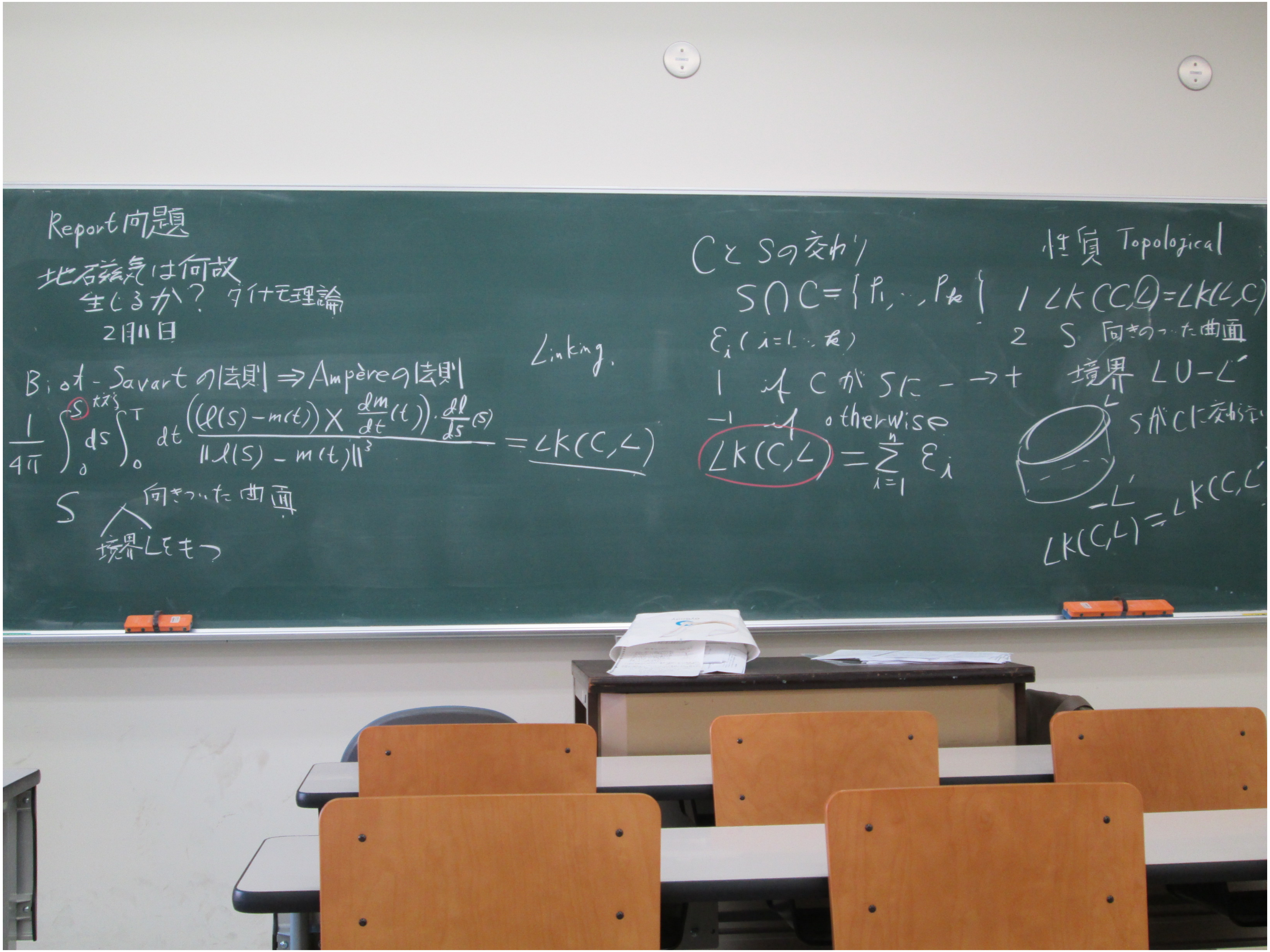
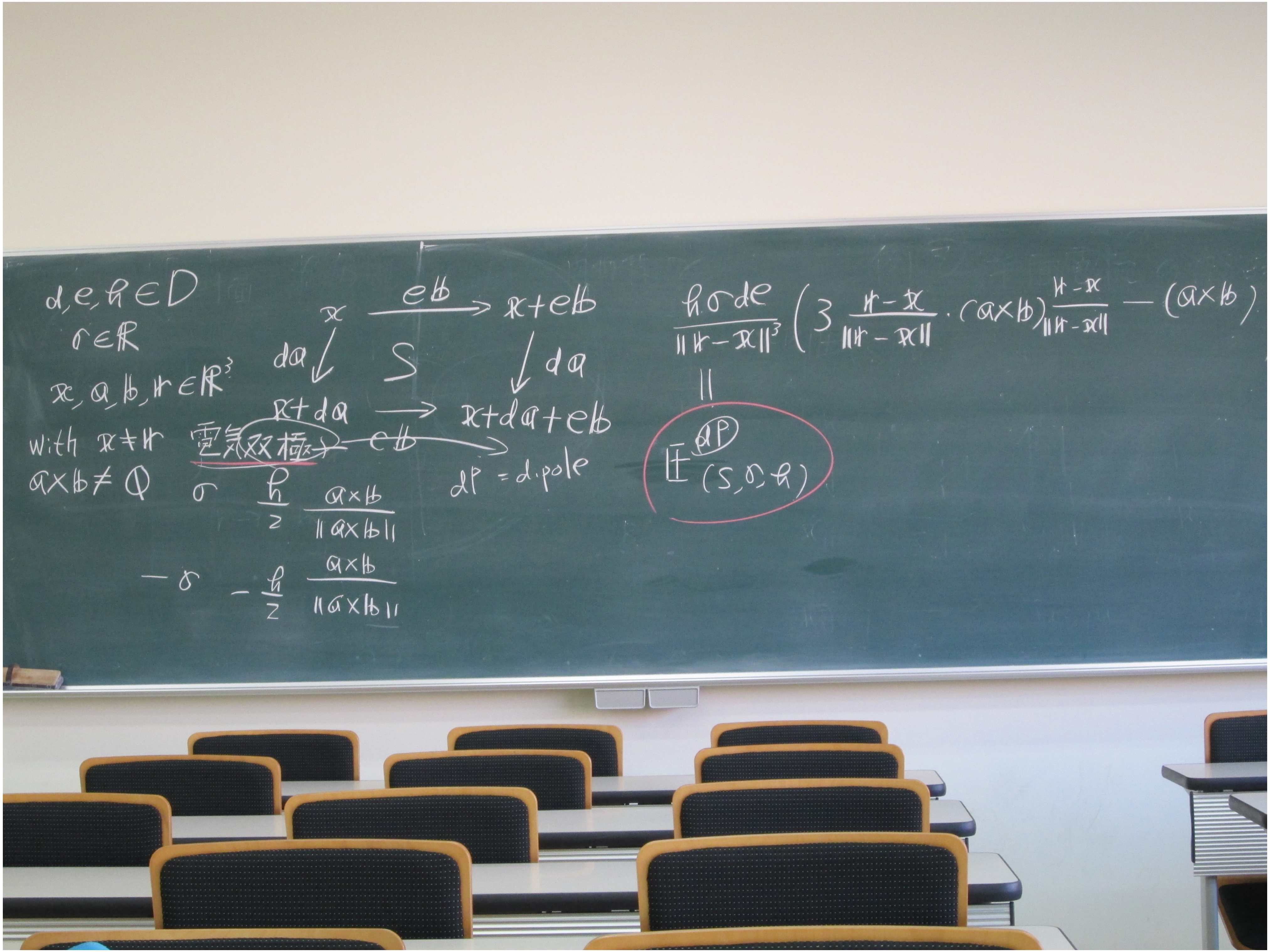
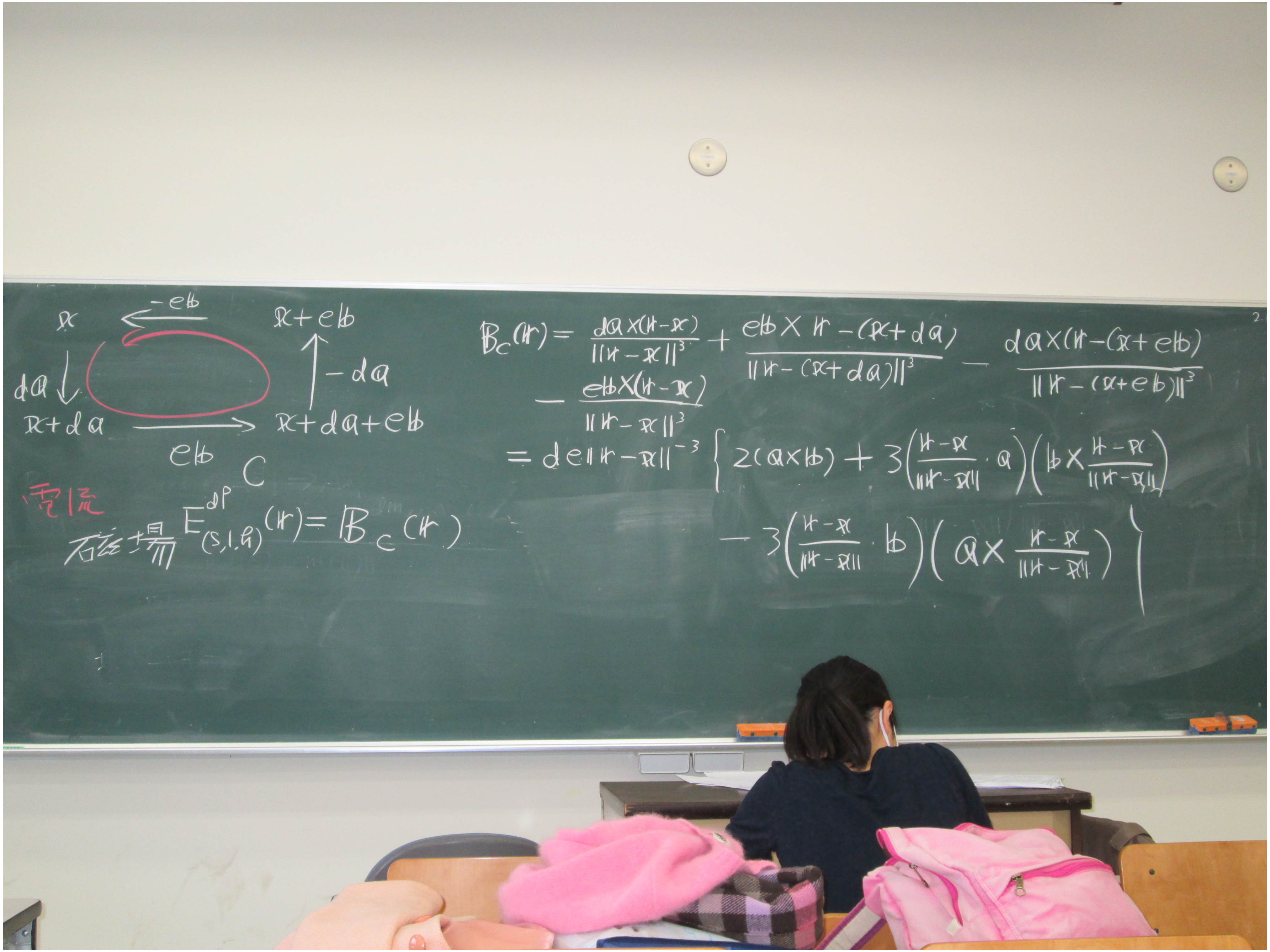
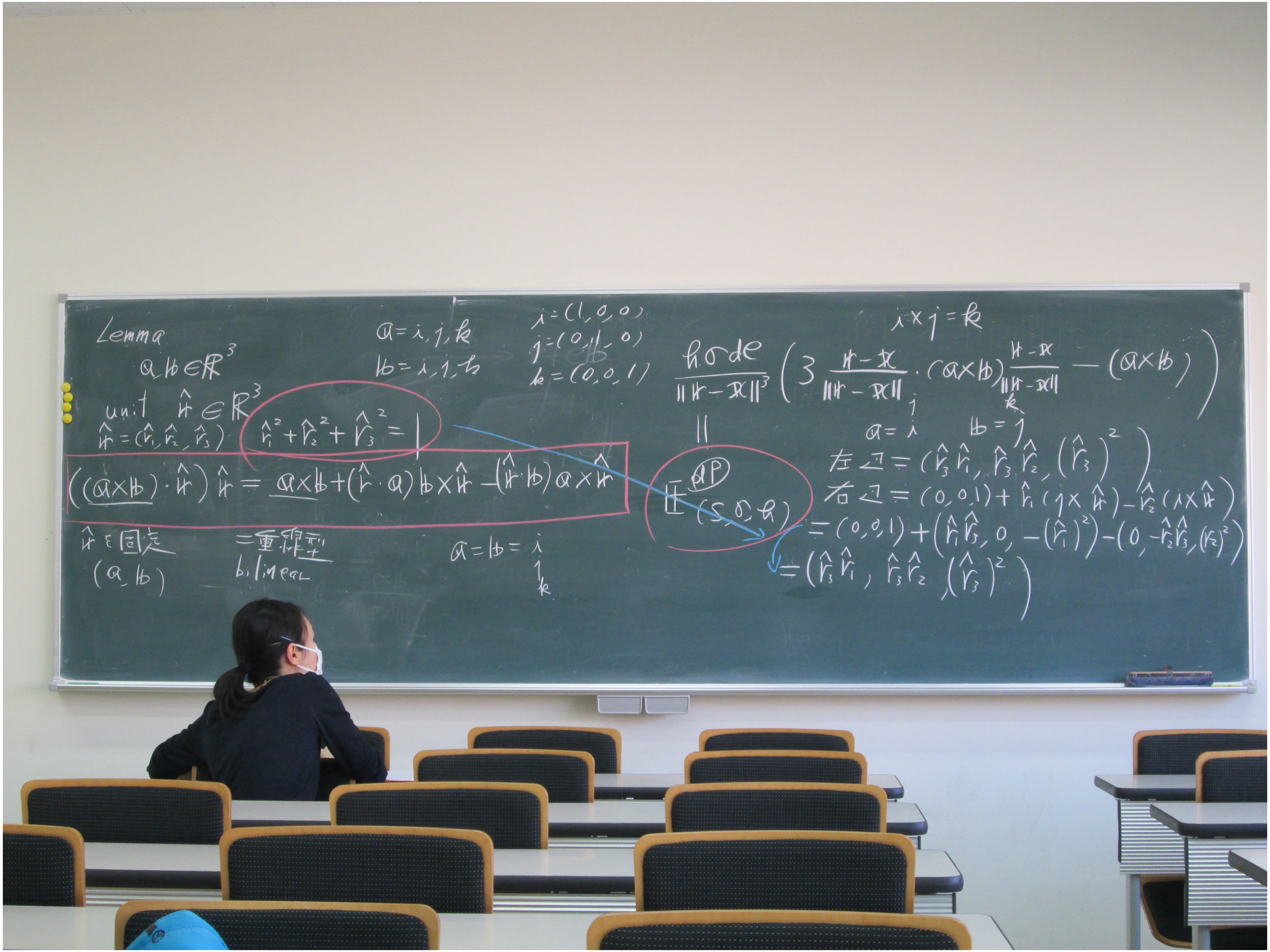

Report問題
地磁気は何故生じるか? ダイナモ理論
2月11日
Biot-Savartの法則 ⇒ Ampèreの法則
$\frac{1}{4\pi}\int_0^S ds \int_0^T dt \frac{((\ell(s)-m(t)) \times \frac{dm}{dt}(t)) \cdot \frac{d\ell}{ds}(s)}{\|\ell(s)-m(t)\|^3} = Lk(C,L)$
S 向きついた曲面
境界Lをもつ
Linking.
CとSの交わり
$S \cap C = \{P_1, \ldots, P_k\}$
ε_i ($i=1 \ldots k$)
1 if CがSに − → +
−1 if otherwise
$Lk(C,L) = \sum_{i=1}^{n} \varepsilon_i$
性質 Topological
1 $Lk(C,L) = Lk(L,C)$
2 S 向きのついた曲面
境界 $L \cup -L'$
SがCに交わらない
$Lk(C,L) = Lk(C,L')$

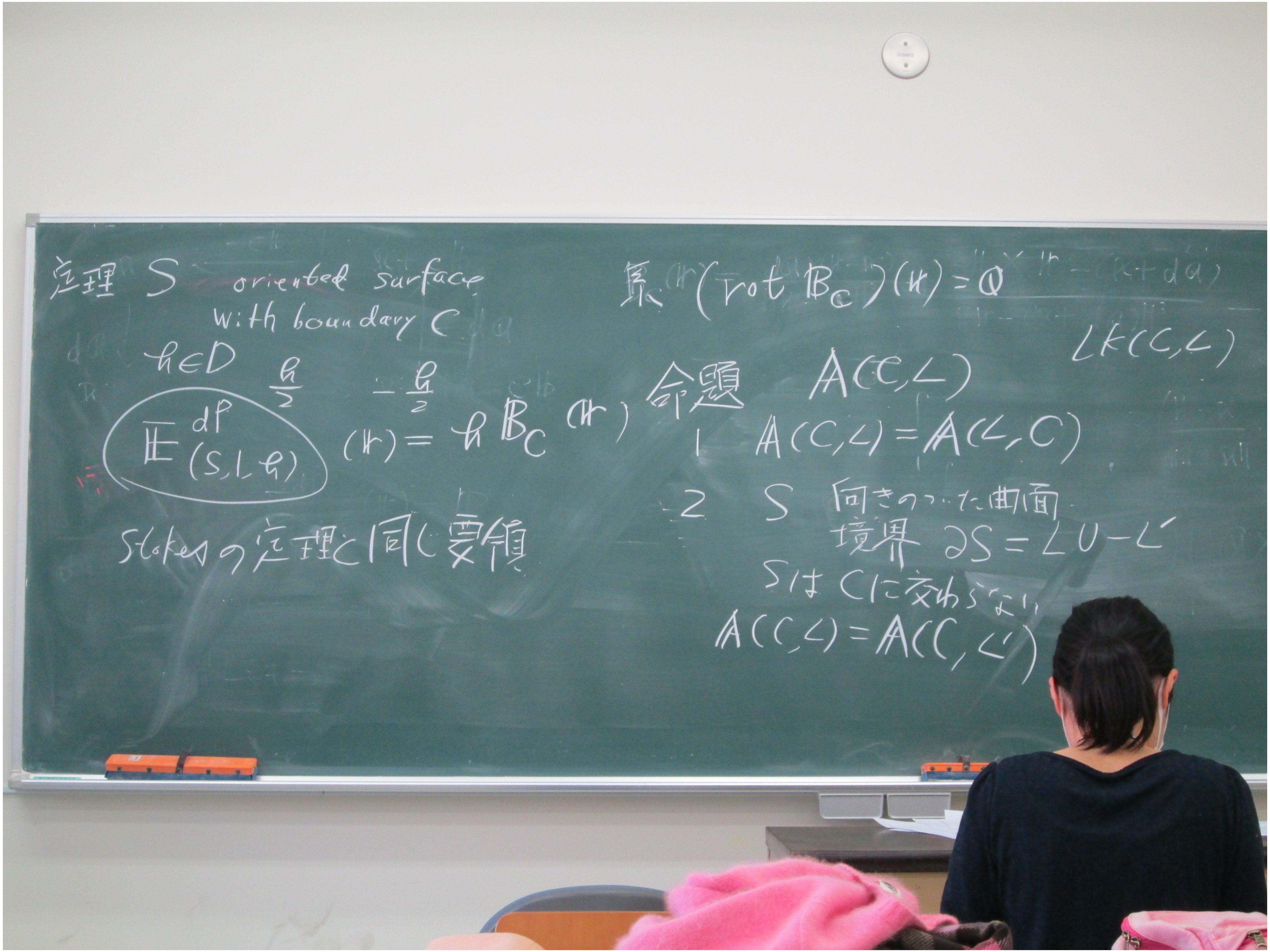


d, e, h ∈ D
r ∈ ℝ
x, a, b, r ∈ ℝ³
with x ≠ r
a×b ≠ 0
x → x+eb
da
S
x+da → x+da+eb
電気双極子
dp = dipole
σ h/2 a×b/‖a×b‖
−σ −h/2 a×b/‖a×b‖
hσde/‖r−x‖³ (3 (r−x)/‖r−x‖ · (a×b) (r−x)/‖r−x‖ − (a×b))
E^dp(S, σ, h)

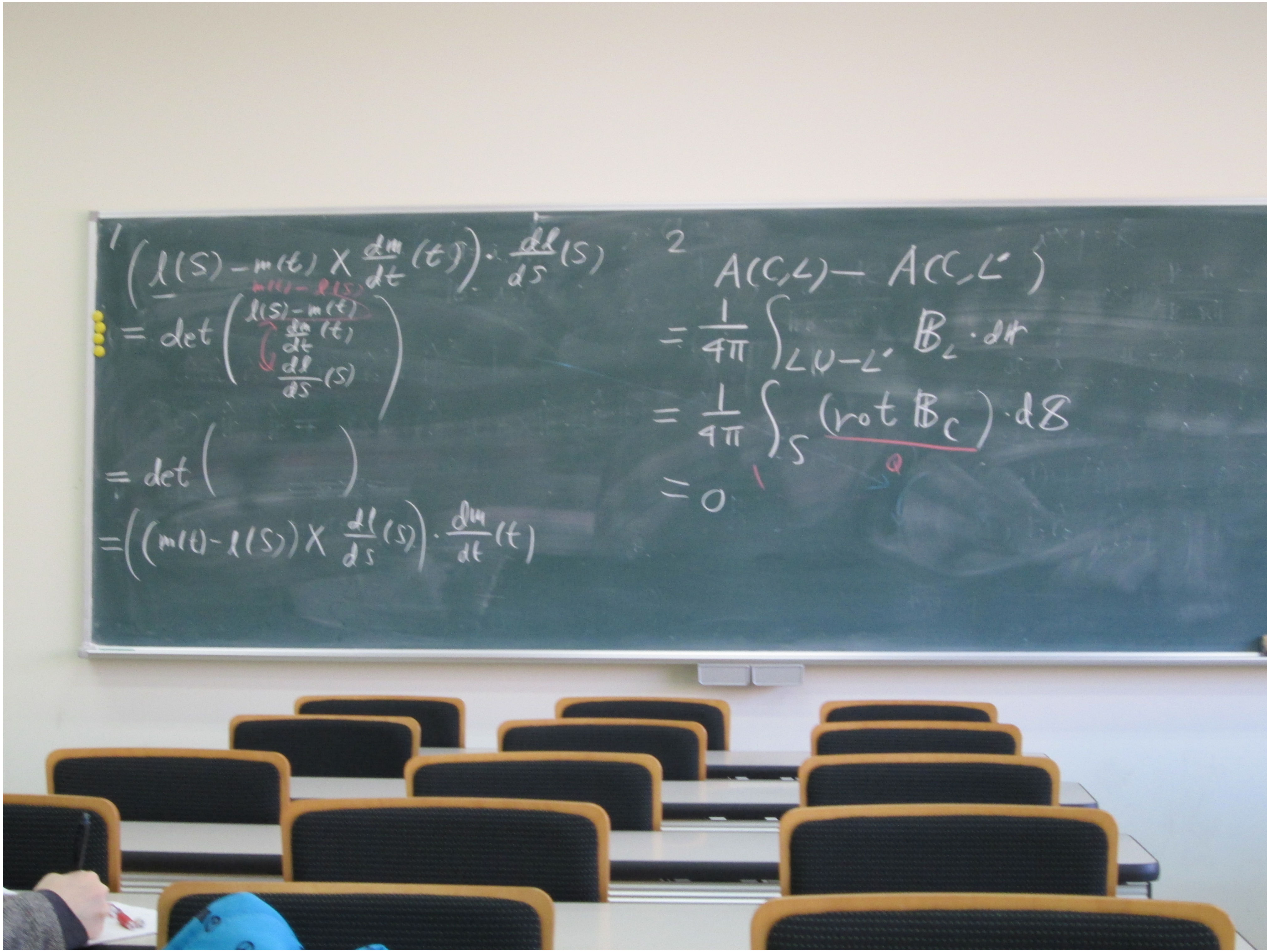


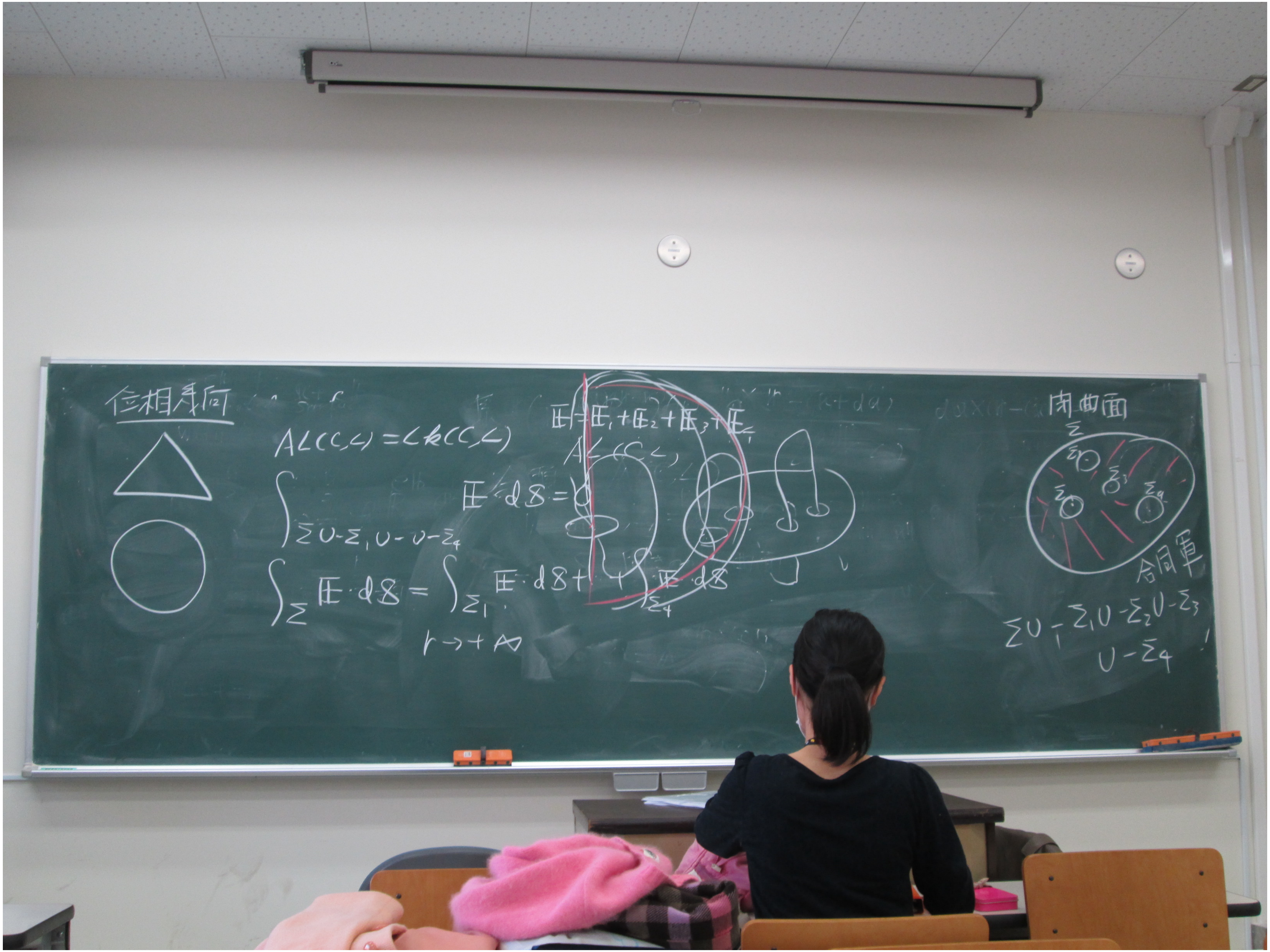
2.
$\mathbf{x} \xleftarrow{-e\mathbf{b}} \mathbf{x}+e\mathbf{b}$
$d\mathbf{a} \downarrow$ $\uparrow -d\mathbf{a}$
$\mathbf{x}+d\mathbf{a} \xrightarrow{e\mathbf{b}} \mathbf{x}+d\mathbf{a}+e\mathbf{b}$
C
電流
磁場 $E^{dP}_{(S,1,R)}(\mathbf{r}) = \mathbf{B}_C(\mathbf{r})$
$\mathbf{B}_C(\mathbf{r}) = \dfrac{d\mathbf{a}\times(\mathbf{r}-\mathbf{x})}{\|\mathbf{r}-\mathbf{x}\|^3} + \dfrac{e\mathbf{b}\times \mathbf{r}-(\mathbf{x}+d\mathbf{a})}{\|\mathbf{r}-(\mathbf{x}+d\mathbf{a})\|^3} - \dfrac{d\mathbf{a}\times(\mathbf{r}-(\mathbf{x}+e\mathbf{b}))}{\|\mathbf{r}-(\mathbf{x}+e\mathbf{b})\|^3} - \dfrac{e\mathbf{b}\times(\mathbf{r}-\mathbf{x})}{\|\mathbf{r}-\mathbf{x}\|^3}$
$= de\|\mathbf{r}-\mathbf{x}\|^{-3}\Big\{2(\mathbf{a}\times\mathbf{b}) + 3\Big(\dfrac{\mathbf{r}-\mathbf{x}}{\|\mathbf{r}-\mathbf{x}\|}\cdot\mathbf{a}\Big)\Big(\mathbf{b}\times\dfrac{\mathbf{r}-\mathbf{x}}{\|\mathbf{r}-\mathbf{x}\|}\Big) - 3\Big(\dfrac{\mathbf{r}-\mathbf{x}}{\|\mathbf{r}-\mathbf{x}\|}\cdot\mathbf{b}\Big)\Big(\mathbf{a}\times\dfrac{\mathbf{r}-\mathbf{x}}{\|\mathbf{r}-\mathbf{x}\|}\Big)\Big\}$


Lemma
a, b ∈ R³
unit r̂ ∈ R³
r̂ = (r̂₁, r̂₂, r̂₃)
r̂₁² + r̂₂² + r̂₃² = 1
((a×b)·r̂) r̂ = a×b + (r̂·a) b×r̂ − (r̂·b) a×r̂
r̂ を固定
(a, b)
二重線型
bilinear
a = i, j, k
b = i, j, k
a = b = i
i = (1, 0, 0)
j = (0, 1, 0)
k = (0, 0, 1)
i × j = k
a = i
b = j
左辺 = (r̂₃r̂₁, r̂₃r̂₂, (r̂₃)²)
右辺 = (0, 0, 1) + r̂₁(j×r̂) − r̂₂(i×r̂)
= (0, 0, 1) + (r̂₁r̂₃, 0, −(r̂₁)²) − (0, −r̂₂r̂₃, (r̂₂)²)
= (r̂₃r̂₁, r̂₃r̂₂, (r̂₃)²)



定理 S oriented surface
with boundary C
Stokesの定理と同じ要領
系 (rot B_C)(r) = 0
lk(C, L)
命題 A(C, L)
1 A(C, L) = A(L, C)
2 S 向きのついた曲面
境界 ∂S = L ∪ −L'
SはCに交わらない
A(C, L) = A(C, L')


1
$\left(\left(l(s) - m(t)\right) \times \frac{dm}{dt}(t)\right) \cdot \frac{dl}{ds}(s)$
$m(t) - l(s)$
$= \det\begin{pmatrix} l(s) - m(t) \\ \frac{dm}{dt}(t) \\ \frac{dl}{ds}(s) \end{pmatrix}$
$= \det\left(\quad \right)$
$= \left(\left(m(t) - l(s)\right) \times \frac{dl}{ds}(s)\right) \cdot \frac{dm}{dt}(t)$
2
$A(C,L) - A(C,L')$
$= \frac{1}{4\pi} \int_{L \cup -L'} B_L \cdot dr$
$= \frac{1}{4\pi} \int_S (\mathrm{rot}\, B_C) \cdot dS$
$= 0$


位相幾何
AL(C,L) = Lk(C,L)
閉曲面