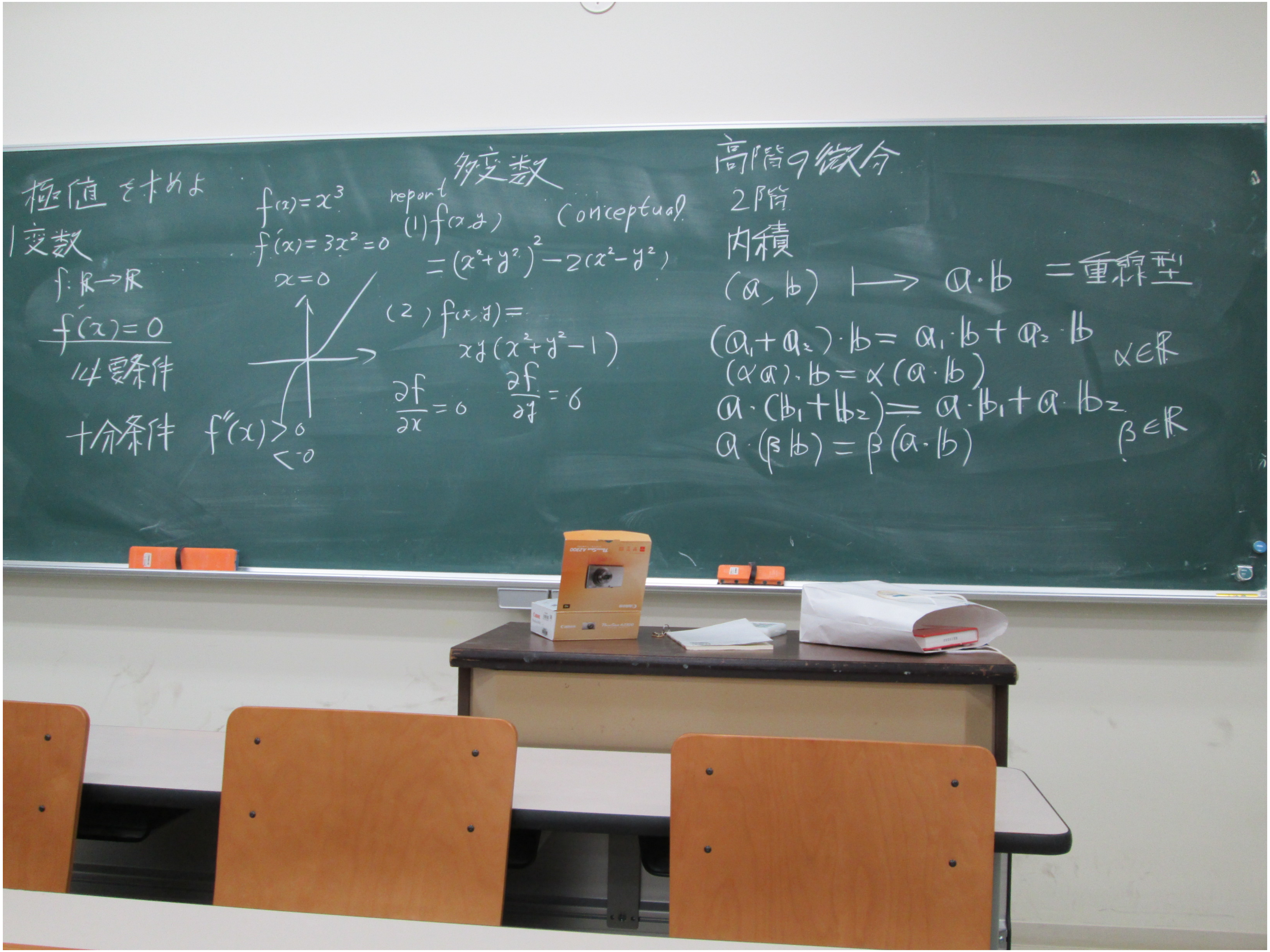
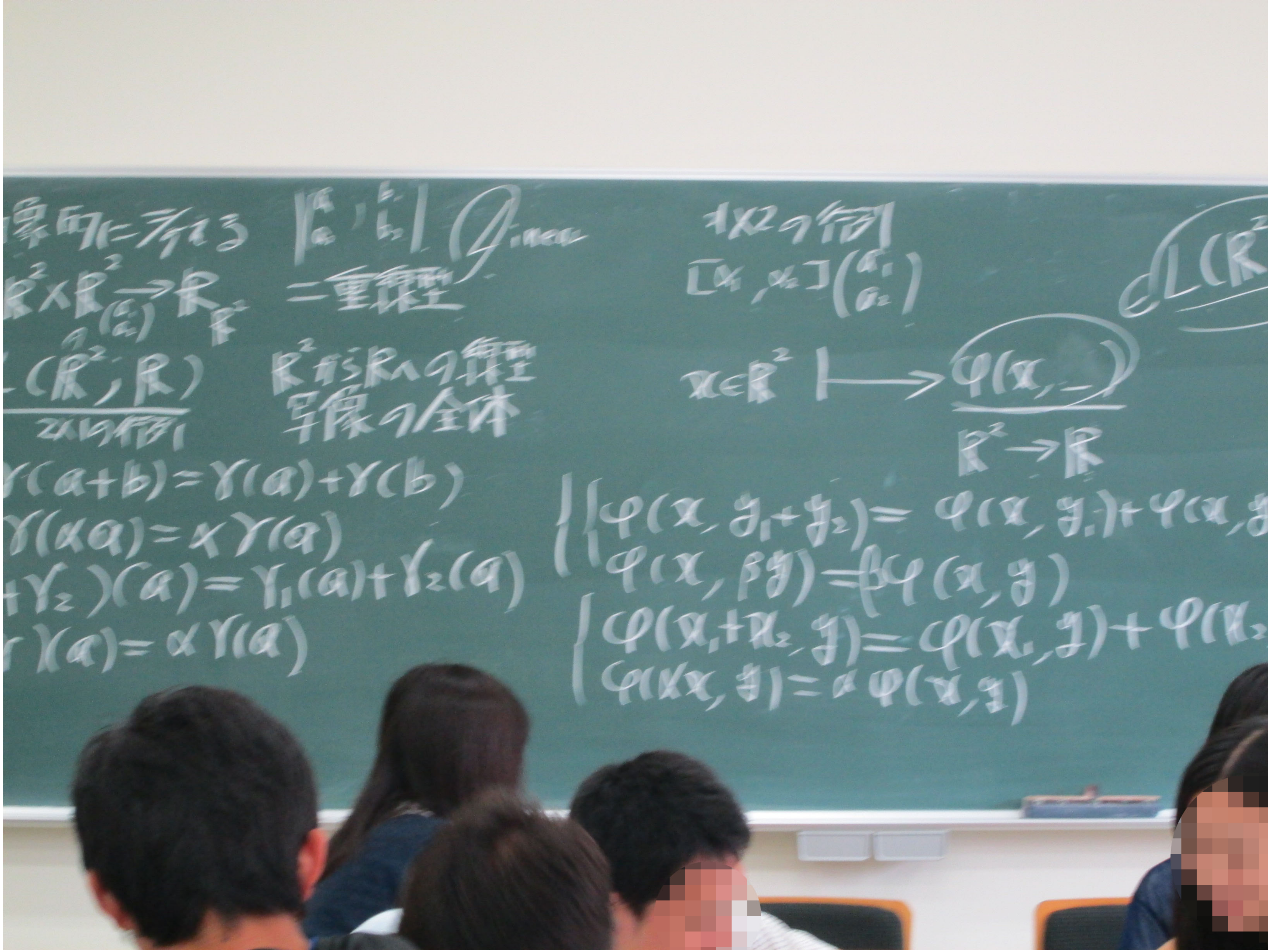
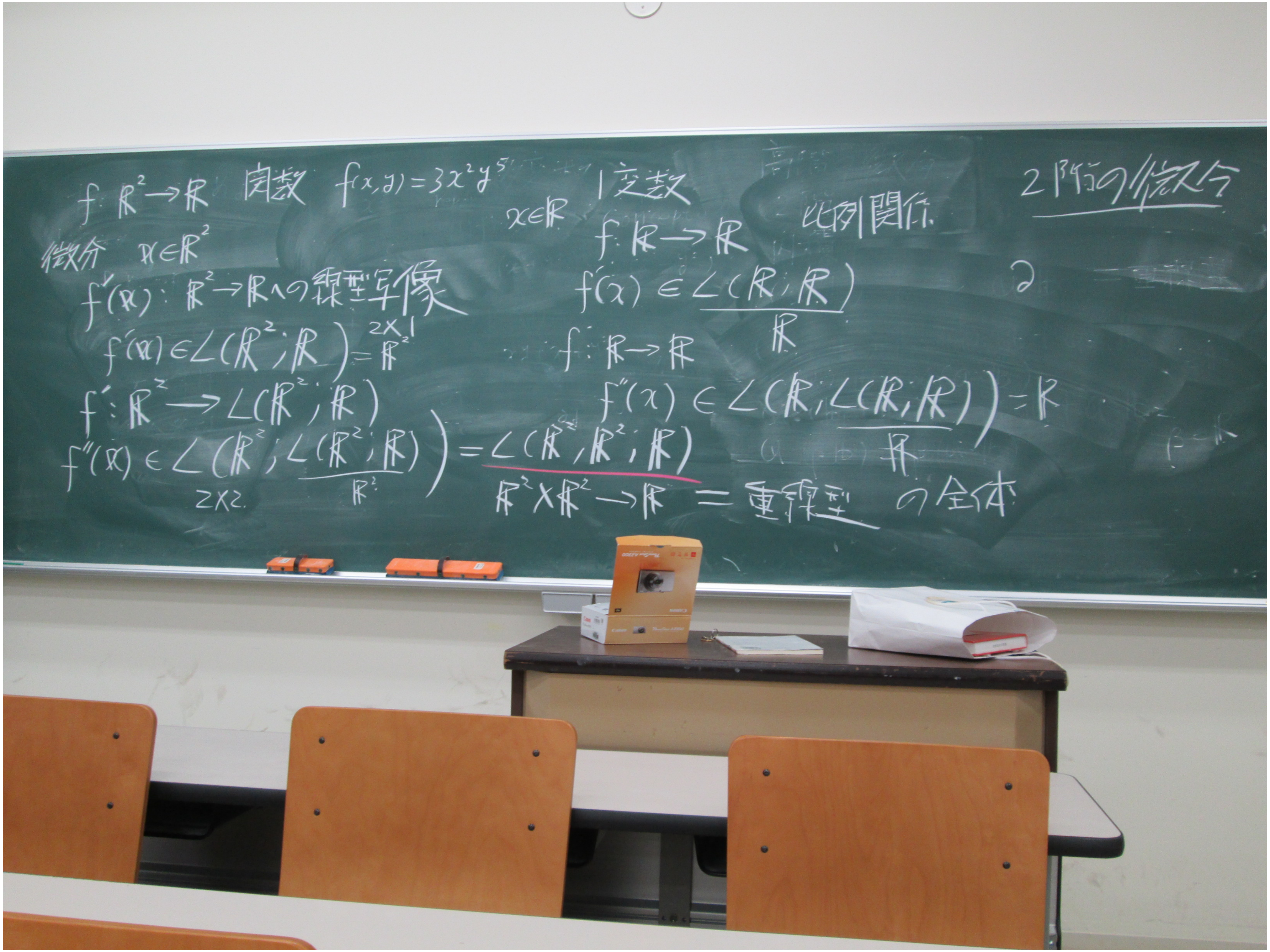
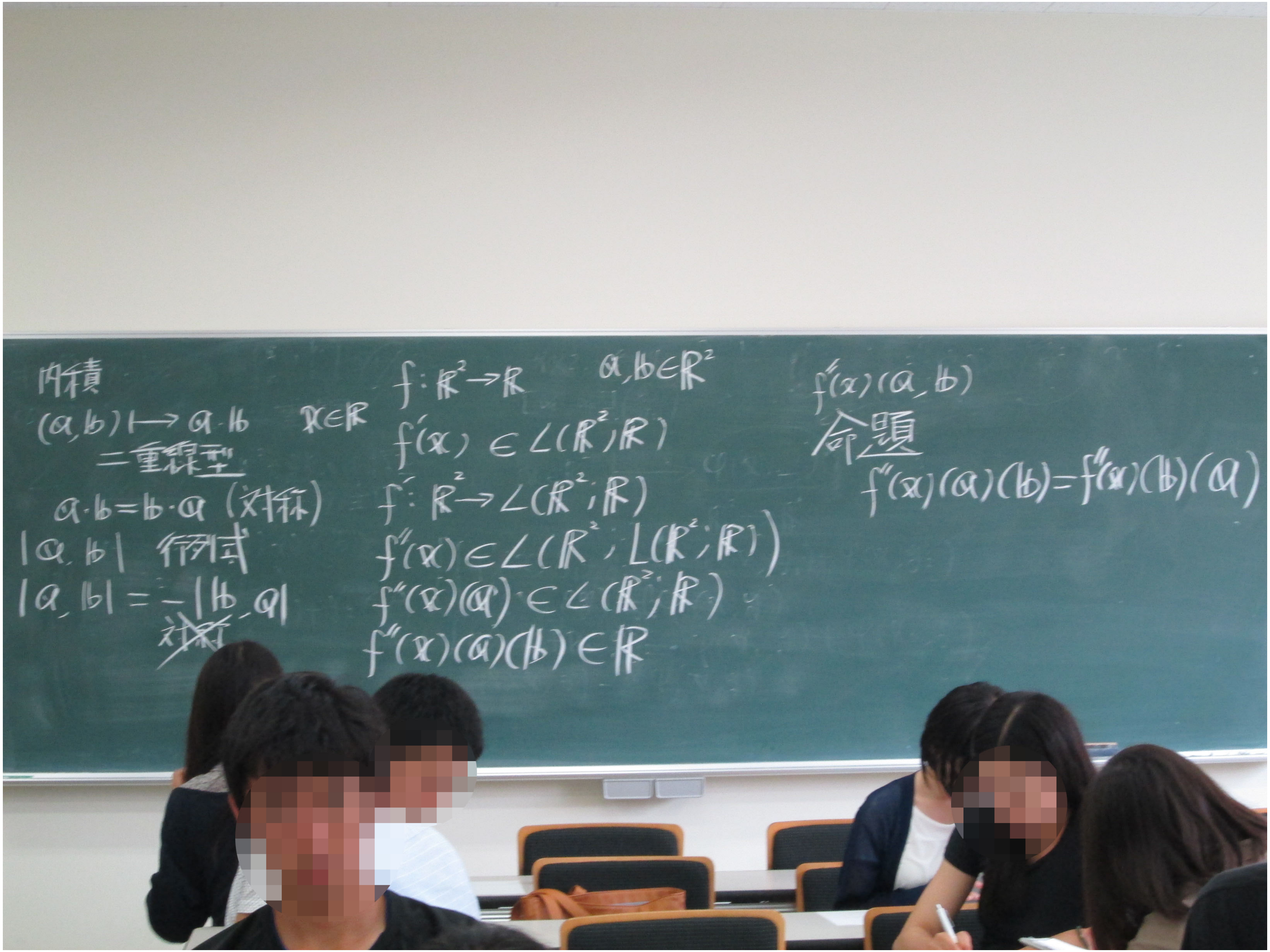

極値を求めよ
1変数
f: R→R
f'(x)=0
必要条件
十分条件 f''(x)>0
<0
f(x)=x^3
f'(x)=3x^2=0
x=0
多変数
report
(1) f(x,y)
=(x^2+y^2)^2−2(x^2−y^2)
conceptual
(2) f(x,y)=
xy(x^2+y^2−1)
∂f/∂x=0 ∂f/∂y=0
高階の微分
2階
内積
(a, b) ⟼ a·b =重線型
(a1+a2)·b = a1·b + a2·b
(αa)·b = α(a·b)
α∈R
a·(b1+b2) = a·b1 + a·b2
a·(βb) = β(a·b)
β∈R


二重線型
1×2の行列
R²からRへの線型
写像の全体
2×1の行列
x∈R² ⟼ φ(x, _)
R² → R
φ(x, y₁+y₂) = φ(x, y₁) + φ(x,
φ(x, βy) = βφ(x, y)
φ(x₁+x₂, y) = φ(x₁, y) + φ(x₂
φ(αx, y) = αφ(x, y)

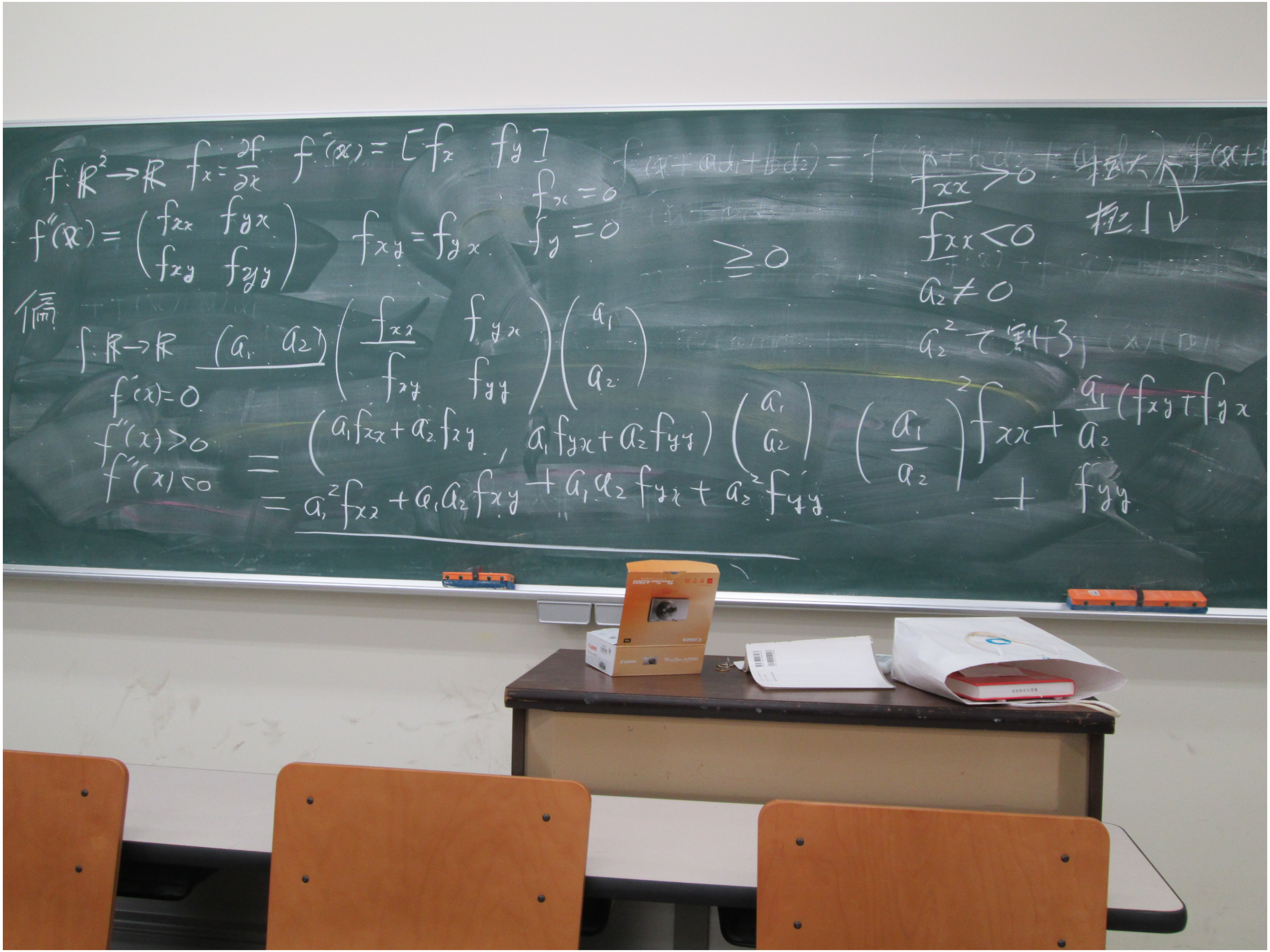

f: R² → R 関数 f(x,y)=3x²y⁵
微分 x∈R²
f′(x): R²→R への線型写像
f′(x)∈L(R²;R) = R² 2×1
f′: R² → L(R²;R)
f″(x) ∈ L(R²;L(R²;R)) = L(R²,R²;R)
2×2
R²×R² → R = 重線型 の全体
x∈R
1変数
f: R→R
比例関係
f′(x) ∈ L(R;R)
f: R→R
f″(x) ∈ L(R;L(R;R)) = R
2階の微分



内積
$(\mathbf{a},\mathbf{b}) \longmapsto \mathbf{a}\cdot\mathbf{b}$
二重線型
$\mathbf{a}\cdot\mathbf{b} = \mathbf{b}\cdot\mathbf{a}$ (対称)
$|\mathbf{a}, \mathbf{b}|$ 行列式
$|\mathbf{a}, \mathbf{b}| = -|\mathbf{b}, \mathbf{a}|$
~~対称~~
$\mathbf{x} \in \mathbb{R}$
$f : \mathbb{R}^2 \to \mathbb{R}$
$f'(\mathbf{x}) \in L(\mathbb{R}^2; \mathbb{R})$
$f' : \mathbb{R}^2 \to L(\mathbb{R}^2; \mathbb{R})$
$f''(\mathbf{x}) \in L(\mathbb{R}^2; L(\mathbb{R}^2; \mathbb{R}))$
$f''(\mathbf{x})(\mathbf{a}) \in L(\mathbb{R}^2; \mathbb{R})$
$f''(\mathbf{x})(\mathbf{a})(\mathbf{b}) \in \mathbb{R}$
$\mathbf{a}, \mathbf{b} \in \mathbb{R}^2$
$f''(\mathbf{x})(\mathbf{a}, \mathbf{b})$
命題
$f''(\mathbf{x})(\mathbf{a})(\mathbf{b}) = f''(\mathbf{x})(\mathbf{b})(\mathbf{a})$


$f(x+ad_1+bd_2) = f(x+ad) +$
$f'(x+ad_1)(b)d_2$
$= f(x)+f'(x)(a)d_1$
$\{f(x)+f''(x)(a)d_1\}(b)d_2$
$f(x)+f'(x)(a)d_1+f'(x)(b)d_2$
$+ f''(x)(a)(b)d_1d_2$
$f(x+ad_1+bd_2) = f(x+bd_2+ad_1) = f(x+bd_2)+$
$f'(x+bd_2)(a)d_1$
$= f(x)+f'(x)(b)d_2+\{f'(x)+f''(x)(b)d_2\}(a)d_1$
$= f(x)+f'(x)(b)d_2+f'(x)(a)d_1+f''(x)(b)(a)d_1d_2$


R²×R²→R への2重線形写像を
e₂ とう表すか？
(a₁ a₂)   b=(b₁ b₂)
+a₂e₂   b₁e₁+b₂e₂
φ(a,b)=φ(a₁e₁+a₂e₂, b₁e₁+b₂e₂)
=a₁b₁φ(e₁,e₁)+a₁b₂φ(e₁,e₂)
+a₂b₁φ(e₂,e₁)+a₂b₂φ(e₂,e₂



$f: \mathbb{R}^2 \to \mathbb{R}$
$f_x = \frac{\partial f}{\partial x}$
$f'(x) = [f_x \;\; f_y]$
$f''(x) = \begin{pmatrix} f_{xx} & f_{yx} \\ f_{xy} & f_{yy} \end{pmatrix}$
$f_{xy} = f_{yx}$
$f_x = 0$
$f_y = 0$
$\geqq 0$
$f_{xx} > 0$
極大
$f_{xx} < 0$
極小
$a_2 \neq 0$
a_2^2 で割る
偏
$f: \mathbb{R} \to \mathbb{R}$
$f'(x) = 0$
$f''(x) > 0$
$f''(x) < 0$
$(a_1 \;\; a_2)\begin{pmatrix} f_{xx} & f_{yx} \\ f_{xy} & f_{yy} \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$
$= (a_1 f_{xx} + a_2 f_{xy},\; a_1 f_{yx} + a_2 f_{yy})\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$
$= a_1^2 f_{xx} + a_1 a_2 f_{xy} + a_1 a_2 f_{yx} + a_2^2 f_{yy}$
$\left(\frac{a_1}{a_2}\right)^2 f_{xx} + \frac{a_1}{a_2}(f_{xy} + f_{yx})$
$+ f_{yy}$

月3　微積分

No.

DATE 6.24

極値を求めよ

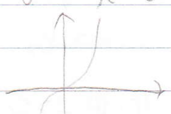
1変数　$f:\mathbb{R}\to\mathbb{R}$

$f'(x)=0$　必要条件

十分条件

$f(x)=x^3$　$f'(x)=3x^2=0$　$x=0$

$f''(x)>0$



多変数

(1) $f(x,y)=(x^2+y^2)^2-2(x^2-y^2)$

(2) $f(x,y)=xy(x^2+y^2-1)$

$\frac{\partial f}{\partial x}=0$　$\frac{\partial f}{\partial y}=0$

高階の微分

内積　$(\mathbf{a},\mathbf{b})\longmapsto \mathbf{a}\cdot\mathbf{b}$　二重線型

$(\mathbf{a}_1+\mathbf{a}_2)\mathbf{b}=\mathbf{a}_1\cdot\mathbf{b}+\mathbf{a}_2\cdot\mathbf{b}$

$(\alpha\cdot\mathbf{a})\mathbf{b}=\alpha(\mathbf{a},\mathbf{b})$　$\alpha\in\mathbb{R}$

$\mathbf{a}(\mathbf{b}_1+\mathbf{b}_2)=\mathbf{a}\cdot\mathbf{b}_1+\mathbf{a}\cdot\mathbf{b}_2$

$\mathbf{a}(\beta\mathbf{b})=\beta(\mathbf{a}\cdot\mathbf{b})$　$\beta\in\mathbb{R}$

抽象的に考える　$\begin{vmatrix}a_1 & b_1\\ a_2 & b_2\end{vmatrix}$

$\varphi:\mathbb{R}^2\times\mathbb{R}^2\to\mathbb{R}$　二重線型

$L(\mathbb{R}^2,\mathbb{R})$　$\mathbb{R}^2$から$\mathbb{R}$への線型写像の全体

$r(\mathbf{a}+\mathbf{b})=r(\mathbf{a})+r(\mathbf{b})$

$r(\alpha\mathbf{a})=\alpha r(\mathbf{a})$

$(r_1+r_2)(\mathbf{a})=r_1(\mathbf{a})+r_2(\mathbf{a})$

$(\alpha r)(\mathbf{a})=\alpha r(\mathbf{a})$

1×2の行列

$[\alpha_1,\alpha_2]\begin{pmatrix}a_1\\ a_2\end{pmatrix}$

$\mathbf{x}\in\mathbb{R}^2\longmapsto\varphi(\mathbf{x},\_)$　$\mathbb{R}^2\to\mathbb{R}$

$\varphi(\mathbf{x},\mathbf{y}_1+\mathbf{y}_2)=\varphi(\mathbf{x},\mathbf{y}_1)+\varphi(\mathbf{x},\mathbf{y}_2)$

$\varphi(\mathbf{x},\beta\mathbf{y})=\beta\varphi(\mathbf{x},\mathbf{y})$

$\varphi(\mathbf{x}_1+\mathbf{x}_2,\mathbf{y})=\varphi(\mathbf{x}_1,\mathbf{y})+\varphi(\mathbf{x}_2,\mathbf{y})$

$\varphi(\alpha\mathbf{x},\mathbf{y})=\alpha\varphi(\mathbf{x},\mathbf{y})$

$f:\mathbb{R}^2\longrightarrow\mathbb{R}$

$f(x,y)=3x^2y^5$

微分　$\mathbf{x}\in\mathbb{R}^2$

$f'(\mathbf{x})$: $\mathbb{R}^2\to\mathbb{R}$への線型写像

$f'(\mathbf{x})\in L(\mathbb{R}^2,\mathbb{R})$

$f':\mathbb{R}^2\to L(\mathbb{R}^2,\mathbb{R})$

1変数

$f:\mathbb{R}\to\mathbb{R}$　$x\in\mathbb{R}$　比例関係

$f'(x)\in \underbrace{L(\mathbb{R},\mathbb{R})}_{\mathbb{R}}$

$f''(\mathbf{x})\in L(\mathbb{R}^2, L(\mathbb{R}^2,\mathbb{R}))$　2×2行列

$=L(\mathbb{R}^2,\mathbb{R}^2;\mathbb{R})$

$\mathbb{R}^2\times\mathbb{R}^2\to\mathbb{R}$　二重線型の全体

二階の微分

内積

$(\mathbf{a},\mathbf{b})\longmapsto\mathbf{a}\cdot\mathbf{b}$　二重線型

$\mathbf{a}\cdot\mathbf{b}=\mathbf{b}\cdot\mathbf{a}$　(対称)

$|\mathbf{a}\cdot\mathbf{b}|$　行列式

$|\mathbf{a}\cdot\mathbf{b}|=-|\mathbf{b}\cdot\mathbf{a}|$　(対称でない)

$f:\mathbb{R}^2\to\mathbb{R}$　$\mathbf{x}\in\mathbb{R}$

$f'(\mathbf{x})\in L(\mathbb{R}^2,\mathbb{R})$

$f':\mathbb{R}^2\to L(\mathbb{R}^2,\mathbb{R})$

$f''(\mathbf{x})\in L(\mathbb{R}^2,L(\mathbb{R}^2,\mathbb{R}))$

$f''(\mathbf{x})(\mathbf{a})\in L(\mathbb{R}^2,\mathbb{R})$

$f''(\mathbf{x})(\mathbf{a})(\mathbf{b})\in\mathbb{R}$

$f''(\mathbf{x})(\mathbf{a},\mathbf{b})$

命題

$\boxed{f''(\mathbf{x})(\mathbf{a})(\mathbf{b})=f''(\mathbf{x})(\mathbf{b})(\mathbf{a})}$

$f(\mathbf{x}+\mathbf{a}d_1+\mathbf{b}d_2)-$

$=f(\mathbf{x}+\mathbf{a}d_1)+f'(\mathbf{x}+\mathbf{a}d_1)(\mathbf{b})d_2$

$=f(\mathbf{x})+f'(\mathbf{x})(\mathbf{a})d_1$

$+\{f'(\mathbf{x})+f''(\mathbf{x})(\mathbf{a})d_1\}(\mathbf{b})d_2$

$=f(\mathbf{x})+f'(\mathbf{x})(\mathbf{a})d_1+f'(\mathbf{x})(\mathbf{b})d_2$

$+f''(\mathbf{x})(\mathbf{a})(\mathbf{b})d_1d_2$

No.

DATE

微積分

$f(x+\mathbb{a}d_1+\mathbb{b}d_2)$

$= f(x+\mathbb{b}d_2+\mathbb{a}b_1)$

$= f(x+\mathbb{b}d_2)+f'(x+\mathbb{b}d_2)(\mathbb{a})d_1$

$= f(x)+f'(x)(\mathbb{b})d_2$

$\quad + \{ f'(x)+f''(x)(\mathbb{b})d_2 \}(\mathbb{a})d_1$

$= f(x)+f'(x)(\mathbb{b})d_2+f'(x)(\mathbb{a})d_1$

$\quad + f''(x)(\mathbb{b})(\mathbb{a})\,d_1d_2$

$\varphi: \mathbb{R}^2\times\mathbb{R}^2 \to \mathbb{R}$ への2重線型写像をどう表すか?

$\mathbb{a}=\begin{pmatrix}a_1\\a_2\end{pmatrix} \qquad \mathbb{b}=\begin{pmatrix}b_1\\b_2\end{pmatrix}$

$a_1\mathbb{e}_1+a_2\mathbb{e}_2 \qquad b_1\mathbb{e}_1+b_2\mathbb{e}_2$

$\varphi(\mathbb{a},\mathbb{b})$

$= \varphi(a_1\mathbb{e}_1+a_2\mathbb{e}_2,\ b_1\mathbb{e}_1+b_2\mathbb{e}_2)$

$= a_1b_1\varphi(\mathbb{e}_1,\mathbb{e}_1)+a_1b_2\varphi(\mathbb{e}_1,\mathbb{e}_2)$

$\quad + a_2b_1\varphi(\mathbb{e}_2,\mathbb{e}_1)+a_2b_2\varphi(\mathbb{e}_2,\mathbb{e}_2)$

$= (a_1, a_2)\begin{pmatrix}\varphi(\mathbb{e}_1,\mathbb{e}_1) & \varphi(\mathbb{e}_1,\mathbb{e}_2)\\ \varphi(\mathbb{e}_2,\mathbb{e}_1) & \varphi(\mathbb{e}_2,\mathbb{e}_2)\end{pmatrix}\begin{pmatrix}b_1\\b_2\end{pmatrix}$

$f:\mathbb{R}^2\to\mathbb{R} \qquad f_x=\frac{\partial f}{\partial x} \qquad f'(x)=[f_x, f_y]$

$f''(x)=\begin{pmatrix}f_{xx} & f_{yx}\\ f_{xy} & f_{yy}\end{pmatrix} \qquad f_{xy}=f_{yx}$

$(a_1, a_2)\begin{pmatrix}f_{xx} & f_{yx}\\ f_{xy} & f_{yy}\end{pmatrix}\begin{pmatrix}a_1\\a_2\end{pmatrix}$

$= a_1^2 f_{xx}+a_1b_2f_{yx}+a_2b_1f_{xy}$

$\quad + a_2^2 f_{yy}$

$f_{xx}>0$ 極小値

$f_{xx}<0$ 極大値

$a_2\neq 0$ a_2^2で割る

$dx\wedge dy\wedge dz$ 行列式

$\mathbb{a}=\begin{pmatrix}a_1\\a_2\end{pmatrix} \qquad \mathbb{b}=\begin{pmatrix}b_1\\b_2\end{pmatrix}$

1次元の線型空間

基底 $dx\wedge dy$ 行列式

$dx\wedge dy(\mathbb{a},\mathbb{b}) = dx(\mathbb{a})dy(\mathbb{b})-dx(\mathbb{b})dy(\mathbb{a})$

$= a_1b_2-a_2b_1 = \begin{vmatrix}a_1 & b_1\\ a_2 & b_2\end{vmatrix}$

$\varphi:\mathbb{R}^2\to\mathbb{R}$

$d\varphi = \frac{\partial\varphi}{\partial x}dx+\frac{\partial\varphi}{\partial y}dy$

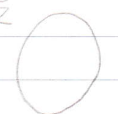
Gaussの法則 (物理) 発散0
Gaussの発散定理 (数学)
クーロンの法則 (物理)

斥力 同符号
引力 異符号

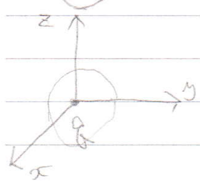
$q_1 q_2$に比例 r^2に反比例

万有引力

閉曲面で囲まれる領域 Ω



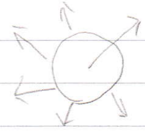
$$\underbrace{\int_\Sigma f \cdot dz}_{=0} = \underbrace{\int_\Omega (\mathrm{div}\, f)\, dV}_{=0}$$

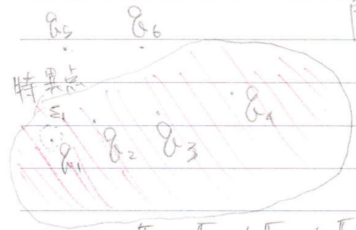


球面 $\begin{cases}\text{原点中心} \\ \text{半径 } a\end{cases}$

接平面

$$4\pi a^2 \times \frac{kq}{a^2} = 4\pi k q$$





閉曲面 Σ
面積分
電場

(重ね合わせ)

$$E = E_1 + E_2 + E_3 + E_4 + E_5 + E_6$$

$$(☆) = \int_\Sigma E \cdot dS = \int_\Sigma E_1 \cdot dS + \int_\Sigma E_2 \cdot dS + \cdots + \underbrace{\int_\Sigma E_6 \cdot dS}_{=0}$$

E_1 $\Sigma \cup \Sigma_1$

$$\int_{\Sigma \cup \Sigma_1} E \cdot dS = 0$$

$$\int_{\Sigma \cup \Sigma_1} E \cdot dS = \int_\Sigma E_1\, dS - \int_{\Sigma_1} E_1\, dS = 0$$

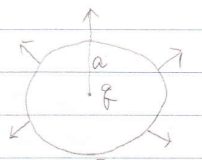
$$\int_\Sigma E_1 \cdot dS = \int_{\Sigma_1} E_1\, dS = 4\pi k q_1$$

$$(☆) = 4\pi k (q_1 + q_2 + q_3 + q_4)$$

経験的真理

$4\pi k (q_1 + q_2 + q_3 + q_4)$

Gaussの法則 $\Longleftarrow$ クーロンの法則



$$4\pi a^2 \times f = 4\pi k q$$

$$f = \frac{kq}{a^2}$$

複素関数論

$f : \mathbb{C} \to \mathbb{C}$　　$\mathbb{C} = \mathbb{R}^2$ $\mathbb{R}$上の線型空間

$f : \mathbb{R}^2 \to \mathbb{R}^2$　　違う

複素数にはかけ算が入っている。

$f(x+iy) = f_1(x+iy) + i f_2(x+iy)$

$f_1 : \mathbb{C} \to \mathbb{R}$　　$f_1(x, y)$

$f_2 : \mathbb{C} \to \mathbb{R}$

$$f'(x,y) = \begin{pmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{pmatrix}$$

$$(a+ib)(x+iy) = (ax - by) + i(bx - ay)$$

$$\begin{pmatrix} a & -b \\ b & a \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\left.\begin{aligned} \frac{\partial f_1}{\partial x} &= \frac{\partial f_2}{\partial y} \\ \frac{\partial f_2}{\partial x} &= -\frac{\partial f_1}{\partial y} \end{aligned}\right\} \text{コーシーリーマンの方程式}$$

複素数の意味で微分可能

ベクトル解析 (3次元)

2次元上での微分形式

0次元の微分形式　$\mathbb{R}^2 \to \mathbb{R}$

1次元の微分形式

$\mathbb{R}^2$の各点に1次の交代形式を対応させる。

$\mathbb{R}^2 \to \mathbb{R}$への線型写像

$$\left.\begin{aligned} dx &: \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \longmapsto a_1 \\ dy &: \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \longmapsto a_2 \end{aligned}\right\} \text{基底}$$

2次の微分形式 $\mathbb{R}^2$の各点に
2次の交代形式を対応させる関数