)d_1} = f''(x)(b)(a)\, d_2 d_1$

$\boxed{f(x+bd_2+ad_1) - f(x+bd_2)}$ 　 $\boxed{f(x+ad_1) - f(x)}$

(左辺) $= \boxed{f(x+ad_1+bd_2) - f(x+ad_1)}\ \boxed{-f(x+bd_2)+f(x)}$

$f(x+ad_1+bd_2) - f(x+ad_1) = f'(x+ad_1)(b)\, d_2$

$f(x+bd_2) - f(x) = f'(x)(b) d_2$

$f'(x+ad_1)(b)d_2 - f'(x)(b)d_2 = \underline{\underline{f''(x)(a)(b)d_1d_2}}$

極値　$f: \mathbb{R} \to \mathbb{R}$　　　$f(x) = x^3$

$f(x,y)$　必要条件　　$f'(x) = 3x^2$

$f'(x) = 0$　　$x = 0$ 極値ではない

<u>増減表</u>　　〈十分条件〉

$f(x,y)$　　$f'(x) = 0$

$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = 0$　　$f''(x) > 0$ (極小値)

$f''(x) < 0$ (極大値)

$f''(x) = 0$ (極値なし)

$x \in \mathbb{R}^2$　任意の0でないベクトル$a \in \mathbb{R}^2$について

$f(x)$がxで極大値

$\Leftrightarrow t \in \mathbb{R} \mapsto f(\underline{x+at})$が$t=0$で極大値

$a = e$　$g'(a)$

$g''(a)$