

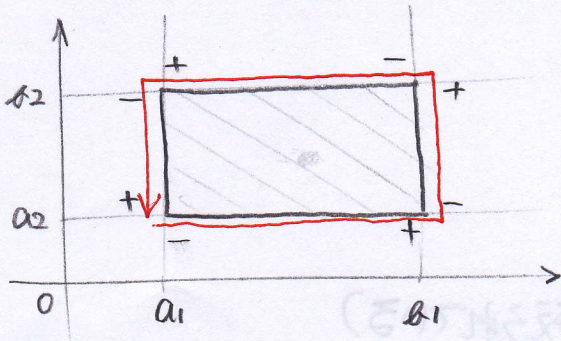
Stokesの定理

曲面 \$\Sigma\$

0次の微分形式 (スカラー場) \$\varphi\$

$$\int_{\Sigma} d(d\varphi) = \int_{\partial\Sigma} d\varphi = \int_{\partial(\partial\Sigma)} \varphi$$

$$\Sigma [a_1, b_1] \times [a_2, b_2] \rightarrow \mathbb{R}^3$$



$$\int_{\partial(\partial\Sigma)} \varphi = 0$$

\$\nabla\$ + ブラ (nabla) $\begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix}$ 作用素のベクトル, 擬似ベクトル

$$\text{grad} \varphi = \nabla \varphi$$

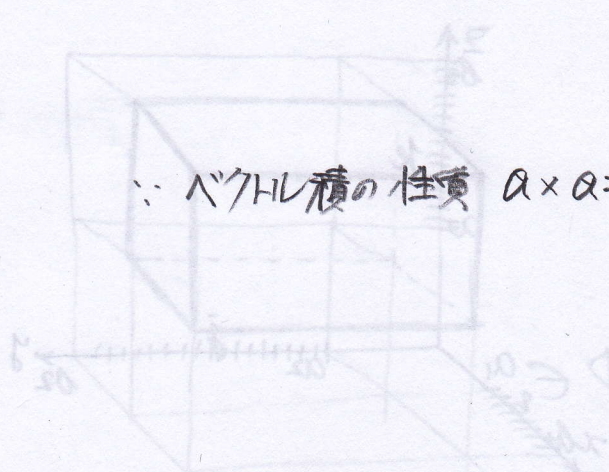
$$\text{rot} f = \nabla \times f$$

$$\text{rot}(\text{grad}) = \nabla \times (\nabla \varphi)$$

$$= (\nabla \times \nabla) \varphi$$

$$= 0$$

\$\therefore\$ ベクトル積の性質 \$a \times a = 0\$



向きつけ

曲線

線

線

曲面

向きつけ ⇒ どちらを表とするか?

閉曲面

面の外側を表とする

空間

右手系
左手系

div.

閉曲面に囲まれた領域 Ω

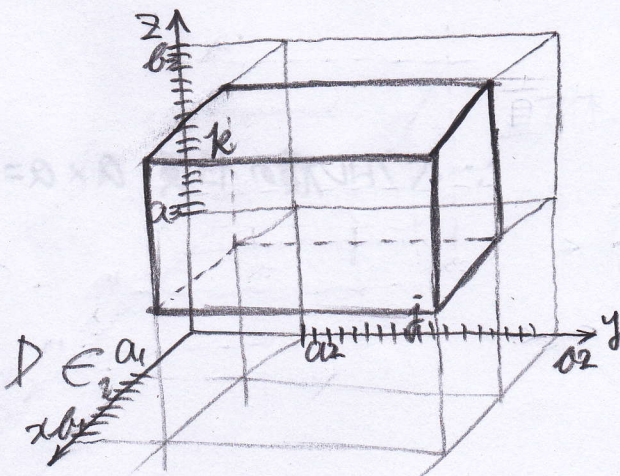
2次の微分形式

$$\int_{\Omega} dw = \int_{\partial\Omega} w \quad (w \text{ が与えられている})$$

こういう3次の微分形式があたらいい!! (願望)

無限小のlevelで成り立てば一般に成り立つ

$$\Omega: [a_1, b_1] \times [a_2, b_2] \times [a_3, b_3] \rightarrow \mathbb{R}^3$$

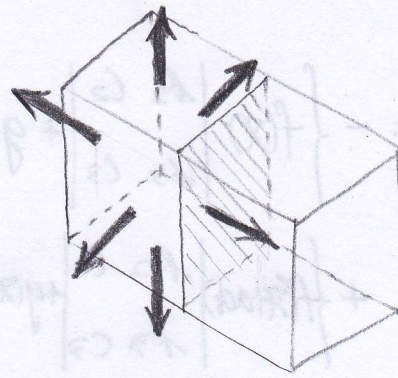


直方体

Ω は Ω_{ijk} に分割される。

$$\int_{\Omega} dw = \sum_{ijk} \int_{\Omega_{ijk}} dw$$

$$= \sum_{ijk} \int_{\partial\Omega_{ijk}} w$$



divergence (発散)

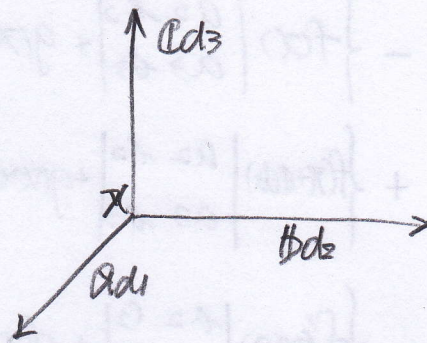
Gaussの発散定理

Lagrange

平行六面体 Ω

$$x, a, b, c \in \mathbb{R}^3$$

$$d_1, d_2, d_3 \in D$$



$$w = f dy \wedge dz + g dz \wedge dx + h dx \wedge dy$$

$$\int_{\Sigma} w = \int_{\partial\Sigma} w$$

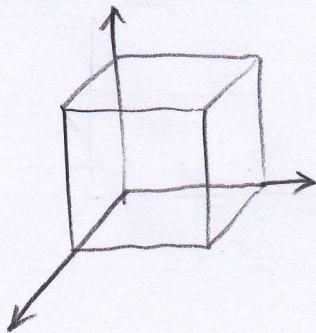
これを計算 面積分を体積分に変換する

が成立するおりに dw を決める <Report>

(計算結果を示しておく...)

$$dw = \left(\frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} + \frac{\partial h}{\partial z} \right) dz \wedge dy \wedge dx$$

面積分の計算 (以前のreport)



立方体

$$\int_{\partial\Omega} \mathbf{r} \cdot d\mathbf{S} = \int_{\Omega} \text{div } \mathbf{r} \, dV$$

$$\mathbf{r} = (x, y, z) \quad f=x, g=y, h=z \quad 1+1+1=3$$

(定数)

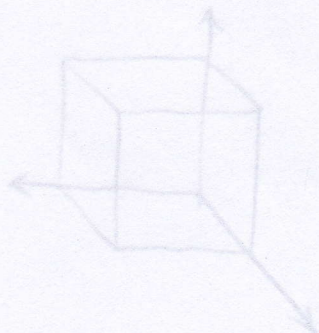
$$= 3 \int_{\Omega} dV$$

↓
立方体の体積

$$\begin{aligned}
\int_{\partial\Omega} w &= - \left\{ f(x) \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} + g(x) \begin{vmatrix} b_3 & c_3 \\ a_1 & c_1 \end{vmatrix} + h(x) \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} \right\} d_2 d_3 \\
&+ \left\{ f(x+ad_1) \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} + g(x+ad_1) \begin{vmatrix} b_3 & c_3 \\ a_1 & c_1 \end{vmatrix} + h(x+ad_1) \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} \right\} d_2 d_3 \\
&+ \left\{ f(x) \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + g(x) \begin{vmatrix} a_3 & c_3 \\ a_1 & c_1 \end{vmatrix} + h(x) \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} \right\} d_1 d_3 \\
&- \left\{ f(x+bd_1) \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + g(x+bd_1) \begin{vmatrix} a_3 & c_3 \\ a_1 & c_1 \end{vmatrix} + h(x+bd_1) \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} \right\} d_1 d_3 \\
&- \left\{ f(x) \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix} + g(x) \begin{vmatrix} a_3 & b_3 \\ a_1 & b_1 \end{vmatrix} + h(x) \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \right\} d_1 d_2 \\
&+ \left\{ f(x+cd_1) \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix} + g(x+cd_1) \begin{vmatrix} a_3 & b_3 \\ a_1 & b_1 \end{vmatrix} + h(x+cd_1) \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \right\} d_1 d_2 \\
&= \left\{ f'(x) \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} + g'(x) \begin{vmatrix} b_3 & c_3 \\ a_1 & c_1 \end{vmatrix} + h'(x) \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} \right\} d_2 d_3 \\
&+ \left\{ f'(x) \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + g'(x) \begin{vmatrix} a_3 & c_3 \\ a_1 & c_1 \end{vmatrix} + h'(x) \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} \right\} d_1 d_3 \\
&+ \left\{ f'(x) \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix} + g'(x) \begin{vmatrix} a_3 & b_3 \\ a_1 & b_1 \end{vmatrix} + h'(x) \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \right\} d_1 d_2
\end{aligned}$$

$$f'(x) = \frac{\partial f}{\partial x}(x) dx + \frac{\partial g}{\partial y}(x) dy + \frac{\partial h}{\partial z}(x) dz \quad (\text{链式计算})$$

<2> 升降问题 I



$$= \left\{ \begin{vmatrix} f(x)(a) \\ g(x)(a) & b & c \\ h(x)(a) \end{vmatrix} + \begin{vmatrix} a & f(x)(b) \\ g(x)(b) & c \\ h(x)(b) \end{vmatrix} + \begin{vmatrix} a & b & f(x)(c) \\ g(x)(c) \\ h(x)(c) \end{vmatrix} \right\} dx dy dz$$

$$\varphi: \mathbb{R}^3 \times \mathbb{R}^3 \times \mathbb{R}^3 \rightarrow \mathbb{R}$$

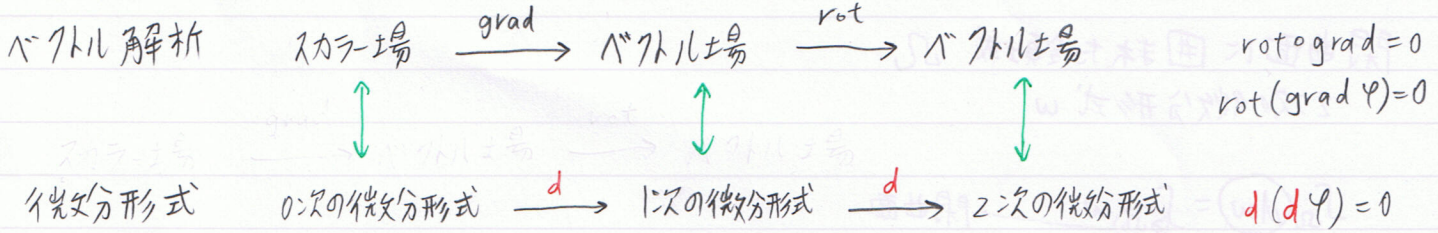
$$\varphi = \underline{\alpha} dx \wedge dy \wedge dz$$

$$a = e_1, b = e_2, c = e_3$$

<以降内題II>

$\varphi(a, b, c)$ 三重線型 交代

12/8 (水) 3限 微積分 (生物学類)



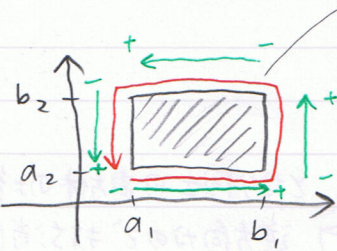
Stokes の定理

曲面 Σ

0次の微分形式 φ
(スカラー場)

$$\int_{\Sigma} d(d\varphi) = \int_{\partial\Sigma} d\varphi = \int_{\partial(\partial\Sigma)} \varphi = 0$$

$$\Sigma : [a_1, b_1] \times [a_2, b_2] \rightarrow \mathbb{R}^3$$



それぞれの点 が 2回 + と - で 出てくるので
全部足し合わせると 0

∇ ナブラ
nabla

$$\begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix}$$

作用素のベクトル
擬似ベクトル

$$\text{grad } \varphi = \nabla \varphi$$

$$\text{rot } \mathbf{f} = \nabla \times \mathbf{f}$$

$$\begin{aligned} \text{rot}(\text{grad } \varphi) &= \nabla \times (\nabla \varphi) \\ &= (\nabla \times \nabla) \varphi \\ &= 0 \end{aligned}$$

ベクトル積の性質
 $\mathbf{a} \times \mathbf{a} = 0$

向きづけ



曲線

曲面

ふろしき

どちらを表にしますか?

空間

右手系

左手系

閉曲線



閉曲面

囲われている領域

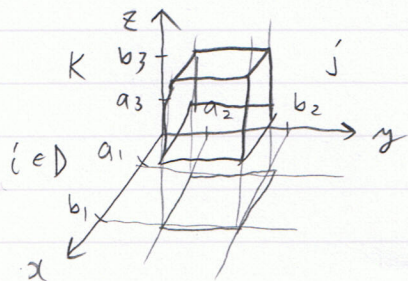
(外側を表とする)

閉曲面に囲まれた領域 Ω
 2次の微分形式 w

$\int_{\partial\Omega} dw = \int_{\partial\Omega} w$ 閉曲面
 こういう3次の微分形式を定めた

無限小の level で成り立つは一般に成り立つ

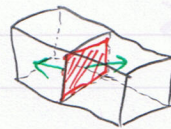
$\Omega : [a_1, b_1] \times [a_2, b_2] \times [a_3, b_3] \rightarrow \mathbb{R}^3$



Ω は Ω_{ijk} に分割される

$$\int_{\Omega} dw = \sum_{ijk} \int_{\Omega_{ijk}} dw$$

$$= \sum_{i,j,k} \int_{\partial\Omega_{ijk}} w$$

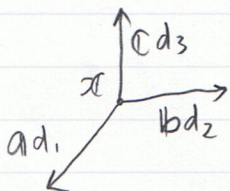


となり同士の直方体の境界面は
 逆方向なので打ち消し合う

打ち消し合わないのは一番外側の大きな
 直方体の境界面

div (ergence) 発散

Gauss の発散定理



$d_1, d_2, d_3 \in D \quad x, a, b, c \in \mathbb{R}^3$

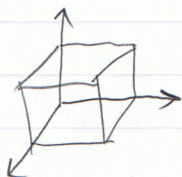
平行六面体 Ω

$w = f dy \wedge dz + g dz \wedge dx + h dx \wedge dy$

$\int_{\Sigma} dw = \int_{\partial\Sigma} w$ が成り立つように dw を決める
 これを計算

$$dw = \left(\frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} + \frac{\partial h}{\partial z} \right) dx \wedge dy \wedge dz$$

面積分の計算 (以前のreport)



立方体

$$\int \mathbf{v} \cdot d\mathbf{S}$$

$$\mathbf{v} = (x, y, z)$$

$$f = x \quad g = y \quad h = z$$

$$1+1+1=3 \text{ (定数関数)}$$

$$\int_{\partial \Omega} \mathbf{v} \cdot d\mathbf{S} = \int_{\Omega} \overset{=3}{\text{div } \mathbf{v}} \cdot dV$$

面積分 体積分

$$\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \quad \mathbf{c} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$$

$$\int_{\partial \Omega} \omega = - \left\{ f(x) \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} + g(x) \begin{vmatrix} b_3 & c_3 \\ b_1 & c_1 \end{vmatrix} + h(x) \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} \right\} d_2 d_3$$

$$+ \left\{ f(x+a_1 d_1) \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} + g(x+a_1 d_1) \begin{vmatrix} b_3 & c_3 \\ b_1 & c_1 \end{vmatrix} + h(x+a_1 d_1) \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} \right\} d_2 d_3$$

$$+ \left\{ f(x) \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + g(x) \begin{vmatrix} a_3 & c_3 \\ a_1 & c_1 \end{vmatrix} + h(x) \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} \right\} d_1 d_3$$

$$- \left\{ f(x+b_1 d_2) \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + g(x+b_1 d_2) \begin{vmatrix} a_3 & c_3 \\ a_1 & c_1 \end{vmatrix} + h(x+b_1 d_2) \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} \right\} d_1 d_3$$

$$- \left\{ f(x) \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix} + g(x) \begin{vmatrix} a_3 & b_3 \\ a_1 & b_1 \end{vmatrix} + h(x) \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \right\} d_1 d_2$$

$$+ \left\{ f(x+c_1 d_3) \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix} + g(x+c_1 d_3) \begin{vmatrix} a_3 & b_3 \\ a_1 & b_1 \end{vmatrix} + h(x+c_1 d_3) \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \right\} d_1 d_2$$

$$= \left\{ f'(x)(a_1) \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} + g'(x)(a_1) \begin{vmatrix} b_3 & c_3 \\ b_1 & c_1 \end{vmatrix} + h'(x)(a_1) \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} \right\} d_1 d_2 d_3$$

$$- \left\{ f'(x)(b_1) \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + g'(x)(b_1) \begin{vmatrix} a_3 & c_3 \\ a_1 & c_1 \end{vmatrix} + h'(x)(b_1) \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} \right\} d_1 d_2 d_3$$

$$+ \left\{ f'(x)(c) \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix} + g'(x)(c) \begin{vmatrix} a_3 & b_3 \\ a_1 & b_1 \end{vmatrix} + h'(x)(c) \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \right\} d_1 d_2 d_3$$

$$\left. \begin{aligned} f'(x) &= \frac{\partial f}{\partial x}(x) dx + \frac{\partial f}{\partial y}(x) dy + \frac{\partial f}{\partial z}(x) dz \\ g'(x) &= \frac{\partial g}{\partial x}(x) dx + \frac{\partial g}{\partial y}(x) dy + \frac{\partial g}{\partial z}(x) dz \\ h'(x) &= \frac{\partial h}{\partial x}(x) dx + \frac{\partial h}{\partial y}(x) dy + \frac{\partial h}{\partial z}(x) dz \end{aligned} \right\} \uparrow \text{代入}$$

$$= \left\{ \begin{vmatrix} f'(x)(a) & b & c \\ g'(x)(a) & & \\ h'(x)(a) & & \end{vmatrix} + \begin{vmatrix} a & f'(x)(b) & c \\ & g'(x)(b) & \\ & h'(x)(b) & \end{vmatrix} + \begin{vmatrix} a & b & f'(x)(c) \\ & & g'(x)(c) \\ & & h'(x)(c) \end{vmatrix} \right\} d_1 d_2 d_3$$

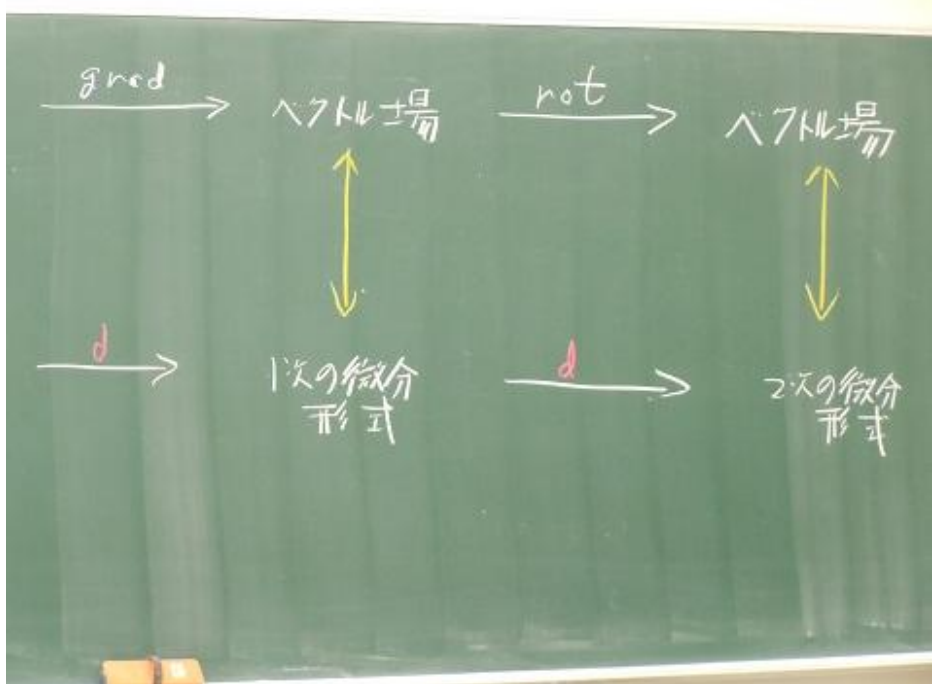
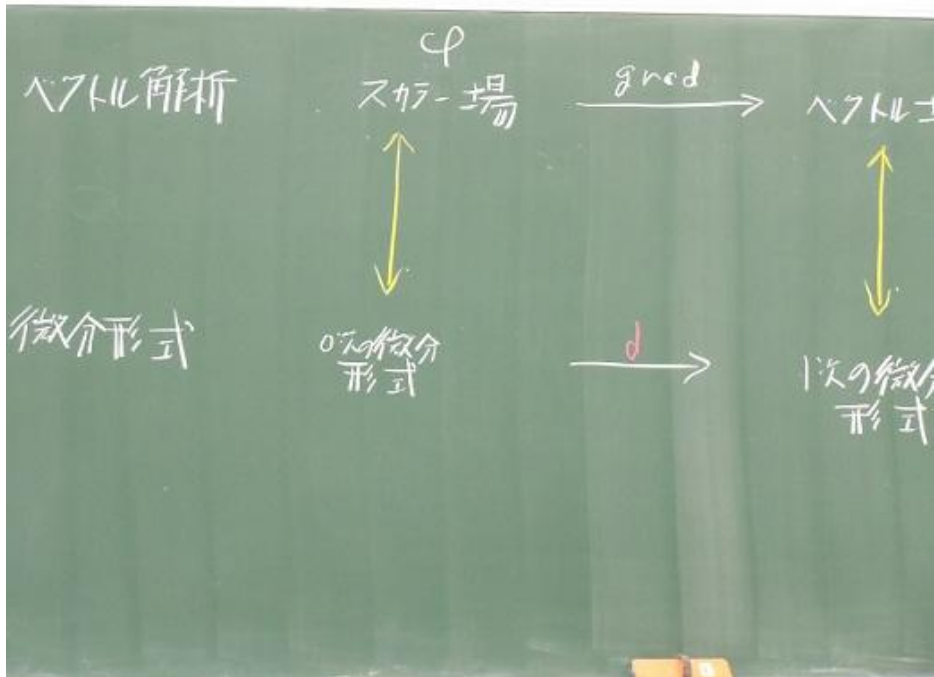
$$\varphi: \mathbb{R}^3 \times \mathbb{R}^3 \times \mathbb{R}^3 \rightarrow \mathbb{R}$$

$\varphi(a, b, c)$ 3重線形
交代

$$\varphi = \alpha dx \wedge dy \wedge dz$$

\uparrow
 α を決めたとき

$$a = e_1, b = e_2, c = e_3 \text{ を代入する}$$



\wedge フォーム場 $\text{rot} \circ \text{grad} = 0$
 $\text{rot}(\text{grad } \varphi) = 0$
 $d(d\varphi) = 0$
 2-次の微分形式

Stokes の定理
 曲面 Σ
 0-次の微分形式 (スカラー場) φ
 $0 = \int_{\Sigma} d(d\varphi)$
 $= \int_{\partial\Sigma} d\varphi = \int_{\partial\Sigma} \varphi = 0$

$\Sigma = [a_1, b_1] \times [a_2, b_2]$

$\Sigma: [a_1, b_1] \times [a_2, b_2] \rightarrow \mathbb{R}^3$

$\nabla + \vec{r}$
 $nabla$

$\text{grad } \varphi =$
 $\text{rot } \vec{f} =$
 $\text{rot}(\text{grad } c$

$= 0$

$\nabla + \vec{r}$
 $nabla$

$\begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix}$

作用素のベクトル
 類似ベクトル
 ベクトル積の外積

$\text{grad } \varphi = \nabla \varphi$
 $\text{rot } \vec{f} = \nabla \times \vec{f}$

$\text{rot}(\text{grad } \varphi) = \nabla \times (\nabla \varphi)$
 $= (\nabla \times \nabla) \varphi$
 $= 0$

$\alpha \times \alpha = 0$

向きつけ

曲線 曲面
向き 向き
向きと向きを表に合わせる

閉曲面

閉曲面

曲面 向き
向きと向きを表に合わせるか?

化学式 空間 rot
糖 右手系 左手系

閉曲面 向き
2次の微分

閉曲面
外側を表

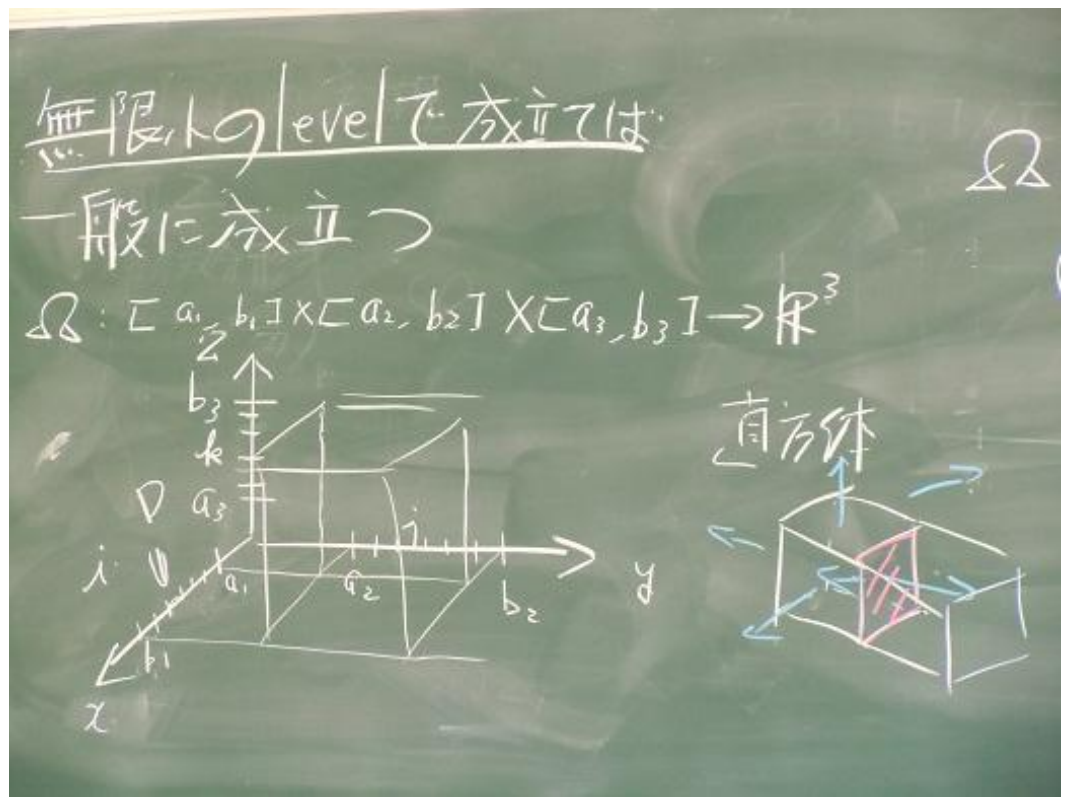
$\int_{\partial \Omega}$

向きと向きを合わせる

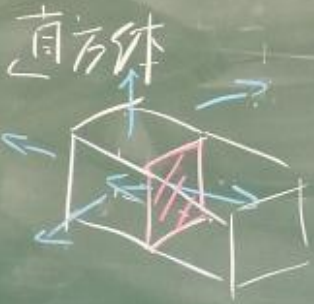
rot 閉曲面に囲まれた div
 領域 Ω
 右手系
 左手系 2-次の微分形式 ω が与えらる...

$\int_{\Omega} d\omega = \int_{\partial\Omega} \omega$

2-次の微分形式が
 あらうよ!!!



Ω は Ω_{ijk} に分割される
 $\rightarrow \mathbb{R}^3$
 直方体



$$\int_{\Omega} d\omega = \sum_{i,j,k} \int_{\Omega_{ijk}} d\omega$$

$$= \sum_{i,j,k} \int_{\partial \Omega_{ijk}} \omega$$

Ω_{ijk} に分割される
 $\omega = \sum_{i,j,k} \int_{\Omega_{ijk}} d\omega$
 $= \sum_{i,j,k} \int_{\partial \Omega_{ijk}} \omega$

div (ergence)
 発散
 Gauss の発散定理
 Lagrange

$x, a, b, c \in \mathbb{R}$ $d_1, d_2, d_3 \in D$



平行六面体 Ω

$$\omega = f dx \wedge dy + g dy \wedge dz + h dz \wedge dx$$

$$\int_{\Sigma} d\omega = \int_{\partial \Sigma} \omega$$

$d\omega \in \mathbb{R}^3$

$$h = (x, y, z)$$

$$f = x \quad g = y \quad h = z$$

$$1+1+1=3 \text{ (定数)}$$

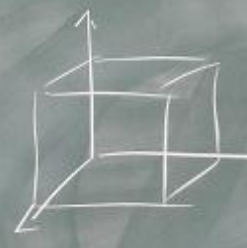
$$d\omega = f dx \wedge dy + g dy \wedge dz + h dz \wedge dx$$

$$d\omega = \left(\frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} + \frac{\partial h}{\partial z} \right) dx \wedge dy \wedge dz$$

ω が成立する
 \leftarrow 二つの条件

面積分の計算 (rep)

立方体



$$\int_{\partial \Omega} h \cdot dS$$

(x, y, z)
 $h = z$
 (定数関数)

$$h dx \wedge dy$$

$$d\omega = \left(\frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} + \frac{\partial h}{\partial z} \right) dx \wedge dy \wedge dz$$

に

4/

面積分の計算 (report)



立方体

$$\int_{\partial \Omega} h \cdot dS = \int_{\Omega} \text{div} h \, dV$$

$$\int_{\partial \Omega} \omega = - \left\{ f(x) \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} + g(x) \begin{vmatrix} b_3 & c_3 \\ b_1 & c_1 \end{vmatrix} \right.$$

$$+ \left\{ f(x + a d_1) \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} + g(x + a d_1) \begin{vmatrix} b_3 & c_3 \\ b_1 & c_1 \end{vmatrix} \right.$$

$$+ \left\{ f(x) \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + g(x) \begin{vmatrix} a_3 & c_3 \\ a_1 & c_1 \end{vmatrix} + \right.$$

$$\left. - \left\{ f(x + b d_2) \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + g(x + b d_2) \begin{vmatrix} a_3 & c_3 \\ a_1 & c_1 \end{vmatrix} \right\} \right.$$

$$\left. - \left\{ f(x) \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix} + g(x) \begin{vmatrix} a_3 & b_3 \\ a_1 & b_1 \end{vmatrix} + h(x) \begin{vmatrix} a_1 & a_2 \end{vmatrix} \right\} \right.$$

$$+ g(x) \left| \begin{array}{c} b_3 \\ b_1 \end{array} \right| \begin{array}{c} c_3 \\ c_1 \end{array} \left| + h(x) \left| \begin{array}{c} b_1 \\ b_2 \end{array} \right| \begin{array}{c} c_1 \\ c_2 \end{array} \right| \left\{ d_2, d_3 \right.$$

$$\left| \begin{array}{c} + g(x + a d_1) \\ g_3 \end{array} \right| \begin{array}{c} c_3 \\ c_1 \end{array} \left| + h(x + a d_1) \left| \begin{array}{c} b_1 \\ b_2 \end{array} \right| \begin{array}{c} c_1 \\ c_2 \end{array} \right|$$

$$\left| \begin{array}{c} a_3 \\ a_1 \end{array} \right| \begin{array}{c} c_3 \\ c_1 \end{array} \left| + h(x) \left| \begin{array}{c} a_1 \\ a_2 \end{array} \right| \begin{array}{c} c_1 \\ c_2 \end{array} \right| \left\{ d_1, d_3 \right.$$

$$f(x + b d_2) \left| \begin{array}{c} a_3 \\ a_1 \end{array} \right| \begin{array}{c} c_3 \\ c_1 \end{array} \left| + h(x + b d_2) \left| \begin{array}{c} a_1 \\ a_2 \end{array} \right| \begin{array}{c} c_1 \\ c_2 \end{array} \right| \left\{ d_1, d_3 \right.$$

$$h(x) \left| \begin{array}{c} a_1 \\ a_2 \end{array} \right| \begin{array}{c} b_1 \\ b_2 \end{array} \left| \left\{ d_1, d_2 \right.$$

$$\left| \begin{array}{c} b_1 \\ b_2 \end{array} \right| \begin{array}{c} c_1 \\ c_2 \end{array} \left| \left\{ d_2, d_3 \right. \quad a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$$

$$\left| \begin{array}{c} c_3 \\ c_1 \end{array} \right| + h(x + a d_1) \left| \begin{array}{c} b_1 \\ b_2 \end{array} \right| \begin{array}{c} c_1 \\ c_2 \end{array} \left| \left\{ d_2, d_3 \right.$$

$$\left| \begin{array}{c} c_1 \\ c_2 \end{array} \right| \left\{ d_1, d_3 \right.$$

$$h(x + b d_2) \left| \begin{array}{c} a_1 \\ a_2 \end{array} \right| \begin{array}{c} c_1 \\ c_2 \end{array} \left| \left\{ d_1, d_3 \right.$$

$$\begin{aligned}
 & + \left\{ f(x + \mathbb{C}d_3) \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix} + g(x + \mathbb{C}d_3) \begin{vmatrix} a_1 & b_1 \\ a_3 & b_3 \end{vmatrix} \right. \\
 = & \left\{ f'(x)(a) \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} + g'(x)(a) \begin{vmatrix} b_3 & c_3 \\ b_1 & c_1 \end{vmatrix} \right. \\
 & - \left\{ f'(x)(b) \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + g'(x)(b) \begin{vmatrix} a_3 & c_3 \\ a_1 & c_1 \end{vmatrix} \right. \\
 & + \left\{ f'(x)(c) \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} + g'(x)(c) \begin{vmatrix} b_3 & c_3 \\ b_1 & c_1 \end{vmatrix} \right. \\
 & \left. \left. \begin{matrix} a_2 & b_2 \\ a_3 & b_3 \end{matrix} \right\} \right.
 \end{aligned}$$

$$\begin{aligned}
 & g(x + \mathbb{C}d_3) \begin{vmatrix} a_3 & b_3 \\ a_1 & b_1 \end{vmatrix} + h(x + \mathbb{C}d_3) \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \left\{ d_1 d_2 \right. \\
 & f'(x)(a) \begin{vmatrix} b_3 & c_3 \\ b_1 & c_1 \end{vmatrix} + h'(x)(a) \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} \left\{ d_1 d_2 d_3 \right. \\
 & f'(x)(b) \begin{vmatrix} a_3 & c_3 \\ a_1 & c_1 \end{vmatrix} + h'(x)(b) \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} \left\{ d_1 d_2 \right. \\
 & f'(x)(c) \begin{vmatrix} b_3 & c_3 \\ b_1 & c_1 \end{vmatrix} + h'(x)(c) \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} \left\{ d_1 d_2 d_3 \right.
 \end{aligned}$$

$$\begin{aligned}
 & (x + \mathbb{C}d_3) \left| \begin{array}{cc} a_1 & b_1 \\ a_2 & b_2 \end{array} \right| \left\{ d_1 d_2 \right. \\
 & (a) \left| \begin{array}{cc} b_1 & c_1 \\ b_2 & c_2 \end{array} \right| \left\{ d_1 d_2 d_3 \right. \\
 & (b) \left| \begin{array}{cc} a_1 & c_1 \\ a_2 & c_2 \end{array} \right| \left\{ d_1 d_2 d_3 \right. \\
 & (c) \left| \begin{array}{cc} b_1 & c_1 \\ b_2 & c_2 \end{array} \right| \left\{ d_1 d_2 d_3 \right.
 \end{aligned}$$

$$\begin{aligned}
 f'(x) &= \frac{\partial f(x)}{\partial x} dx \\
 &+ \frac{\partial f(x)}{\partial y} dy \\
 &+ \frac{\partial f(x)}{\partial z} dz
 \end{aligned}$$

$$= \left| \begin{array}{cc|c} f'(x)(a) & & \mathbb{C} \\ g'(x)(a) & b & \\ h'(x)(a) & & \end{array} \right| + \left| \begin{array}{cc|c} f'(x)(b) & & \mathbb{C} \\ g'(x)(b) & a & \\ h'(x)(b) & & \end{array} \right|$$

$\varphi: \mathbb{R}^3 \times \mathbb{R}^3 \times \mathbb{R}^3 \rightarrow \mathbb{R}$ $\varphi = \alpha dx \wedge dy \wedge dz$
 $\varphi(a, b, c)$ 三重积 c_1 $h(x) \left| \begin{array}{cc} a_1 & b_1 \\ a_2 & b_2 \end{array} \right|$
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$$\begin{aligned}
 & (x) \left| \begin{array}{cc} a_2 & b_2 \\ a_3 & b_3 \end{array} \right| + g(x) \left| \begin{array}{cc} a_3 & b_3 \\ a_1 & b_1 \end{array} \right| + h(x) \left| \begin{array}{cc} a_1 & b_1 \\ a_2 & b_2 \end{array} \right| \left\{ d_1 d_2 d_3 \right. \\
 & + g(x + b d_2) \left| \begin{array}{cc} a_3 & c_3 \\ a_1 & c_1 \end{array} \right| + \dots
 \end{aligned}$$

$$\begin{array}{l} f'(x)(b) \\ g'(x)(b) \\ h'(x)(b) \end{array} + \left(\begin{array}{c} \text{C} \\ + \\ \text{a} \quad b \end{array} \right) \begin{array}{l} f'(x)(a) \\ g'(x)(a) \\ h'(x)(a) \end{array}$$

$$\varphi = x dx + y dy + z dz \quad a = \mathbb{P}_1, \quad b = \mathbb{P}_2$$

$$\begin{array}{l} c_1 \quad h(x) \quad \left| \begin{array}{l} a_1 \quad c_1 \\ a_2 \quad c_2 \end{array} \right| \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} d_1 d_3 \\ (x + b d_2) \quad \left| \begin{array}{l} a_3 \quad c_3 \\ a_1 \quad c_1 \end{array} \right| + h(x + b d_2) \quad \left| \begin{array}{l} a_1 \quad c_1 \\ a_2 \quad c_2 \end{array} \right| \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} d_1 \\ (x) \quad \left| \begin{array}{l} a_1 \quad b_1 \\ a_2 \quad b_2 \end{array} \right| \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} d_1 d_2 \end{array}$$

$$\begin{array}{l} a \quad b \quad \left. \begin{array}{l} f'(x)(a) \\ g'(x)(a) \\ h'(x)(a) \end{array} \right\} d_1 d_2 d_3 \end{array}$$

$$a = \mathbb{P}_1, \quad b = \mathbb{P}_2, \quad c = \mathbb{P}_3$$

$$\left(\begin{array}{c} d_1 d_3 \\ + b d_2 \end{array} \right) \left| \begin{array}{l} a_1 \quad c_1 \\ a_2 \quad c_2 \end{array} \right| \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} d_1 d_3$$