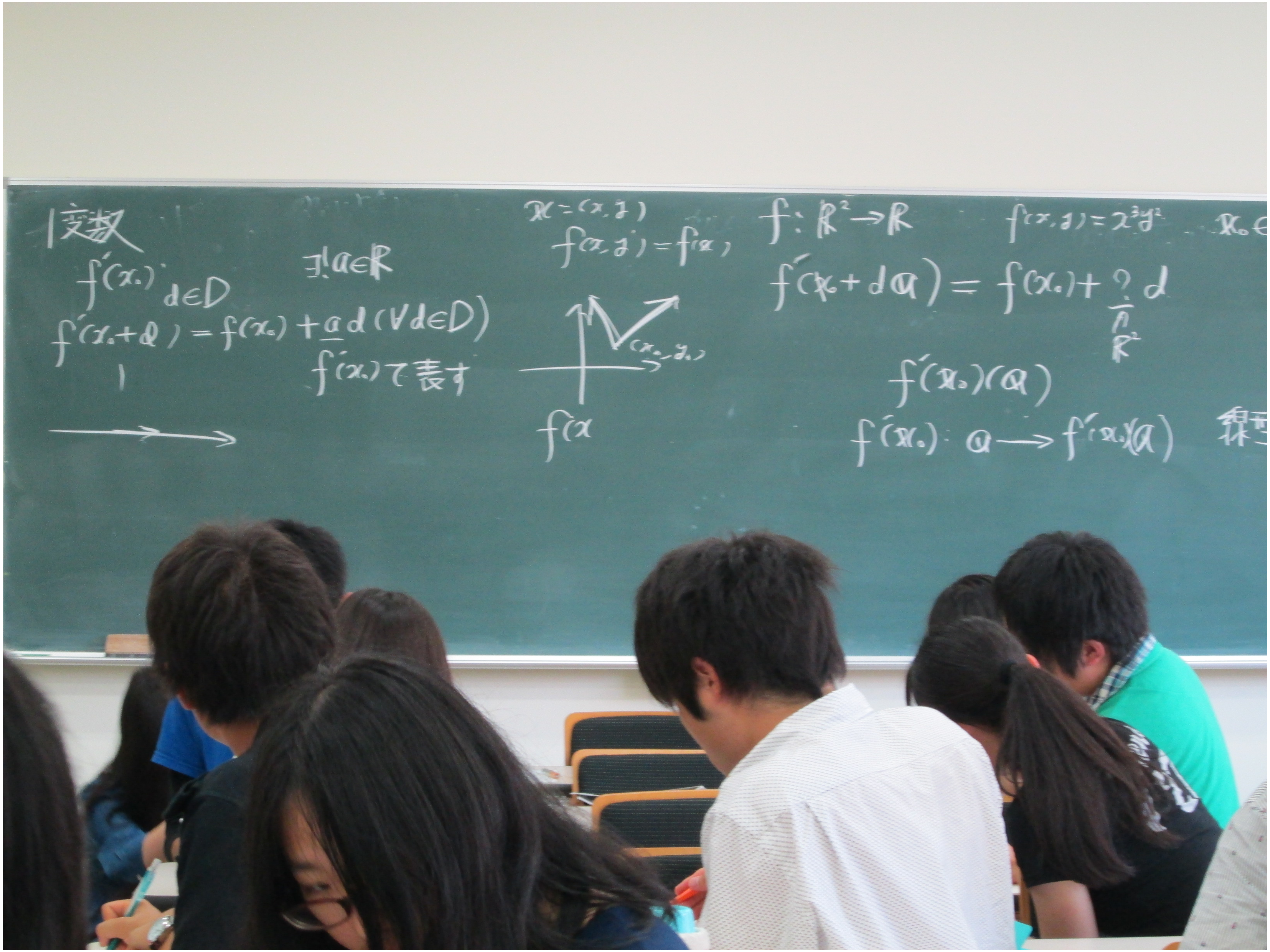
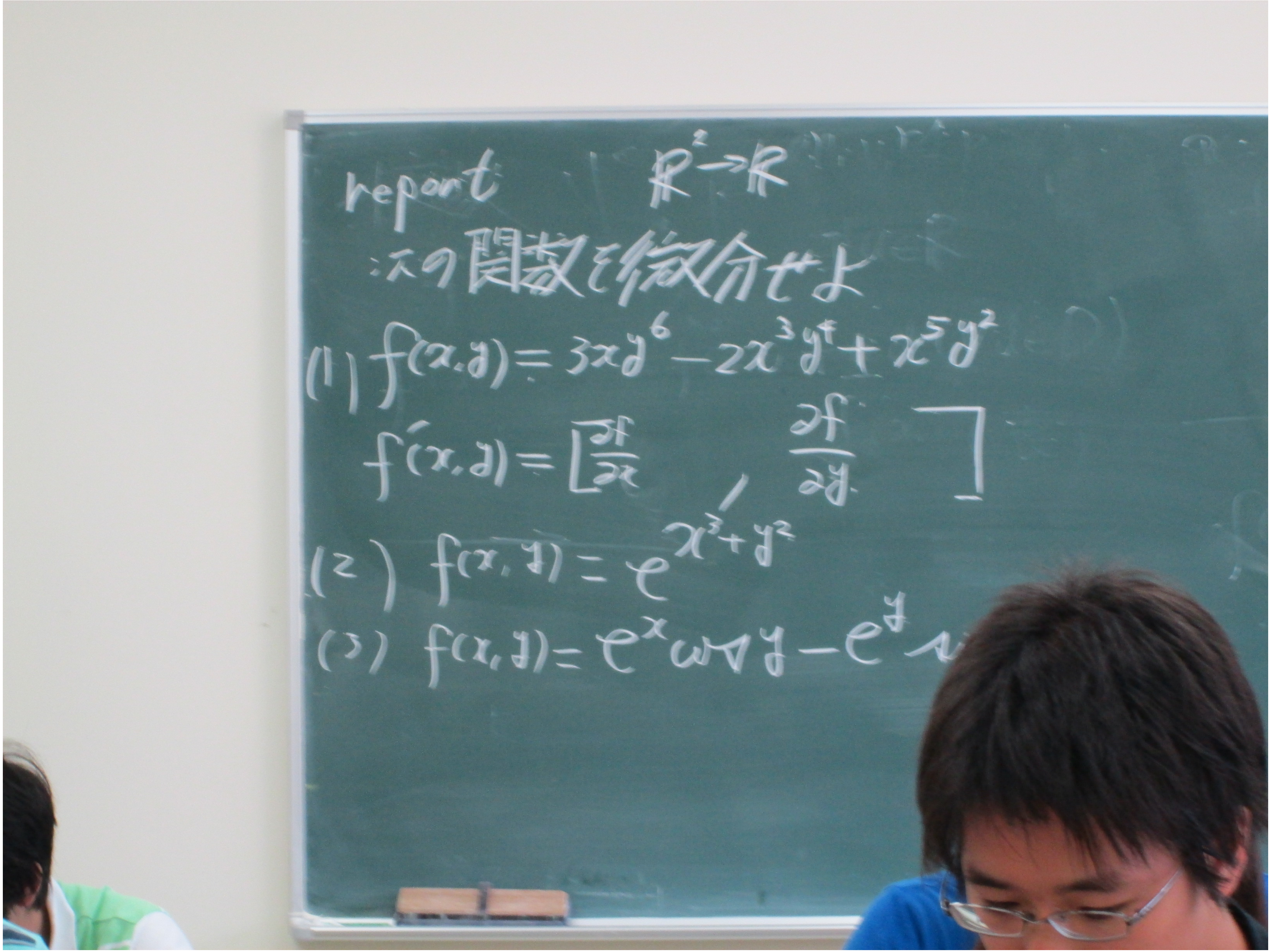
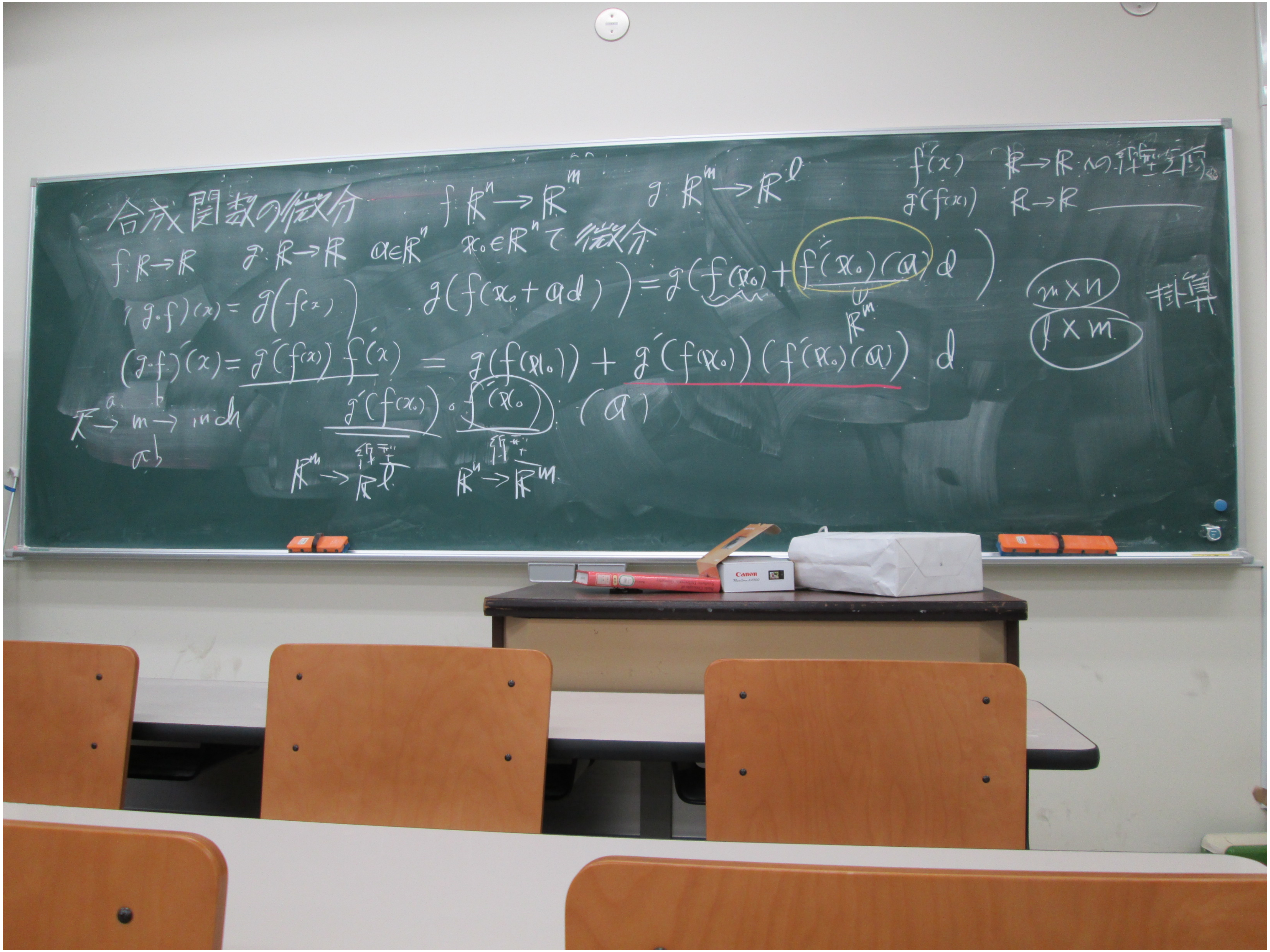
微分
変数
f'(x₀)
点x₀でのfの微分係数
y = x²
x₀ = 3
f'(x₀) = 6
y = f(x)
Δy = g(Δx)
Δy = f'(x₀)Δx
Δx ⟼ Δy
f'(3)(a) = 6a
a ⟼ 6a
f'(x₀)
R → R 線型関数
比例関係
f(x, y)
R² → R
(a₁, a₂)
線型関数
写像
f'(x, y)(a)
L(R;R)
= R
RからRへの線型関数の全体
空間
真っすぐ
f'(x, y): R² → Rへの線型関数
1×2の行列
Canon

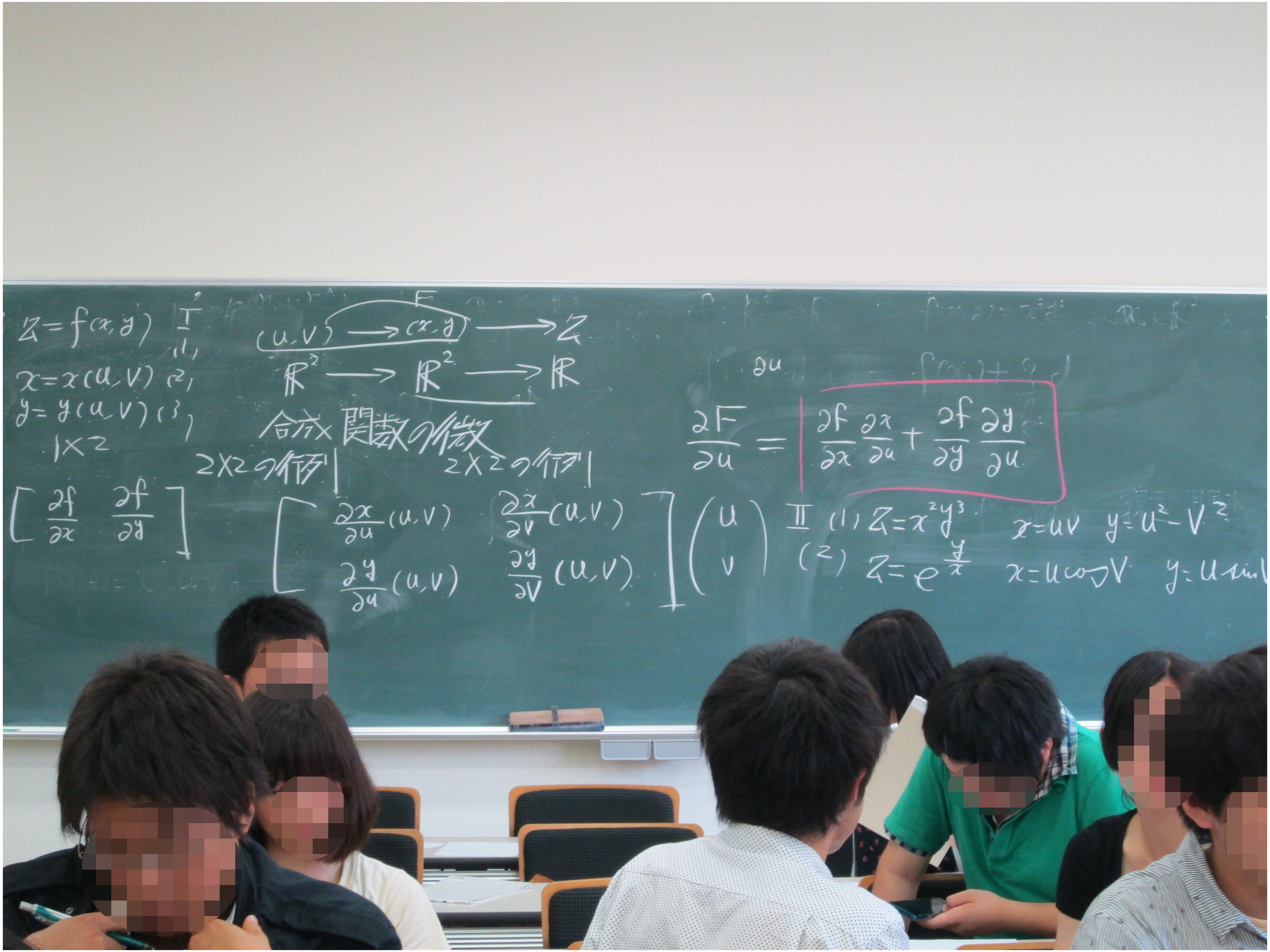

変数
∃!a∈R
f'(x₀)で表す
f: R² → R
f(x, y) = x³y²


f(x,y)
$\mathbb{R}^2 \to \mathbb{R}$
(x_0, y_0)で微分
$f'(x_0, y_0) = [0, 0]$
$a = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = a_1 e_1 + a_2 e_2$
$f'(x_0, y_0)(a)$
$= f'(x_0, y_0)(a_1 e_1 + a_2 e_2)$
$= a_1 f'(x_0, y_0)(e_1) + a_2 f'(x_0, y_0)(e_2)$
$f\left(\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} + d e_1\right) = f\begin{pmatrix} x_0 + d \\ y_0 \end{pmatrix}$
$\forall d \in D$
Canon


report
R^2 → R
次の関数を微分せよ
(1) f(x,y) = 3xy^6 − 2x^3y^4 + x^5y^2
f'(x,y) = [∂f/∂x , ∂f/∂y]
(2) f(x,y) = e^(x^3+y^2)
(3) f(x,y) = e^x cos y − e^y



合成関数の微分
f: Rⁿ → Rᵐ
g: Rᵐ → Rˡ
f: R→R
g: R→R
a∈Rⁿ
x₀∈Rⁿ で微分
(g∘f)(x) = g(f(x))
g(f(x₀+ad)) = g(f(x₀) + f'(x₀)(a) d)
(g∘f)'(x) = g'(f(x)) f'(x) = g(f(x₀)) + g'(f(x₀))(f'(x₀)(a)) d
g'(f(x₀)) ∘ f'(x₀) (a)
Rᵐ → Rˡ
Rⁿ → Rᵐ
m×n
l×m
掛算
f'(x)
R→R
g'(f(x))
R→R


z = f(x, y)
x = x(u, v)
y = y(u, v)
(u, v) → (x, y) → z
R² → R² → R
合成関数の微
1×2
2×2の行列
2×2の行列
∂F/∂u = ∂f/∂x ∂x/∂u + ∂f/∂y ∂y/∂u
Ⅱ (1) z = x²y³ x = uv y = u² − v²
(2) z = e^(y/x) x = u cos v

No.

月 3限 微積分

DATE

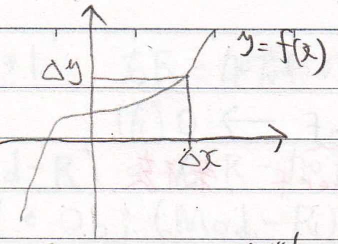
昔　生物

博物学 ⟶ 遺伝子工学

メンデル　　トリポジス

微分

1変数



$f'(x_0)$

$f'(x) : \mathbb{R} \to \mathbb{R}$ 線型関数

$\triangle y = g(\triangle x)$

$\triangle y = f'(x_0)\triangle x$

$\triangle x \mapsto \triangle y$

点xでのfの微分係数　　$f'(x)(a) = 6a$

$y = x^2 \quad x_0 = 3 \quad f'(x_0) = 6 \qquad a \mapsto 6a$

$L(\mathbb{R},\mathbb{R}) = \mathbb{R}$　RからRへの線型関数の全体

2変数

$f'(x,y) : \mathbb{R}^2 \longrightarrow \mathbb{R}$ への線型関数　1×2の行列　[○ ○]

1変数

$f'(x_0) \qquad d \in D \qquad \exists! a \in \mathbb{R}$

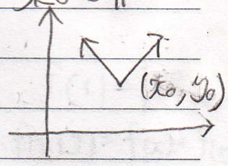
$f'(x_0+d) = f(x_0) + a\,d \quad (\forall d \in D)$

$f'(x_0)$で表す

2変数

$x = (x,y) \qquad\qquad f : \mathbb{R}^2 \to \mathbb{R} \qquad f(x,y) = x^3y^2 \qquad x_0 \in \mathbb{R}^2$

$f(x,y) = f(x) \qquad\qquad f'(x_0 + d\,a) = f(x_0) + \underset{\mathbb{R}^2}{?}\,d$



この?に　$f'(x_0)(a)$

$f'(x_0)\cdot a \to f'(x)(a)$　　　　線型関数

$f(x,y) \quad \mathbb{R}^2 \to \mathbb{R}$

(x_0, y_0)で微分　　$a = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = a_1 e_1 + a_2 e_2$

$f'(x_0, y_0) = [\,○\,,\,○\,]$

$f'(x_0,y_0)(a) = f'(x_0,y_0)(a_1e_1 + a_2e_2)$

$= a_1 f'(x_0,y_0)(e_1) + a_2 f'(x_0,y_0)(e_2)$

$= [f'(x_0,y_0)(e_1),\ f'(x_0,y_0)(e_2)]\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$

$f\left(\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} + d\,e_1\right) = f\begin{pmatrix} x_0+d \\ y_0 \end{pmatrix} = f\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} + f'(x_0,y_0)(e_1)\,d$

$\frac{\partial f}{\partial x}(x_0,y_0)$　fのx方向の偏微分　$\forall d \in D$　　$\frac{\partial f}{\partial x}(x_0,y_0), \frac{\partial f}{\partial y}(x_0,y_0)$

月 3限

No.

DATE

report 次の関数を微分せよ

(1) $f(x,y) = 3xy^6 - 2x^3y^4 + x^5y^2$

$f'(x,y) = [\ \circ\ ,\ \circ\ ]$

$[3y^6 - 6x^2y^4 + 5x^4y^2,\ 18xy^5 - 8x^3y^3 + 2x^5y]$

(2) $f(x,y) = e^{x^3+y^2}$

$= [\quad , \quad]$

(3) $f(x,y) = e^x \cos y - e^y \sin x = [\quad , \quad]$

合成関数の微分

$(g \circ f)(x) = g(f(x))$ $\quad (g \circ f)'(x) = g'(f(x)) \cdot f'(x)$

$f: \mathbb{R}^n \to \mathbb{R}^m$ $\quad g: \mathbb{R}^m \to \mathbb{R}^l$ $\quad a \in \mathbb{R}^n,\ x_0 \in \mathbb{R}^n$ で微分

$g(f(x_0 + ad)) = g(f(x_0) + f'(x_0)(a)d) \rightarrow \in \mathbb{R}^m$

$= g(f(x_0)) + g'(f(x_0))(f'(x_0)(a))d$

$\underset{\mathbb{R}^n \to \mathbb{R}^l}{g'(f(x_0))} \cdot \underset{\mathbb{R}^n \to \mathbb{R}^m}{f'(x_0)(a)}$

$f'(x)$ $\mathbb{R} \to \mathbb{R}$ への線型写像

$g'(f(x))$ $\mathbb{R} \to \mathbb{R}$ ‖

mm $\xrightarrow{10}$ cm $\xrightarrow{100}$ m (1000)

$z = f(x,y)$ $\quad (u,v) \xrightarrow{F} (x,y) \to z$

$x = x(u,v)$ $\quad \mathbb{R}^2 \to \mathbb{R}^2 \to \mathbb{R}$ $\quad \mathbb{R}^2 \to \mathbb{R}^2$ 2x2の行列

$y = y(u,v)$ 合成関数の微分 $\begin{bmatrix} \frac{\partial x}{\partial u}(u,v) & \frac{\partial x}{\partial v}(u,v) \\ \frac{\partial y}{\partial u}(u,v) & \frac{\partial y}{\partial v}(u,v) \end{bmatrix} \begin{pmatrix} u \\ v \end{pmatrix}$

$\mathbb{R}^2 \to \mathbb{R}$ 1x2の行列 $\left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$

$\frac{\partial F}{\partial u} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial u}$ $\quad \frac{\partial F}{\partial v} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial v}$

レポートII

(1) $z = x^2y^3$ $\quad x = uv,\ y = u^2 - v^2$

(2) $z = e^{\frac{y}{x}}$ $\quad x = u\cos v$ $\quad y = u \sin v$

No.

DATE . 6 . 17　外書　月6限

Stokesの定理

1次の微分形式

$w \xrightarrow{微分} dw$　2次の微分形式

・閉曲線 γ を縁とする面 Σ

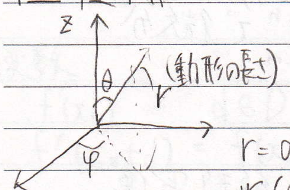
$$\int_\gamma w = \int_\Sigma dw$$

$w \leftrightarrow \mathbb{f}$ ベクトル場

$$\int_\gamma \mathbb{f}\cdot d\mathbb{r} = \int_\Sigma (\mathrm{rot}\,\mathbb{f})\,dS$$

面積分の計算

極座標　球面の表面積



$0 \le r < \infty$

$0 \le \theta \le \pi$

$0 \le \varphi \le 2\pi$

$r = a$ (>0という定数)

$\mathbb{r}(\theta, \varphi)$

$$\int_0^{2\pi}\int_0^{\pi}\left|\frac{\partial \mathbb{r}}{\partial\theta}\times\frac{\partial \mathbb{r}}{\partial\varphi}\right| d\theta\, d\varphi$$

$x = a\sin\theta\cos\varphi$

$y = a\sin\theta\sin\varphi$　$z = a\cos\theta$

$\frac{\partial \mathbb{r}}{\partial\theta} = (a\cos\theta\cos\varphi,\ a\cos\theta\sin\varphi,\ -a\sin\theta)$

$\frac{\partial \mathbb{r}}{\partial\varphi} = (-a\sin\theta\sin\varphi,\ a\sin\theta\cos\varphi,\ 0)$

$\frac{\partial \mathbb{r}}{\partial\theta}\times\frac{\partial \mathbb{r}}{\partial\varphi} = (a^2\sin^2\theta\cos\varphi,\ a^2\sin^2\theta\sin\varphi,\ a^2\sin\theta\cos\theta)$

$|\ | = a^2\sqrt{\sin^4\theta\cos^2\varphi + \sin^4\theta\sin^2\varphi + \sin^2\theta\cos^2\theta}$

$\cdots = 4\pi a^2$

Gaussの発散定理

2次の微分形式

$w \xrightarrow{微分} dw$　3次の微分形式

閉曲面 Σ に囲まれる領域 Ω

$$\int_\Sigma w = \int_\Omega dw$$

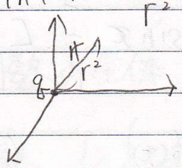
静電気学

q_1　q_2　異(同)

クーロンの法則　q_1, q_2 に比例

r^2 に反比例　万有引力の法則　(逆二乗)

$|\mathbb{r}| = r$　$\frac{1}{r^2}\frac{\mathbb{r}}{r} = \frac{\mathbb{r}}{r^3}$　$kq\frac{\mathbb{r}}{r^3}$



$\mathrm{div}\,\mathbb{f} = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}$

$\mathbb{r} = (x, y, z)$

$\frac{\mathbb{r}}{r^3} = (x(x^2+y^2+z^2)^{-\frac{3}{2}},\ y(x^2+y^2+z^2)^{-\frac{3}{2}},\ z(x^2+y^2+z^2)^{-\frac{3}{2}})$

$\mathrm{div}\frac{\mathbb{r}}{r^3} = (x^2+y^2+z^2)^{-\frac{3}{2}} - 3x^2(x^2+y^2+z^2)^{-\frac{5}{2}}$

$+ (x^2+y^2+z^2)^{-\frac{3}{2}} - 3y^2(x^2+y^2+z^2)^{-\frac{5}{2}}$

$+ (x^2+y^2+z^2)^{-\frac{3}{2}} - 3z^2(x^2+y^2+z^2)^{-\frac{5}{2}}$

$= 0$

逆k乗

$\mathrm{div}(\mathbb{r}\,r^k)$　(kは実数)

$r = (x^2+y^2+z^2)^{\frac{1}{2}}$　$r^k = (x^2+y^2+z^2)^{\frac{k}{2}}$

$\mathbb{r}\,r^k = (x(x^2+y^2+z^2)^{\frac{k}{2}},\ y(x^2+y^2+z^2)^{\frac{k}{2}},\ z(x^2+y^2+z^2)^{\frac{k}{2}})$

$\mathrm{div}\,\mathbb{r}\,r^k = (x^2+y^2+z^2)^{\frac{k}{2}} + 2x^2\cdot\left(\frac{k}{2}\right)(x^2+y^2+z^2)^{\frac{k}{2}-1} + \cdots$

$= 3(x^2+y^2+z^2)^{\frac{k}{2}} + k(x^2+y^2+z^2)(x^2+y^2+z^2)^{\frac{k}{2}-1}$

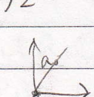
$= (3+k)(x^2+y^2+z^2)^{\frac{k}{2}}$

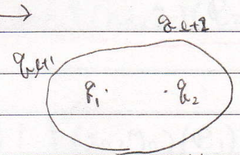
$3+k = 0$　$k = -3$　逆二乗

Gaussの定理

$$\int_\Sigma \mathbb{f}\cdot d\mathbb{S} = \int_\Omega (\mathrm{div}\,\mathbb{f})\,dV$$

$4\pi a^2\ kq\frac{1}{a^2} = 4\pi kq$





いくつかの正電荷

$\mathbb{E}_{q_1} + \mathbb{E}_{q_2} + \cdots + \mathbb{E}_{q_{l+1}} + \mathbb{E}_{q_{l+2}}$