微分幾方程式

$D_1 = D = \{ d \in \mathbb{R} \mid d^2 = 0 \}$

$D_2 = \{ d \in \mathbb{R} \mid d^3 = 0 \}$

$\vdots$

$D_n = \{ d \in \mathbb{R} \mid d^n = 0 \}$

$(d_1, \cdots, d_n) \in D^n \mapsto d_1 + \cdots + d_n \in D_n$ 全射（天下り）

$D^n = \underbrace{D \times \cdots \times D}_{n個}$

$N \to N \quad n \mapsto n^2$ 関数（全射ではない）

$2^2 = 4$

$\left.\begin{matrix} 5 \\ 6 \end{matrix}\right\}$ 自然数の２乗ではない

$\mathbb{R}_+ = \{ r \in \mathbb{R} \mid r \geq 0 \}$

$\mathbb{R}_+ \to \mathbb{R}_+ \quad r \mapsto r^2$（全射）

$\sqrt{5}^2 = 5$

（無限次の）

多項式

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2}x^2 + \frac{f^{(3)}(0)}{3!}x^3 + \cdots + \frac{f^{(n)}(0)}{n!}x^n + \cdots$$

$$f(x+\Delta x) = f(x) + f'(x)\Delta x + \frac{f''(x)}{2}(\Delta x)^2 + \cdots + \frac{f^{(n)}(x)}{n!}(\Delta x)^n + \cdots$$

$d \in D_n \quad d^{n+1} = 0$

$$f(d) = f(0) + f'(0)d + \frac{f''(0)}{2}d^2 + \cdots + \frac{f^{(n)}(0)}{n!}d^n$$

一般的には、

$$f(x+\Delta x) = f(x) + f'(x)\Delta x + \frac{f''(x)}{2!}(\Delta x)^2 + \frac{f^{(n)}(x)}{n!}(\Delta x)^n + \frac{f^{(n+1)}(\theta)}{(n+1)!}(\Delta x)^{n+1} \quad x \leq \theta \leq x+\Delta x$$

Taylorの展開

任意 $f : \mathbb{R} \to \mathbb{R}$

$x \in \mathbb{R} \quad d \in D_n$

$$f(x+d) = f(x) + f'(x)d + \cdots + \frac{f^{(n)}(x)}{n!}d^n$$

上のものが証明されたとすると…

$x = 0 \quad d \in D_n$

$$f(d) = f(0) + f'(0)d + \frac{f''(0)}{2}d^2 + \cdots + \frac{f^{(n)}(0)}{n!}d^n$$

☆ $x' = x$（微分方程式）

$x'' = x' = x$

$x''' = x'' = x' = x$

$C = x(0) = x'(0) = x''(0) = x'''(0) = \cdots$

$d \in D_n$

$$x(d) = x(0) + x'(0)d + \frac{x''(0)}{2}d^2 + \frac{x'''(0)}{3!}d^3 + \cdots + \frac{x^{(n)}(0)}{n!}d^n$$

$$= C\left\{1 + d + \frac{d^2}{2} + \frac{d^3}{3!} + \cdots + \frac{d^n}{n!}\right\}$$

☆ $x'' = -x$

$x''' = -x'$

$x'''' = -x'' = x$

$C_1 = x(0) = -x''(0) = x''''(0) = -x^{(6)}(0) = x^{(8)}(0)$

$C_2 = x'(0) = -x^{(3)}(0) = x^{(5)}(0) = -x^{(7)}(0) = x^{(9)}(0)$

$$x(d) = C_1\left\{1 - \frac{d^2}{2} + \frac{d^4}{4!} - \frac{d^6}{6!} \cdots\right\} + C_2\left\{d - \frac{d^3}{3!} + \frac{d^5}{5!} - \frac{d^7}{7!} + \frac{d^9}{9!} \cdots\right\}$$

$f : \mathbb{R} \to \mathbb{R}$ $\quad f' : \mathbb{R} \to \mathbb{R}$

$$f(x+d) = f(x) + f'(x)d$$

$d_1, d_2 \in D$

$$f(\underline{x+d_1} + d_2) = f(x+d_1) + f'(x+d_1)d_2$$

$$= f(x) + f'(x)d_1 + \{f'(x) + f''(x)d_1\}d_2$$

$$= f(x) + f'(x)(d_1+d_2) + f''(x)d_1d_2$$

$$d_1d_2 = \frac{(d_1+d_2)^2}{2}$$

$d \in D_2$
$d_1, d_2 \in D$
$d = d_1 + d_2$

$$= f(x) + f'(x)(d_1+d_2) + \frac{f''(x)}{2}(d_1+d_2)^2$$

$$= f(x) + f'(x)d + \frac{f''(x)}{2}d^2$$

$d_1, d_2, d_3 \in D$

$$f(x+d_1+d_2+d_3)$$

$$= f(x+d_1+d_2) + f'(x+d_1+d_2)d_3$$

$$= f(x) + f'(x)(d_1+d_2) + f''(x)d_1d_2 + \{f'(x) + f''(x)(d_1+d_2) + f'''(x)d_1d_2\}d_3$$

$$= f(x) + f'(x)(d_1+d_2+d_3) + f''(x)(d_1d_2+d_1d_3+d_2d_3) + f'''(x)d_1d_2d_3$$

$$d_1d_2 + d_1d_3 + d_2d_3 = \frac{(d_1+d_2+d_3)^2}{2} \qquad d_1d_2d_3 = \frac{(d_1+d_2+d_3)^3}{3!}$$

$d_1 + d_2 + d_3 = d \in D_3$

$\neq$

$$= f(x) + f'(x)d + \frac{f''(x)}{2}d^2 + \frac{\cancel{6}\, f^{(3)}(x)}{3!}d^3$$

$X_1, \cdots, X_k$ 変数

$$\left.\begin{aligned}
\sigma_k^1(X_1, \cdots, X_k) &= X_1 + \cdots + X_k \\
\sigma_k^2(X_1, \cdots, X_k) &= \sum_{i \neq j} X_i X_j \\
&\vdots \\
\sigma_k^k(X_1, \cdots, X_k) &= X_1 X_2 \cdots X_k
\end{aligned}\right\} \text{$X_1, \cdots, X_k$ の基本対称式}$$

対称式

$k=2$ X_1, X_2

$$2X_1^2 + X_2^2 \xrightarrow{X_1, X_2 \text{ いれかえる}} 2X_2^2 + X_1^2$$

もとの式とちがう → 対称ではない

$$X_1^2 + X_2^2$$

X_1 と X_2 をいれかえても式が同じになる → 対称式

$$X_1^2 + X_2^2 = (X_1 + X_2)^2 - 2X_1X_2$$

基本対称式

(上の基本対称式の整式で書ける)

任意の対称式は基本対称式の整式に書ける

report I 来週の月曜日 D705

命題

$$\sigma_{k+1}^{n}(x_1, \cdots, x_{k+1}) = \underline{\sigma_k^n(x_1, \cdots, x_k)} + \underline{x_{k+1}}\, \sigma_k^{n-1}(x_1, \cdots, x_k)$$

$k=2,\ n=2$

$$(x_1x_2 + x_1x_3 + x_2x_3) = \quad x_1x_2 \quad\quad x_3\ (x_1 + x_2)$$

$$\left(C_r^{n+1} = C_r^n + C_{r-1}^n \right)$$

$\underbrace{\circ\ \circ \cdots\cdots \bullet}_{n+1\ 個}$ ●をふくむのとふくまないものに分ける

命題

$$f(x + d_1 + \cdots + d_k) = \sum_{n\geq 0}^{k} f^{(n)}(x)\, \sigma_k^n(d_1, \cdots, d_k)$$

証明 (by induction on k)

$$f(x + d_1 + \cdots + d_k + d_{k+1}) = f(x + d_1 + \cdots + d_k) + d_{k+1} f'(x)(x + d_1 + \cdots + d_k)$$

$$= \sum_{n=0}^{k} f^{(n)}(x)\, \sigma_k^n(d_1, \cdots, d_k) + d_{k+1} \sum_{n=0}^{k} f^{(n+1)}(x)\, \sigma_k^n(d_1, \cdots, d_k)$$

$$= \sum_{n=0}^{k+1} f^{(n)}(x) \left\{ \sigma_k^n(d_1, \cdots, d_k) + d_{k+1}\, \sigma_k^{n-1}(d_1, \cdots, d_k) \right\}$$

$$= \sum_{n=0}^{k+1} f^{(n)}(x)\, \sigma_{k+1}^n(d_1, \cdots, d_k, d_{k+1})$$

命題　任意の n, k について、 $d_1, \cdots, d_k \in D$

$$(d_1 + d_2 + \cdots + d_k)^n = n!\,\sigma_k^n(d_1, \cdots, d_k)$$

proof (by induction on k)

Report II.

$d_1 + d_2 + \cdots + d_k = d$ とおけば Taylor の定理になる

微分方程式

$D_1 = D = \{ d \in \mathbb{R} \mid d^2 = 0 \}$

$D_2 = \{ d \in \mathbb{R} \mid d^3 = 0 \}$

$\vdots$

$D_n = \{ d \in \mathbb{R} \mid d^{n+1} = 0 \}$

$(d_1, \ldots, d_n) \in D^n \longmapsto d_1 + \cdots + d_n \in \cancel{D_n}\ D_{n}$ 全射（天下り）

$\underbrace{D \times \cdots \times D}_{n\text{個}}$

$\mathbb{R}_+ \longrightarrow \mathbb{R}_+$

$N \longrightarrow N$

$\mathbb{R}_+ = \{ r \in \mathbb{R} \mid r \geq 0 \}$

$r \longmapsto r^2$ （全射です）

$n \longmapsto n^2$ 関数（全射ではない）

$2^2 = 4$ $\sqrt{5}$ 5

5

6

(無限次の)多項式

$$f(x)=f(0)+f'(0)x+\frac{f''(0)}{2}x^2+\frac{f^{(3)}(0)}{3!}x^3+\cdots+\frac{f^{(n)}(0)}{n!}x^n+\cdots$$

$$f(x+\Delta x)=f(x)+f'(x)\Delta x+\frac{f''(x)}{2}(\Delta x)^2+\cdots+\frac{f^{(n)}(x)}{n!}(\Delta x)^n+\cdots$$

$d\in D_n$ $\quad d^{n+1}=0$

$$f(d)=f(0)+f'(0)d+\frac{f''(0)}{2}d^2+\cdots+\frac{f^{(n)}(0)}{n!}d^n \quad //$$

Taylorの定理

任意 $f:\mathbb{R}\to\mathbb{R}$

$x\in\mathbb{R}$ $\quad d\in D_n$

$$f(x+d)=f(x)+f'(x)d+\cdots\frac{f^{(n)}(x)}{n!}d^n$$

$$x=0 \quad d \in D_n$$

$$f(d) = f(0) + f'(0)d + \frac{f''(0)}{2} d^2 + \quad + \frac{f^{(n)}(0)}{n!} d^n$$

$x' = x$ （微分方程式）

$$x'' = x' = x$$

$$x''' = x'' = x' = x$$

$$= x(0) = x'(0) = x''(0) = x'''(0) =$$

$$x'' = -x$$

$$x''' = -x'$$

$$x'''' = -x'' = x$$

$$x''''' = x'''' = -x''' = x'$$

$$d \in D_n$$

$$x(d) = x(0) + x'(0)d + \frac{x''(0)}{2} d^2 + \frac{x'''(0)}{3!} d^3 + \quad + \frac{x^{(n)}(0)}{n!} d^n$$

$$= C\left\{1 + d + \frac{d^2}{2} + \frac{d^3}{3!} + \quad + \frac{d^n}{n!}\right\} \qquad d^{n+1} = 0$$

$$C_1 = x(0) = -x''(0) = x''''(0) = -x^{(6)}(0) = x^{(8)}(0)$$

$$C_2 = x'(0) = -x^{(3)}(0) = x^{(5)}(0) = -x^{(7)}(0) = x^{(9)}(0)$$

$$x(d) = C_1\left\{1 - \frac{d^2}{2} + \frac{d^4}{4!} - \frac{d^6}{6!} \cdots\right\}$$

$$+ C_2\left\{d - \frac{d^3}{3!} + \frac{d^5}{5!} - \frac{d^7}{7!} + \frac{d^9}{9!} \cdots\right\}$$

$f: \mathbb{R} \to \mathbb{R}$ $\quad f': \mathbb{R} \to \mathbb{R}$ $\quad d \in D_2$

$f(x+d) = f(x) + f'(x)d$ $\quad d_1, d_2 \in D$ $\quad d_1 d_2 = \frac{(d_1+d_2)^2}{2}$

$d_1, d_2 \in D$ $\quad d = d_1 + d_2$

$$f(\underline{x+d_1}+d_2) = f(x+d_1) + \underline{f'(x+d_1)}\, d_2$$

$$= f(x) + f'(x)d_1 + \{f'(x) + f''(x)d_1\} d_2$$

$$= f(x) + f'(x)(d_1+d_2) + f''(x)\, d_1 d_2$$

$$= f(x) + f'(x)(d_1+d_2) + f''(x) \ldots)^2$$

Taylor の定理

任意 $f: \mathbb{R} \to \mathbb{R}$

$x \in \mathbb{R}$ $\quad d \in D_n$

$$f(x+d) = f(x) + f'(x)d + \ldots d^n$$

$$d_1, d_2, d_3 \in D \qquad f' : \mathbb{R} \to \mathbb{R}$$

$$f(x+d_1+d_2+d_3)$$
$$= f(x+d_1+d_2) + f'(x+d_1+d_2)\,d_3$$
$$= f(x) + f'(x)(d_1+d_2) + f''(x)\,d_1 d_2$$
$$+ \left\{ f'(x) + f''(x)(d_1+d_2) + f'''(x)\,d_1 d_2 \right\} d_3$$
$$= f(x) + f'(x)(d_1+d_2+d_3) + f''(x)(d_1d_2 + d_1d_3 + d_2d_3)$$
$$+ f'''(x)\,d_1 d_2 d_3$$

$$d_1d_2 + d_1d_3 + d_2d_3 = \frac{(d_1+d_2+d_3)^2}{2}$$

$$d_1 d_2 d_3 = \frac{(d_1+d_2+d_3)^3}{3!}$$

$$d_1 + d_2 + d_3 = d \in D_3$$

$$= f(x) + f'(x)\,d + \frac{f''(x)}{2} d^2 + \frac{f^{(3)}(x)}{3!} d^3$$

$X_1, \ldots, X_k$ 変数　$k=2$

X_1, X_2　対称ではない

$2X_1^2 + X_2^2$　$2X_2^2 + X_1^2$

$\sigma_k^1(X_1, \ldots, X_k) = X_1 + \cdots + X_k$

$\sigma_k^2(X_1, \ldots, X_k) = \sum_{i \neq j} X_i X_j$

$\sigma_k^k(X_1, \ldots, X_k) = X_1 X_2 \cdots X_k$

整式

$X_1, \ldots, X_k$ の

基本対称式

$X_1^2 + X_2^2 = (X_1 + X_2)^2 - 2X_1X_2$

任意の対称式は基本対称式の整...

Newton の定理

Taylor の定理

任意　$f: \mathbb{R} \to \mathbb{R}$

$x \in \mathbb{R}$　$d \in D_n$

$$f(x+d) = f(x) + f'(x)d + \cdots + \frac{f^{(n)}(x)}{n!}d^n$$

$$f(x+\Delta x) = f(x) + f'(x)\Delta x + \frac{f''(x)}{\ldots}\Delta x^2 \ldots$$

$$\frac{f^{(n)}(\ldots)}{\ldots}$$

誤差項

$\sqrt{2}$

口伝

$\leq \theta \leq$

命題 $n=2$ $k=2$

$C^{n+1}_{r} = C^{n}_{r} + C^{n}_{r-1}$

証明して下さい。 analogy

月 D705

$n+1$個

$$\boxed{\sigma^{n}_{k+1}(X_1, \ldots, X_{k+1}) = \sigma^{n}_{k}(X_1, \ldots, X_k) + X_{k+1}\,\sigma^{n-1}_{k}(X_1, \ldots, X_k)}$$

report I

$X_1X_2 + X_1X_3 + X_2X_3 = X_1X_2 + X_3 \times (X_1 + X_2)$

X_1, X_7

命題

$$f(x+d_1+\cdots+d_k)=\sum_{n=0}^{k} f^{(n)}(x)\,\sigma_k^n(d_1,\ldots,d_k)$$

証明 by induction on k $\qquad f'$

$$f(x+d_1+\cdots+d_k+d_{k+1})=f(x+d_1+\cdots+d_k)+d_{k+1}f'(x+d_1+\cdots+d_k)$$

$$=\sum_{n=0}^{k} f^{(n)}(x)\,\sigma_k^n(d_1,\ldots,d_k)+d_{k+1}\sum_{n=0}^{k} f^{(n+1)}(x)\,\sigma_k^n(d_1,\ldots,d_k)$$

$$\sum f^{(n)}(x)\left\{\sigma_k^n(d_1,\ldots,d_k)+d_{k+1}\,\sigma_k(d_1,\ldots,d_k)\right\}$$

$$(d_1,\ldots,d_k,d_{k+1})$$

Taylor の定理

任意 $f:\mathbb{R}\to\mathbb{R}$

$x\in\mathbb{R}$ $\qquad d\in D_n$

$$f(x+d)=f(x)+f'(x)d+\cdots+\frac{f^{(n)}(x)}{n!}d^n$$

$\sqrt{2}$

口伝

誤差

命題 任意 n, k $d_1, \cdots, d_k \in D$

$d \in D_k$

$(d_1 + d_2 + \cdots + d_k)^n = n!\, \sigma_k^n (d_1, \cdots, d_k)$

proof by induction on k II