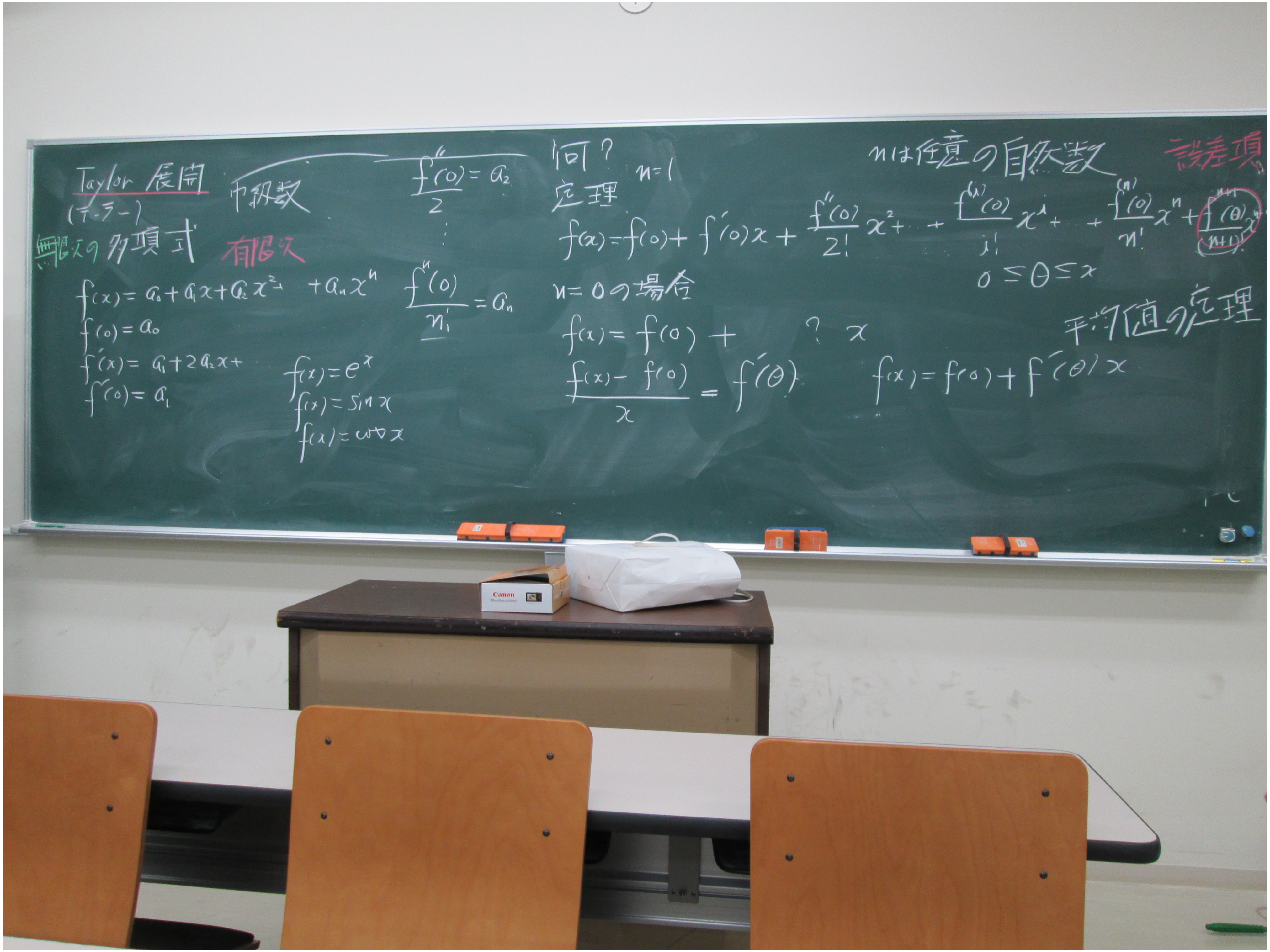
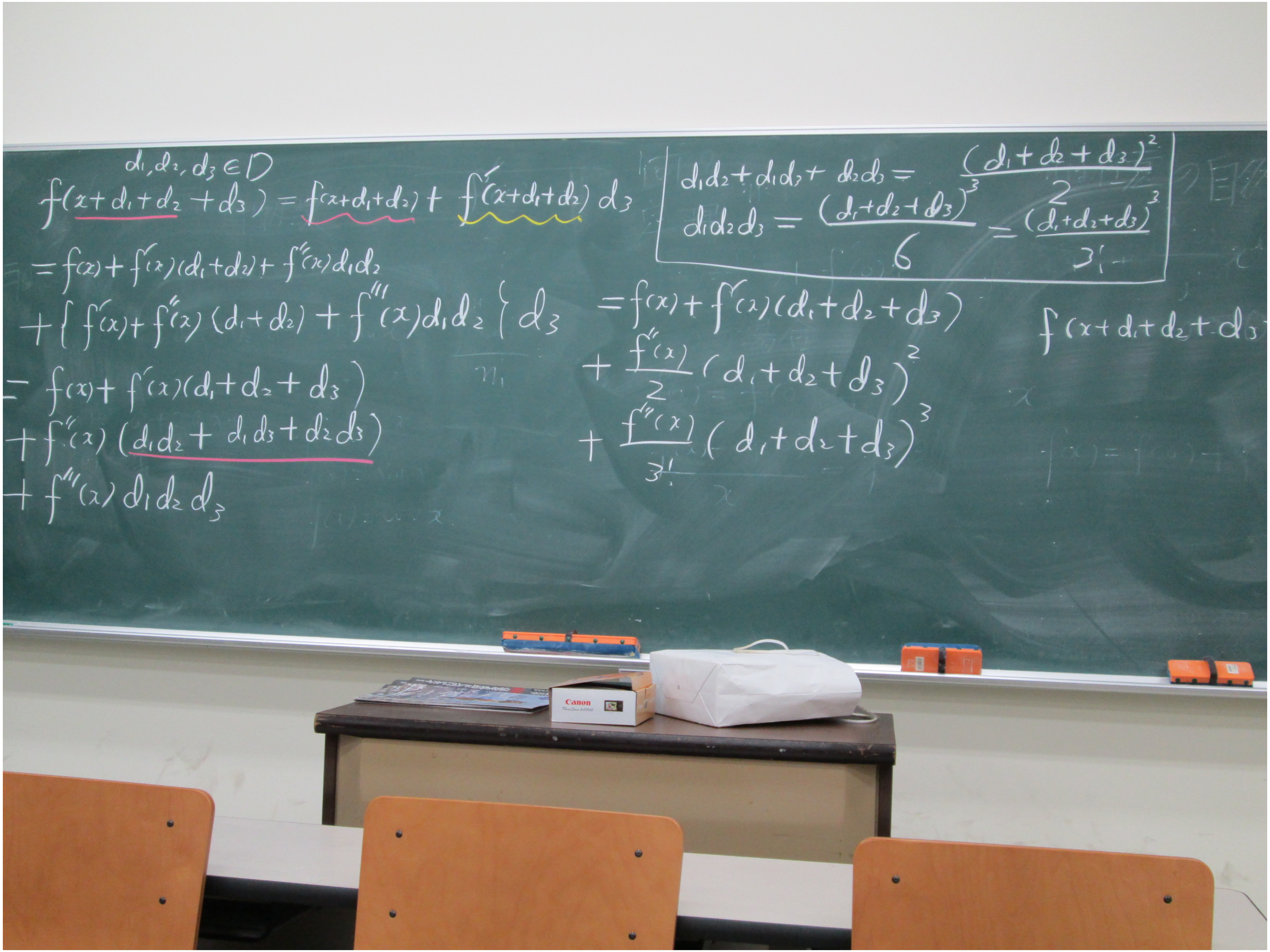
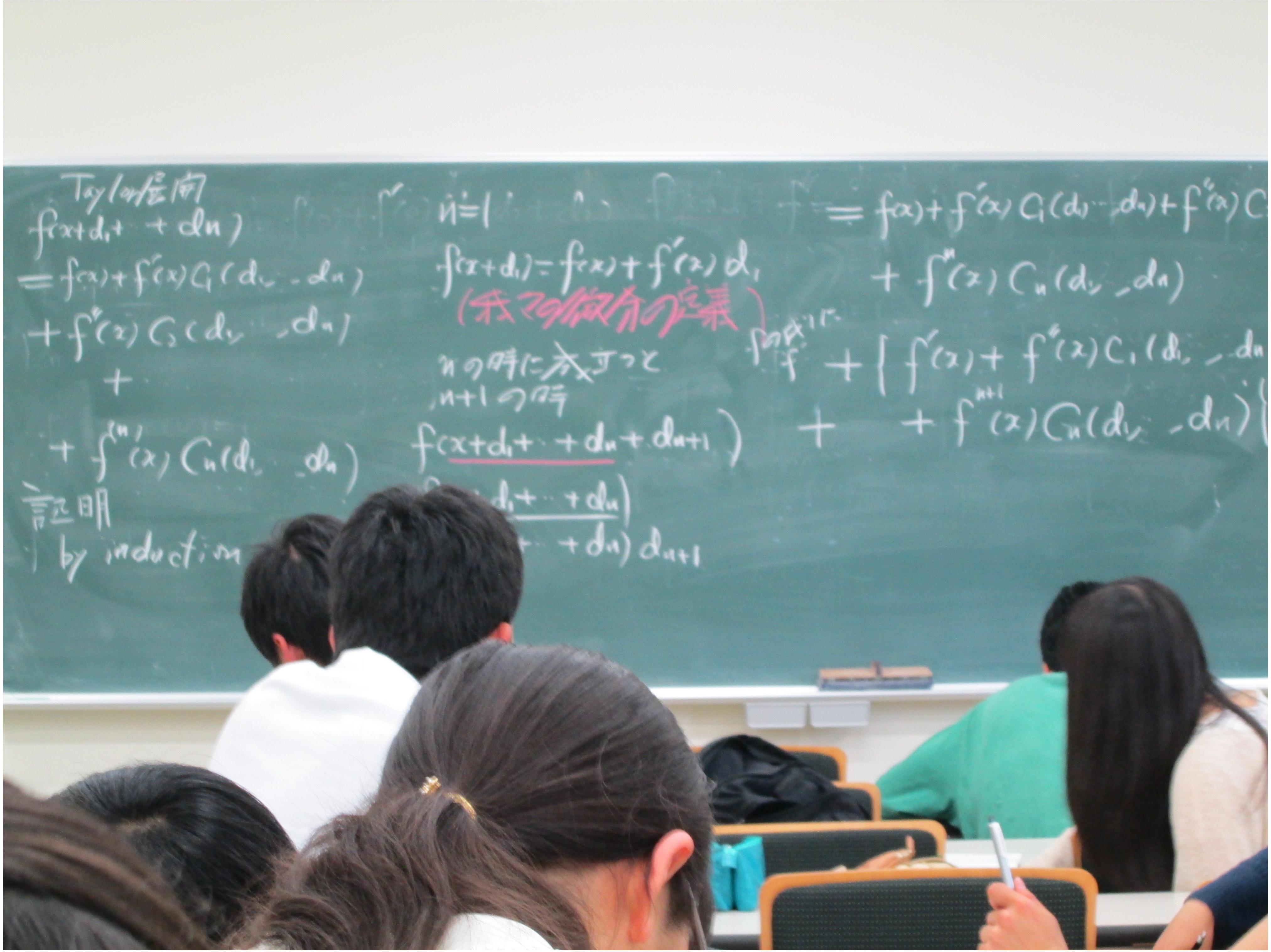

Taylor 展開
(テーラー)
無限次の多項式
巾級数
有限次
$\frac{f''(0)}{2} = a_2$
$f(x) = a_0 + a_1x + a_2x^2 \quad + a_nx^n$
$\frac{f^{(n)}(0)}{n!} = a_n$
$f(0) = a_0$
$f'(x) = a_1 + 2a_2x +$
$f'(0) = a_1$
$f(x) = e^x$
$f(x) = \sin x$
$f(x) = \cos x$
何？
定理
$n=1$
nは任意の自然数
誤差項
$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \cdots + \frac{f^{(\lambda)}(0)}{\lambda!}x^\lambda + \cdots + \frac{f^{(n)}(0)}{n!}x^n + \frac{f^{(n+1)}(\theta)}{(n+1)!}x^{n+1}$
$0 \leq \theta \leq x$
n=0の場合
$f(x) = f(0) + \quad ? \quad x$
$\frac{f(x) - f(0)}{x} = f'(\theta)$
平均値の定理
$f(x) = f(0) + f'(\theta)x$
Canon

$D = \{ d \in \mathbb{R} \mid d^2 = 0 \} = D_1$
$D_n = \{ d \in \mathbb{R} \mid d^{n+1} = 0 \}$
$d_1, \dots, d_n \in D = D_1 \Longrightarrow d_1 + \dots + d_n \in D_n$
$d_1 \in D$ $f(x + d_1) = f(x) + f'(x) d_1$
$d_1, d_2 \in D$
$f(x + d_1 + d_2) = f(x + d_1) + f'(x + d_1) d_2$
$= f(x) + f'(x) d_1 + \{ f'(x) + f''(x) d_1 \} d_2$
$= f(x) + f'(x)(d_1 + d_2) + f''(x) d_1 d_2$
$= f(x) + f'(x)(d_1 + d_2) + \frac{f''(x)}{2}(d_1 + d_2)^2$
$f(x + d) = f(x) + f'(x) d + \frac{f''(x)}{2} d^2$
$f : \mathbb{R} \to \mathbb{R}$
$d_1 d_2 = \frac{(d_1 + d_2)^2}{2}$
$d_1 + d_2 = d \in D_2$
誤差項なし


$d_1, d_2, d_3 \in D$
$f(x+d_1+d_2+d_3) = f(x+d_1+d_2) + f'(x+d_1+d_2)\, d_3$
$= f(x) + f'(x)(d_1+d_2) + f''(x)\, d_1 d_2$
$+ \{ f'(x) + f''(x)(d_1+d_2) + f'''(x)\, d_1 d_2 \} d_3$
$= f(x) + f'(x)(d_1+d_2+d_3)$
$+ f''(x)(d_1 d_2 + d_1 d_3 + d_2 d_3)$
$+ f'''(x)\, d_1 d_2 d_3$
$d_1 d_2 + d_1 d_3 + d_2 d_3 = \frac{(d_1+d_2+d_3)^2}{2}$
$d_1 d_2 d_3 = \frac{(d_1+d_2+d_3)^3}{6} = \frac{(d_1+d_2+d_3)^3}{3!}$
$= f(x) + f'(x)(d_1+d_2+d_3)$
$+ \frac{f''(x)}{2}(d_1+d_2+d_3)^2$
$+ \frac{f'''(x)}{3!}(d_1+d_2+d_3)^3$
$f(x+d_1+d_2+d_3$
Canon


$f(d_1+d_2+d_3)=f(0)+f'(0)(d_1+d_2+d_3)$
$+\frac{f''(0)}{2}(d_1+d_2+d_3)^2+\frac{f'''(0)}{3!}(d_1+d_2+d_3)^3$
$f(0)=C$
$f'=f \quad f''=f' \quad f'''=f''$
$f'(0)=f(0)=C$
$f''(0)=f'(0)=f(0)=C$
$f'''(0)=f''(0)=$
C
$f''=-f \quad f'''=-f' \quad f''''=-f''=f$
$f(0)=C_1$
$f'(0)=C_2$
$f''(0)=-f(0)=-C_1$
$f'''(0)=-f'(0)=-C_2$
$f''''(0)=-f''(0)=f(0)=C_1$


Taylorの定理の証明
f(x+d1+ +dn)
=f(x)+f'(x)(d1+ +dn)
言葉の証明
整式
f(x,y)=3x2+y
3y2+x
f(x,y)=x2+y2
y2+x2
=(x+y)2−2xy
対称式
基本対称式
x1, xn
集中力
C0(x0, ,xn)=1
C1(x1, ,xn)=x1+ +xn 1次
C2(x1, ,xn)= Σ 1≤i<j≤n xi xj 2次
Cn(x1, ,xn)=x1 xn
Newtonの定理
C0(x,y)=1
C1(x,y)=x+y
C2(x,y)=xy
Canon


Taylor展開
証明
by induction
n=1
(我々の微分の定義)
n+1の時

= f(x) + f'(x){ C_1(d_1, ..., d_n) + d_{n+1} { C_r(d_1, ..., d_{n+1})
+ f''(x){ C_2(d_1, ..., d_n) + d_{n+1} C_1(d_1, ..., d_n) }
Σ_{1≤i<j≤n} d_i d_j + d_{n+1}(d_1 + ... + d_n)
C_2(d_1, ..., d_{n+1})
一般に
C_{k+1}(d_1, ..., d_n) + d_{n+1} C_k(d_1, ..., d_n) = C_{k+1}(d_1, ..., d_{n+1})
組合せ
{n+1}C_{r+1} = {n}C_{r+1} + {n}C_r
Newton