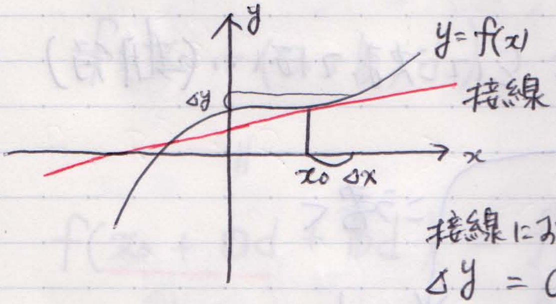
微積分　2学期（2コマ目）

多変数の微分

曲がっている ⇒ まっすぐ



2ンスえ

接線においては

$\Delta y = a \Delta x$（比例関係）

線型写像　代数的にとり扱う

$\mathbb{R} \to \mathbb{R}$　線型代数

$\mathbb{R}^2 \to \mathbb{R}$

$(\Delta x, \Delta y) \mapsto \Delta z$

Kock（コック）-Lawvere（ラビアー）の公理

$$\boxed{\begin{array}{l} \forall f : D \to \mathbb{R} \\ (\exists! a \in \mathbb{R})(\forall d \in D) \quad (\underbrace{f(d) = f(0) + ad}_{\text{直線}}) \end{array}}$$

$f : \mathbb{R} \to \mathbb{R} \quad x_0 \in \mathbb{R}$

$d \in D \mapsto f(x_0 + d) \in \mathbb{R}$

~~∃a!~~ $(\exists! a \in \mathbb{R})(\forall d \in D)$

$f(x_0 + d) = f(x_0) + \underline{a}d$

$f'(x_0)$: f の x_0 における微分係数

$f : \mathbb{R}^n \to \mathbb{R}^m$

$$\left(\begin{array}{l} f : D \to \mathbb{R}^m \begin{pmatrix} f_1 \\ \vdots \\ f_m \end{pmatrix} \\ \quad f_i : D \to \mathbb{R} \\ (\exists! a \in \mathbb{R}^m)(\forall d \in D) \quad (f(d) = f(0) + ad) \end{array} \right)$$

$x_0 \in \mathbb{R}^n$でfを微分

$a \in \mathbb{R}^n \to \mathbb{R}^m$への線型写像として決まってほしい(期待)

$f'(x_0)(a)$ こう書く

$d \in D \mapsto f(x_0 + ad) \in \mathbb{R}^m$

$$f(x_0 + ad) = f(x_0) + \bigcirc d$$

$\cap$

$\mathbb{R}_m$

$f'(x_0) : \mathbb{R}^n \to \mathbb{R}^m$ 線型?

$a, b \in \mathbb{R}^n$

1) $f'(x_0)(a + b) = f'(x_0)(a) + f'(x_0)(b)$

2) $f'(x_0)(\alpha a) = \alpha f'(x_0)(a)$

2)からやる

ひっつける

$$f(x_0 + \alpha a d) = f(x_0) + f'(x_0)(\alpha a) d$$

$$\|$$

$$f(x_0 + a(\underset{\substack{\uparrow \\ D}}{\alpha d})) = f(x_0) + f'(x_0)(a)(\alpha d)$$

$d \in D \to \alpha d \in D$ ($\because (\alpha d)^2 = \alpha^2 d^2 = 0$)

よって $f'(x_0)(\alpha a) d = f'(x_0)(a)(\alpha d)$

より成立

お知らせ $\frac{10}{26}$ (水) は休講

次に 1) をやる。

$$f(x_0+(a+b)d) = f(x_0) + f'(x_0)(a+b)d$$

$$\parallel$$

$$f(\underline{x_0 + a}d + bd) = f(x_0+ad) + f'(x_0+ad)(b)d$$

$$= f(x_0) + f'(x_0)(a)d$$

$$+ (f'(x_0) + f''(x_0)(a)d)(b)d$$

$f'(x_0)(b)d$

dを2回かけるので0になる

$$= f(x_0) + \{f'(x_0)(a) + f'(x_0)(b)\}d$$

よって $f'(x_0)(a+b)d = \{f'(x_0)(a) + f'(x_0)(b)\}d$

より成立

$$f : \mathbb{R}^n \to \mathbb{R}^m$$

$x_0 \in \mathbb{R}^n$ $f'(x_0) : \mathbb{R}^n \to \mathbb{R}^m$ 線型写像

行列 $m\left\{\overbrace{\left(\quad\quad\right)}^{n}\right.$ $m\times n$ の行列

$e_1 = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$ n個

$e_2 = \begin{pmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{pmatrix}$

$\vdots$

$e_n = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}$

$$f(x_0 + \underbrace{e_1 d}_{\begin{pmatrix} d \\ 0 \\ \vdots \\ 0 \end{pmatrix}}) = f(x_0) + f'(x_0)(e_1)d$$

1変数

偏微分 $\frac{\partial f}{\partial x_1}(x_0) \in \mathbb{R}^m$

$$m \times n \left[\frac{\partial f}{\partial x_1}(x_0) \frac{\partial f}{\partial x_2}(x_0) \cdots \frac{\partial f}{\partial x_n}(x_0) \right]$$

$$\underset{(x_1, \cdots, x_n)}{f : \mathbb{R}^n} \to \underset{(y_1, \cdots, y_m)}{\mathbb{R}^m}$$

$$y_1 = y_1(x_1, \cdots, x_n)$$

$$\vdots$$

$$y_m = y_m(x_1, \cdots, x_n)$$

$$\begin{pmatrix} \frac{\partial y_1}{\partial x_1} & \frac{\partial y_1}{\partial x_2} & \cdots & \frac{\partial y_1}{\partial x_n} \\ & \vdots & & \\ \frac{\partial y_m}{\partial x_1} & \frac{\partial y_m}{\partial x_2} & \cdots & \frac{\partial y_m}{\partial x_n} \end{pmatrix}$$

合成関数の微分

$g : \mathbb{R}^m \to \mathbb{R}^l$

$g \circ f$ 合成 $: \mathbb{R}^n \to \mathbb{R}^l$

~~x_0~~ $x_0 \in \mathbb{R}^n$ $\quad a \in \mathbb{R}^n$

$$g \circ f(x_0 + a d) = g(f(x_0 + a d))$$

$$= g(f(x_0) + \underbrace{f'(x_0)(a)}_{\in \mathbb{R}^m} d)$$

$$= g(f(x_0)) + \underbrace{g'(f(x_0))(f'(x_0)(a))}_{= \left(g'(f(x_0)) \circ f'(x_0) \right)(a)} d$$

参 $\left(\begin{array}{l} x \to y \\ y \to z \\ \\ \dfrac{dz}{dx} = \dfrac{dz}{dy} \dfrac{dy}{dx} \end{array} \right)$

$$\mathbb{R}^n \to \mathbb{R}^m \to \mathbb{R}^l$$

$$\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \quad \begin{pmatrix} y_1 \\ \vdots \\ y_m \end{pmatrix} \quad \begin{pmatrix} z_1 \\ \vdots \\ z_l \end{pmatrix}$$

掛け算

$$\begin{pmatrix} \dfrac{\partial y_1}{\partial x_1} & \cdots & \dfrac{\partial y_1}{\partial x_n} \\ \vdots & & \vdots \\ \dfrac{\partial y_m}{\partial x_1} & \cdots & \dfrac{\partial y_m}{\partial x_n} \end{pmatrix} \quad \begin{pmatrix} \dfrac{\partial z_1}{\partial y_1} & \cdots & \dfrac{\partial z_1}{\partial y_m} \\ \vdots & & \\ \dfrac{\partial z_l}{\partial y_1} & \cdots & \dfrac{\partial z_l}{\partial y_m} \end{pmatrix}$$

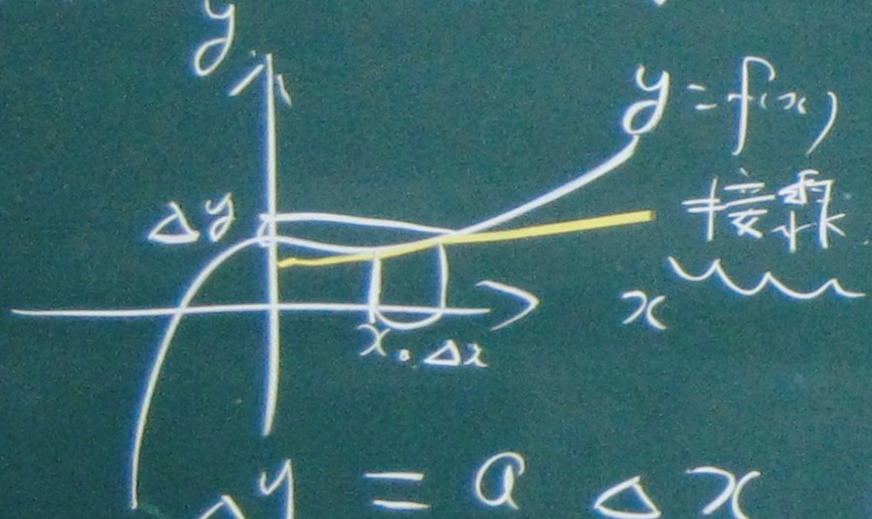
偏微分

$$\boxed{\frac{\partial z_k}{\partial x_i} = \frac{\partial z_k}{\partial y_1}\frac{\partial y_1}{\partial x_i} + \frac{\partial z_k}{\partial y_2}\frac{\partial y_2}{\partial x_i} + \cdots + \frac{\partial z_k}{\partial y_m}\frac{\partial y_m}{\partial x_i}}$$

変数の微分　　巾零無限小

2変数　3次元

曲がってる ⇒ まっすぐ



2次元

$\Delta y = a\,\Delta x$ (比例関係)

線型写像　　代数的

$\mathbb{R} \to \mathbb{R}$　　線型代…

$\mathbb{R}^2 \to \mathbb{R}$　　コック　　ラビアー

$(\Delta x, \Delta y) \mapsto \Delta z$

Kock-Lawvereの公理

$$\boxed{\begin{array}{l}(\forall f : D \longrightarrow \mathbb{R})\\ (\exists !\, a \in \mathbb{R})(\forall d \in D)\\ (\underline{f(d) = f(0) + ad})\end{array}}$$

直線

$f : \mathbb{R} \to \mathbb{R}$　　$x_0 \in \mathbb{R}$

$d \in D \mapsto f(x_0 + d) \in \mathbb{R}$

$(\exists !\, a \in \mathbb{R})(\forall d \in D)$

$(f(x_0 + d) = f(x_0) + \underline{a}\,d)$

$f'(x_0)$

fのx_0における微分係数

$f : \mathbb{R}^n \to \mathbb{R}^m$

$f : D \to \mathbb{R}^m \qquad \begin{pmatrix} f_1 \\ \vdots \\ f_m \end{pmatrix}$

$f_i : D \to \mathbb{R}$

$(\exists!\, a \in \mathbb{R}^m)(\forall d \in D)$

$(f(d) = f(0) + a d)$

$x_0 \in \mathbb{R}^n$ で f を微分

$a \in \mathbb{R}^n \to \mathbb{R}^m$ への線型写像（期待）

$f'(x_0)(a)$ こう書く

$d \in D \mapsto f(x_0 + a d) \in \mathbb{R}^m$

$f(x_0 + a d) = f(x_0) + \bigcirc$

$f'(x_0): \mathbb{R}^n \to \mathbb{R}^m$ 線形？

$a, b \in \mathbb{R}^n$

1) $f'(x_0)(a+b) = f'(x_0)(a) + f'(x_0)(b)$

$\alpha \in \mathbb{R}$

2) $f'(x_0)(\alpha a) = \alpha f'(x_0)(a)$

2) からやる

ひっつける

$$f(x_0 + \alpha a d) = f(x_0) + f'(x_0)(\alpha a)\, d$$

$$\|$$

$$f(x_0 + a(\alpha d)) = f(x_0) + f'(x_0)(a)(\alpha d)$$

$d \in D \to \alpha d \in D$ 次に1)をやります

$(\alpha d)^2 = \alpha^2 d^2 = 0$

f'

$$f(x_0 + (a+b)d) = f(x_0) + f'(x_0)(a+b)\, d$$

$$\|$$

$$f(x_0 + ad + bd) = f(x_0 + ad) + f'(x_0 + ad)(b)\, d$$

$d^2 = 0$

$$= f(x_0) + f'(x_0)(a)d + \left(f'(x_0) + f''(x_0)(a)d\right)(b)\, d$$

$$f'(x_0)(b)d = f(x_0) + \{f'(x_0)(a) + f'(x_0)(b)\}d$$

10月 24日〜26日 (23日〜27日)

$f: \mathbb{R}^n \to \mathbb{R}^m$

$x_0 \in \mathbb{R}^n$

$f'(x_0): \mathbb{R}^n \to \mathbb{R}^m$ 線型写像

行列

$m\left(\overbrace{\left(\quad\right)}^{n}\right)$ $m \times n$の行列

$\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$ n個

$\mathbf{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$ $\mathbf{e}_n = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}$

$m \times n$ $\left[\frac{\partial f}{\partial x_1}(x_0) \; \frac{\partial f}{\partial x_2}(x_0) \; \cdots \; \frac{\partial f}{\partial x_n}(x_0) \right]$

変数

$f(x_0 + \underline{\mathbf{e}_1 d}) = f(x_0) + f'(x_0)(\mathbf{e}_1)d$

$\begin{pmatrix} d \\ 0 \\ \vdots \\ 0 \end{pmatrix}$ 偏微分 $\frac{\partial f}{\partial x_1}(x_0) \in \mathbb{R}^m$

$f: \mathbb{R}^n \to \mathbb{R}^m$

$(x_1, \dots, x_n) \quad (y_1, \dots, y_m)$

$y_1 = y_1(x_1, \dots, x_n)$

$\vdots$

$y_m = y_m(x_1$

$$\begin{pmatrix} \frac{\partial y_1}{\partial x_1} & \frac{\partial y_1}{\partial x_2} & \cdots & \frac{\partial y_1}{\partial x_n} \\ \vdots & \vdots & & \\ \frac{\partial y_m}{\partial x_1} & \frac{\partial y_m}{\partial x_2} & \cdots & \frac{\partial y_m}{\partial x_n} \end{pmatrix}$$

合成関数の微分

$g: \mathbb{R}^m \to \mathbb{R}^l$

$x.$

合成

$\to \mathbb{R}^l$

$x_0 \in \mathbb{R}^n \quad a \in \mathbb{R}^n$

$$g \circ f(x_0 + a d) = g(f(x_0 + a d))$$

$$= g(f(x_0) + \underbrace{f'(x_0)(a)}_{\in \mathbb{R}^m} d)$$

$$= g(f(x_0)) + \underline{g'(f(x_0))(f'(x_0)(a))}\, d$$

$$(\underline{g'(f(x_0))} \circ \underline{f'(x_0)})(a)$$

$x \to y$

$y \to z$

$$\frac{dz}{dx} = \frac{dz}{dy} \frac{dy}{dx}$$

掛け算
$\mathbb{R}^n \longrightarrow \mathbb{R}^m \longrightarrow \mathbb{R}^l$
$\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$ n $\begin{pmatrix} y_1 \\ \vdots \\ y_m \end{pmatrix}$ $\begin{pmatrix} z_1 \\ \vdots \\ z_l \end{pmatrix}$
$m \begin{pmatrix} \frac{\partial y_1}{\partial x_1} & \cdots & \frac{\partial y_1}{\partial x_n} \\ \vdots & & \vdots \\ \frac{\partial y_m}{\partial x_1} & \cdots & \frac{\partial y_m}{\partial x_n} \end{pmatrix}$
1×1
m
$l \begin{pmatrix} \frac{\partial z_1}{\partial y_1} & \cdots & \frac{\partial z_1}{\partial y_m} \\ \vdots & & \vdots \\ & & \frac{\partial z_l}{\partial y_m} \end{pmatrix}$
1×1 1×1
$\frac{\partial z_k}{\partial x_i} =$
$\frac{\partial z_1}{\partial y_1} \frac{\partial y_1}{\partial x_1} +$