

线性代数

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

线性变换

$$f: \mathbb{R}^2 \rightarrow \mathbb{R}^2$$

← 2x2 的行列式
表示 = 面积扩大
系数
线性
变换

$$x, y \in \mathbb{R}^2$$

$$(1) f(x+y) = f(x) + f(y)$$

$$(2) f(\alpha x) = \alpha f(x)$$

$$\alpha \in \mathbb{R} \quad e_1, e_2$$

$$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = x_1 e_1 + x_2 e_2 \quad (\text{线性组合})$$

$$f(x) = f(x_1 e_1 + x_2 e_2) = x_1 f(e_1) + x_2 f(e_2) = x_1 \begin{pmatrix} a_{11} \\ a_{21} \end{pmatrix} + x_2 \begin{pmatrix} a_{12} \\ a_{22} \end{pmatrix} =$$

$$f(e_1) = \begin{pmatrix} a_{11} \\ a_{21} \end{pmatrix}$$

$$f(e_2) = \begin{pmatrix} a_{12} \\ a_{22} \end{pmatrix}$$

$$a_{11}, a_{21}, e_1 \in \mathbb{R}$$

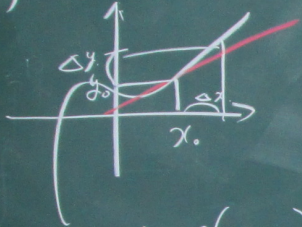
$e_1, e_2 \in \text{基底}$

2x2 的行列式

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

変数

$$f: \mathbb{R} \rightarrow \mathbb{R}$$



$$\Delta y = g(\Delta x)$$

接線: 傾き

$$\Delta y = a \Delta x \quad (a: \text{傾き})$$

$$f'(x_0)$$

f の x_0 における
微分係数

$$\mathbb{R} \rightarrow \mathbb{R}^3$$

$$f(x) = \begin{pmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \end{pmatrix}$$

成分に成分

$$f'(x) = \begin{pmatrix} f_1'(x) \\ f_2'(x) \\ f_3'(x) \end{pmatrix}$$

$$f_i: \mathbb{R} \rightarrow \mathbb{R}$$



$f: \mathbb{R}^n \rightarrow \mathbb{R}^m$ $\forall A$ for all
 \exists Existence
 $f: D \rightarrow \mathbb{R}^m$
 $f(d) = \begin{pmatrix} f_1(d) \\ \vdots \\ f_m(d) \end{pmatrix}$
 $f(d) = f(x_0) + \underline{a}d$ ($\forall d \in D$)
 $f: \mathbb{R} \rightarrow \mathbb{R}$

$f_1(d) = f_1(x_0) + a_1 d$
 $f_2(d) = f_2(x_0) + a_2 d$
 $f(d) = \begin{pmatrix} f_1(d) \\ f_2(d) \\ \vdots \\ f_m(d) \end{pmatrix} = \begin{pmatrix} f_1(x_0) + a_1 d \\ f_2(x_0) + a_2 d \\ \vdots \\ f_m(x_0) + a_m d \end{pmatrix}$
 $= \begin{pmatrix} f_1(x_0) \\ f_2(x_0) \\ \vdots \\ f_m(x_0) \end{pmatrix} + \begin{pmatrix} a_1 \\ \vdots \\ a_m \end{pmatrix} d$
 $= f(x_0) + \underline{a}d$

$D = \{d \in \mathbb{R} \mid d^2 = 0\}$
 $f: D \rightarrow \mathbb{R}$
 $f(d) = f(x_0) + \underline{a}d$ ($\forall d \in D$)
 $f: \mathbb{R} \rightarrow \mathbb{R}$

$f(x+d) = f(x_0+d) \in \mathbb{R}$
 $x_0 \in \mathbb{R}$

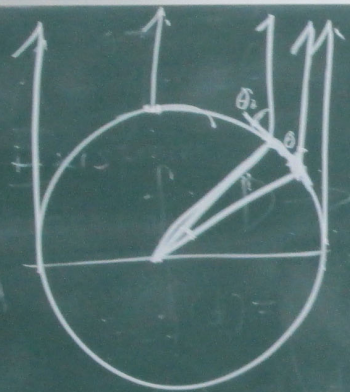


$\mathbb{R}^n \rightarrow \mathbb{R}^m$
 $x_0 \in \mathbb{R}^n$ (微分)
 $a \in \mathbb{R}^n$ ε 固定 $\exists ! b \in \mathbb{R}^m$
 $d \in D \mapsto \frac{f(x_0 + ad) - f(x_0)}{d} \in \mathbb{R}^m$
 $\frac{f(x_0) + b d - f(x_0)}{d}$
 $f'(x_0) a$

$f(x_0)(a)$
 $\alpha \in \mathbb{R}$
 $f(x_0)(a_1 + a_2) = f(x_0)(a_1) + f(x_0)(a_2)$
 $f(x_0)(\alpha a) = \alpha f(x_0)(a)$
 $f(x_0): \mathbb{R}^n \rightarrow \mathbb{R}^m$ 写像
 线性

$\mathbb{R} \rightarrow \mathbb{R}$ 数
 1×1
 証明終了





イテステネス

$$f'(x_0)(\alpha d) = \alpha f'(x_0)(d) \quad \text{for } d \in D \Rightarrow \alpha d \in D$$

$$f(x_0 + \alpha d) = f(x_0) + \frac{f'(x_0)(\alpha d)}{d}$$

$$\theta_0 \quad f(x_0 + \alpha(\alpha d)) = f(x_0) + \frac{f'(x_0)(\alpha)(\alpha d)}{\alpha}$$



$$f(x_0 + (a_1 + a_2)d) = f(x_0 + a_1 d + a_2 d) = f(x_0 + a_1 d) + f'(x_0 + a_1 d)(a_2 d)$$

$$= f(x_0) + f'(x_0)(a_1 d) + f'(x_0 + a_1 d)(a_2 d)$$

$$= f(x_0) + f'(x_0)(a_1) + f'(x_0)(a_2) + f'(x_0)(a_2)$$

$$f'(\underline{x}) \in \mathbb{R}^m$$

$$\mathbb{R}^n$$

