球の表面積(一変数).

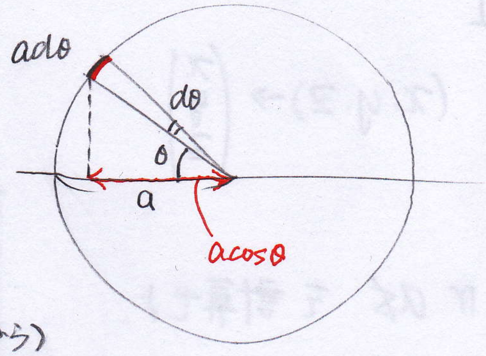
上半分 $0 \leqq \theta \leqq \frac{\pi}{2}$

$$2\int_0^{\frac{\pi}{2}} 2\pi a^2 \cos\theta \, d\theta$$

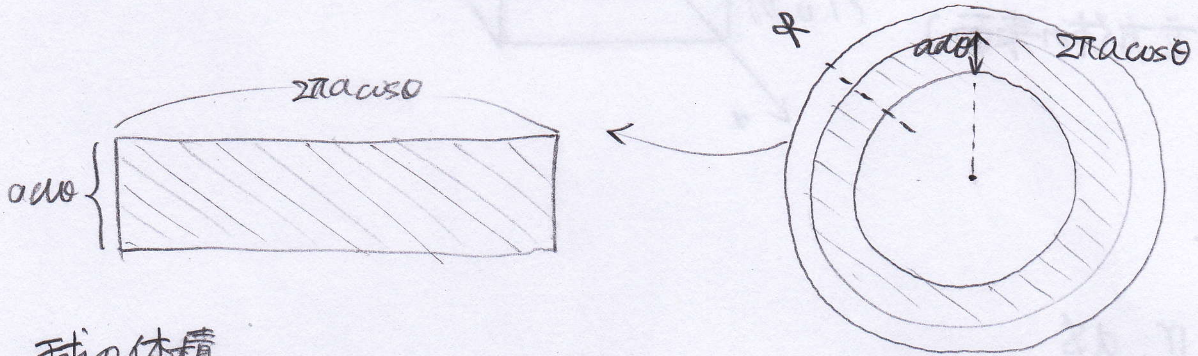
$$=4\pi a^2 \left[\sin\theta\right]_0^{\frac{\pi}{2}}$$

$$=4\pi a^2$$

(断面)

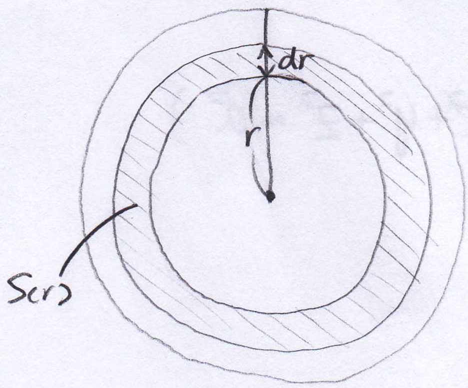


(上から)



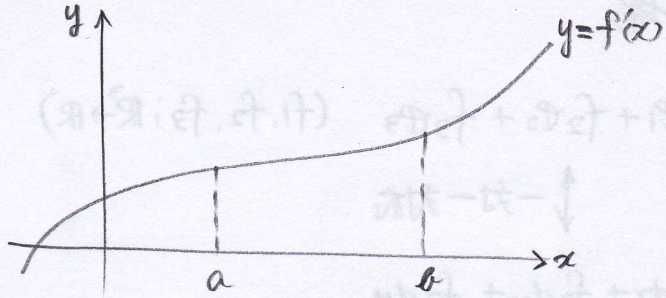
球の体積

$$\int_0^a S(r)\, dr = \cdots = \frac{4}{3}\pi a^3$$

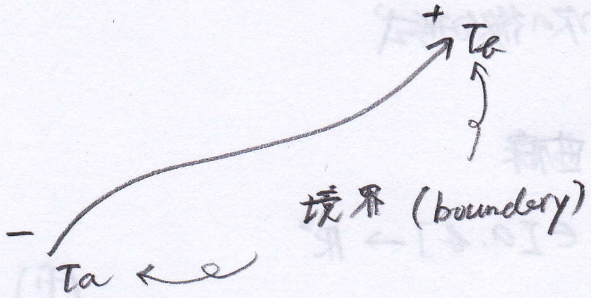


微積分学の基本定理

$$\int_a^b f(x)\,dx = f(b) - f(a)$$



曲線 $\tau : [a, b] \to \mathbb{R}^3$



スカラー場 = 0次の微分形式

$\varphi : \mathbb{R}^3 \to \mathbb{R}$

$d\varphi$: 1次の微分形式

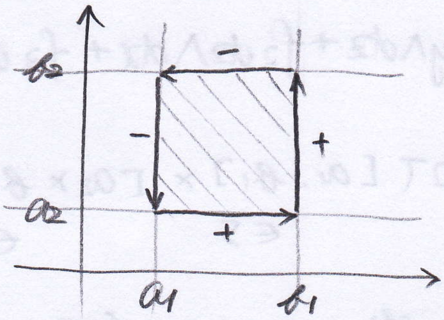
$\mathbb{R}^3 \to L(\mathbb{R}^3 ; \mathbb{R})$

→ $\mathbb{R}^3$から$\mathbb{R}$への線型写像の全体

$\varphi \xrightarrow{\text{微分}} d\varphi$

0次, 1次の微分形式

$$\int_\tau d\varphi = \varphi(\tau(b)) - \varphi(\tau(a)) = \int_{\partial\tau} \varphi$$



$\omega \xrightarrow{\text{微分}} d\omega$

1次, 2次の微分形式

$$\int_\tau d\omega = \int_{\partial\tau} \omega$$

↓

こういうものがあればいいな!(願望)

ベクトル場

$f_1\mathbb{e}_1+f_2\mathbb{e}_2+f_3\mathbb{e}_3 \quad (f_1, f_2, f_3 : \mathbb{R}^3\to\mathbb{R})$

$\updownarrow$ 一対一対応

$f_1dx+f_2dy+f_3dy$

1次の微分形式

T: 曲線

$t\in[a,b]\to\mathbb{R}^3$

$$\int_T \mathbb{f}\, d\mathbb{r} \qquad \mathbb{f}=\begin{pmatrix}f_1\\f_2\\f_3\end{pmatrix}$$

$$=\int_a^b \mathbb{f}(T(t))\,T'(t)\,dt$$

(水の)流れの場

$f_1\,dy\wedge dz+f_2\,dz\wedge dx+f_3\,dx\wedge dy=\omega$

曲面 T $\underset{\in s}{[a_1, b_1]}\times\underset{\in t}{[a_2\times b_2]}\to\mathbb{R}^3$

$$\int_{a_2}^{b_2}\int_{a_1}^{b_1}\mathbb{f}(T(s,t))\left(\frac{\partial T}{\partial s}\times\frac{\partial T}{\partial t}\right)ds\,dt=\int_T \mathbb{f}\,d\mathbb{S}$$

$$\int_{a_2}^{b_2}\int_{a_1}^{b_1}\omega(T(s,t))\left(\frac{\partial T}{\partial s}\times\frac{\partial T}{\partial t}\right)ds\,dt=\int_T \omega$$

1次の微分形式

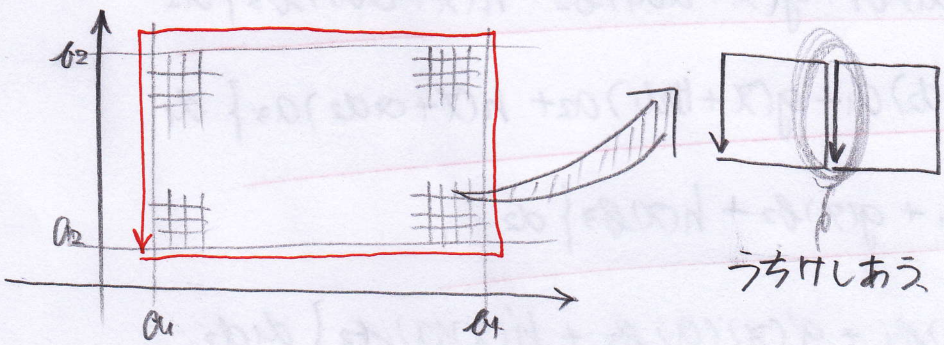
$$w = f\,dx + g\,dy + h\,dz \xrightarrow{\text{微分}} dw$$

曲面 $\tau : [a_1, b_1] \times [a_2, b_2] \rightarrow \mathbb{R}^3$

$$\int_\tau dw = \int_{\partial\tau} w$$ 願望

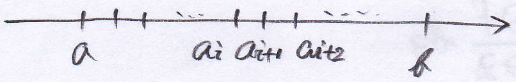
↑

無限小のlevelで成立すると仮定



$$\int_\tau dw = \sum\sum dw = w$$

$$F' = f$$



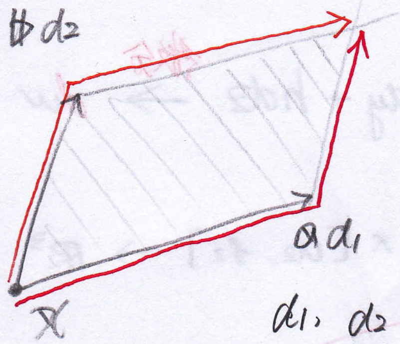
$$dif(a_i) = \int_{a_i}^{a_{i+1}} f(x)\,dx = \cancel{F(a_{i+1})} - F(a_i)$$

$$\int_{a_{i+1}}^{a_{i+2}} f(x)\,dx = F(a_{i+2}) - \cancel{F(a_{i+1})}$$

1次の微分形式

$w = f dx + f dy + f dz$

$a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix},\ b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$



$d_1, d_2 \in D$

$a, b, x \in \mathbb{R}^3$

$$\{f(x)a_1 + g(x)a_2 + h(x)a_3\}$$

$$+ \{f(x+ad_1)b_1 + g(x+ad_1)b_2 + h(x+ad_1)b_3\} d_2$$

$$- \{f(x+bd_2)a_1 + g(x+bd_1)a_2 + h(x+ad_2)a_3\} d_1$$

$$- \{f(x)b_1 + g(x)b_2 + h(x)b_3\} d_2$$

$$= \{f'(x)(a)b_1 + g'(x)(a)b_2 + h'(x)(a)b_2\} d_1 d_2$$

$$- \{f'(x)(b)a_1 + g'(x)(b)a_2 + h'(x)(b)a_3\} d_1 d_2$$

$$\left(f'(x) = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial z} dz \right)$$

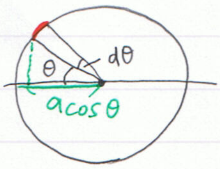
$$\therefore f'(x)(b) = \frac{\partial f}{\partial x} b_1 + \frac{\partial f}{\partial y} b_2 + \frac{\partial f}{\partial z} b_3$$

<Report : 以降計算> (水曜まで)

11/17(水) 3限　　微積分(生物学類)

球の表面積

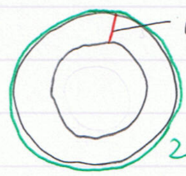
半径 a



上半分

$0 \leq \theta \leq \frac{\pi}{2}$

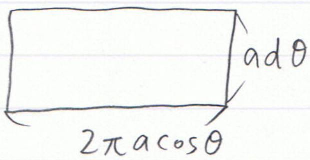
上から見ると



$a d\theta$ ← 弧の長さ = 半径 × 角度

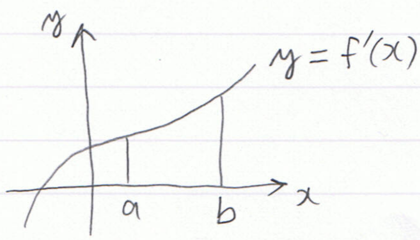
$2\pi a\cos\theta$

長方形



$$2\int_0^{\frac{\pi}{2}} 2\pi a^2 \cos\theta \, d\theta = 4\pi a^2 \left[\sin\theta\right]_0^{\frac{\pi}{2}} = 4\pi a^2$$

微積分学の基本定理



$$\boxed{\int_a^b f'(x)\,dx = f(b) - f(a)}$$

曲線 $\tau : [a,b] \to \mathbb{R}^3$

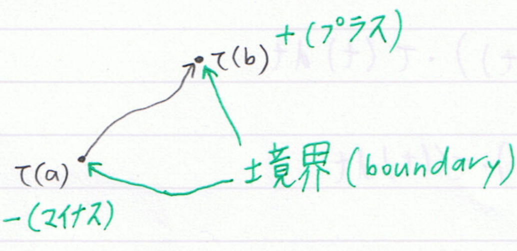
スカラー場 = 0次の微分形式

$\varphi : \mathbb{R}^3 \to \mathbb{R}$

$d\varphi$: 1次の微分形式

$\mathbb{R}^3 \to \underline{L(\mathbb{R}^3; \mathbb{R})}$

$\mathbb{R}^3$ から $\mathbb{R}$ への線型写像全体



$$\int_\tau d\varphi = \varphi(\tau(b)) - \varphi(\tau(a)) = \int_{\partial\tau} \varphi$$

境界での積分

$\varphi \xrightarrow{\text{微分}} d\varphi$

NO.

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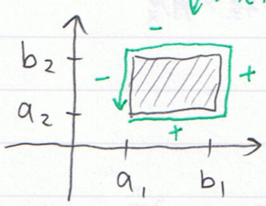
$$w \xrightarrow{\text{微分}} dw$$

1次の微分形式　　　　2次の微分形式

曲面 $\tau : [a_1, b_1] \times [a_2, b_2] \longrightarrow \mathbb{R}^3$

$\int_\tau dw = \int_{\partial\tau} w$ が成り立つように dw を定めたい

境界



ベクトル場（力の場）

$f_1, f_2, f_3 : \mathbb{R}^3 \to \mathbb{R}$

$f_1 e_1 + f_2 e_2 + f_3 e_3$

$\Updownarrow$ 1対1 対応

$f_1 dx + f_2 dy + f_3 dz$（1次の微分形式）

τ：曲線

$t \in [a, b] \longrightarrow \mathbb{R}^3$　　　　$\mathbb{f} = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix}$

$\int_\tau \mathbb{f} \cdot d\mathbb{r}$　　線積分

$= \int_a^b \mathbb{f}(\tau(t)) \cdot \tau'(t)\, dt$

$\int_a^b w(\tau(t)) \cdot \tau'(t)\, dt$

流水の場

$f_1\, dy \wedge dz + f_2\, dz \wedge dx + f_3\, dx \wedge dy$

曲面 $\tau : [\underset{s}{\underset{\Psi}{a_1, b_1}}] \times [\underset{t}{\underset{\Psi}{a_2, b_2}}] \longrightarrow \mathbb{R}^3$

$\int_\tau \mathbb{f} \cdot dS = \int_{a_2}^{b_2}\int_{a_1}^{b_1} \mathbb{f}(\tau(s,t)) \cdot \left(\frac{\partial\tau}{\partial s} \times \frac{\partial\tau}{\partial t}\right) ds\, dt$

$\int_\tau w = \int_{a_2}^{b_2}\int_{a_1}^{b_1} w(\tau(s,t)) \left(\frac{\partial\tau}{\partial s}, \frac{\partial\tau}{\partial t}\right) ds\, dt$

1次の微分形式

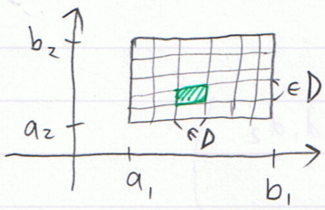
$$w = f\,dx + g\,dy + h\,dz \xrightarrow{d(微分)} dw$$

曲面 $\tau : [a_1, b_1] \times [a_2, b_2] \to \mathbb{R}^3$

$\int_\tau dw = \int_{\partial\tau} w$ となるようにしたい

無限小の level で成立したと仮定する

仮定した時、一般の場合に成り立つか?

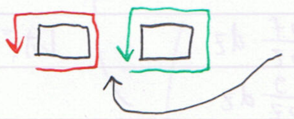


面積分は微小な長方形を足し合わせる

$\int_\tau dw = \sum\sum$

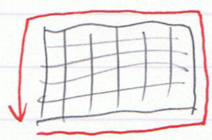
境界

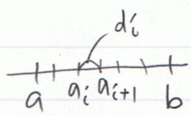
となりあう長方形を考えると



$\uparrow + \downarrow$ は打ち消し合う

打ち消し合わない部分は一番外側





$$d_i f(a_i) = \int_{a_i}^{a_{i+1}} f(x)dx = F(\cancel{a_{i+1}}) - F(a_i)$$

$$\int_{a_{i+1}}^{a_{i+2}} f(x)dx = F(a_{i+2}) - \cancel{F(a_{i+1})}$$ 打ち消し合う

打ち消し合わないのが 両端 (a と b)

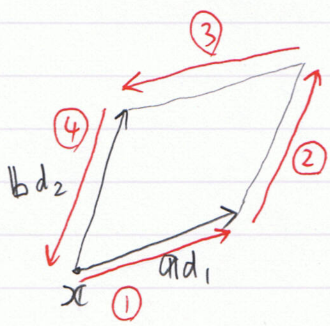
$$F(b) - F(a)$$

1次の微分形式

$w = f\,dx + g\,dy + h\,dz$

$a, b, x \in \mathbb{R}^3$

$d_1, d_2 \in D$



$a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}, \quad b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

4つの積分として考える

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$$\{f(x)a_1 + g(x)a_2 + h(x)a_3\}d_1 \cdots ①$$

$$+ \{f(x+ad_1)b_1 + g(x+ad_1)b_2 + h(x+ad_1)b_3\}d_2 \cdots ②$$

$$- \{f(x+bd_2)a_1 + g(x+bd_2)a_2 + h(x+bd_2)a_3\}d_1 \cdots ③$$

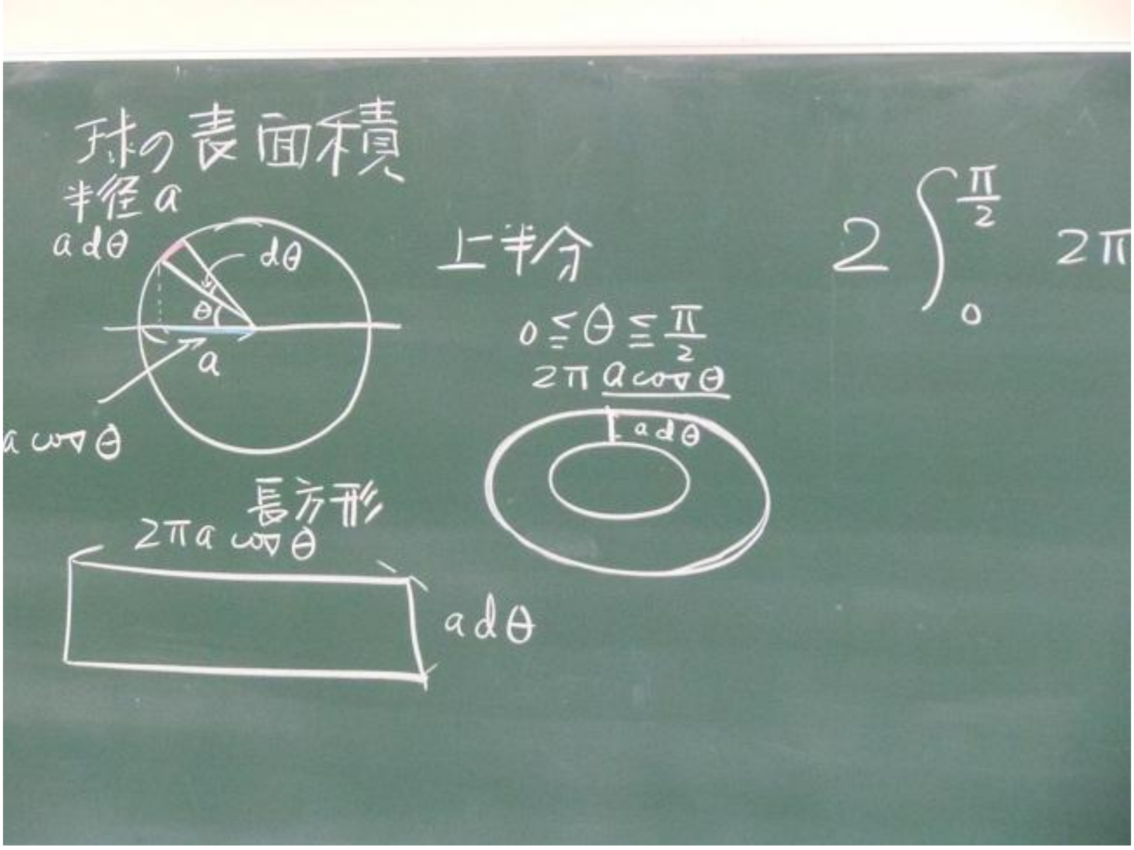
$$- \{f(x)b_1 + g(x)b_2 + h(x)b_3\}d_2 \cdots ④$$

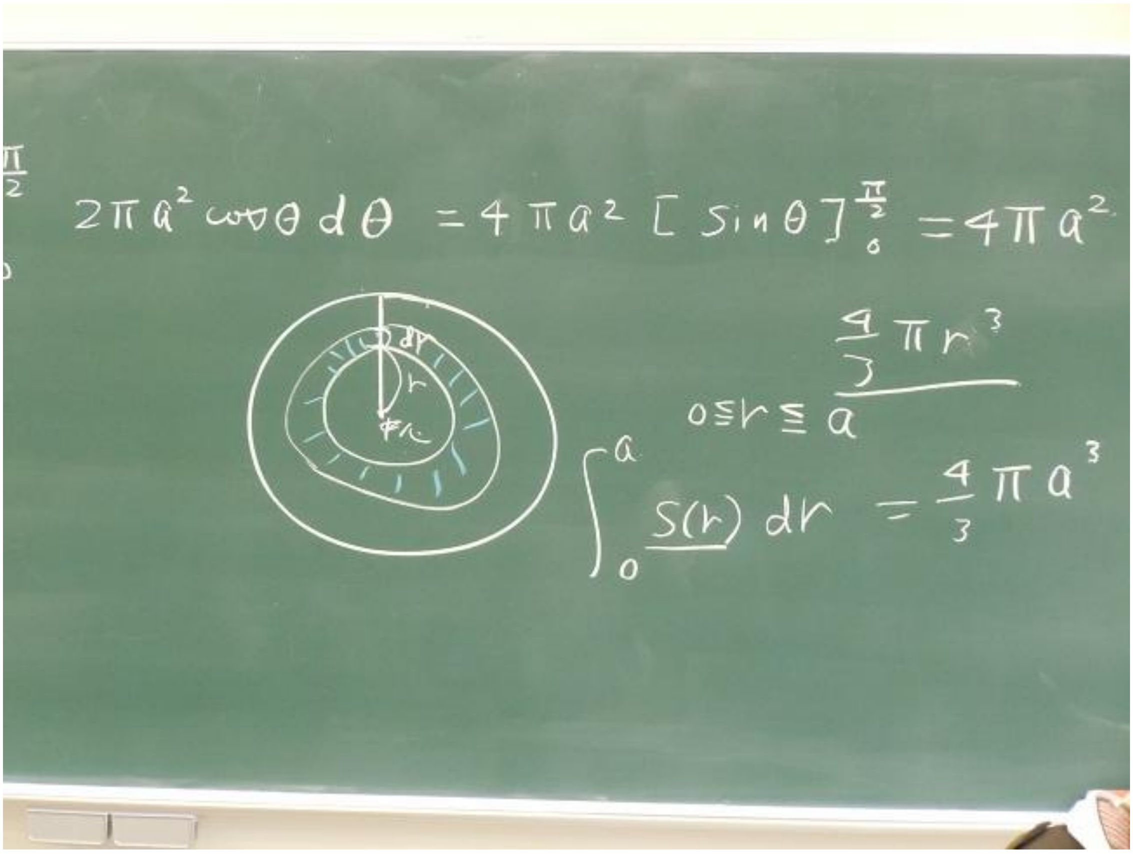
{②+④}+{③−①} のように整理すると

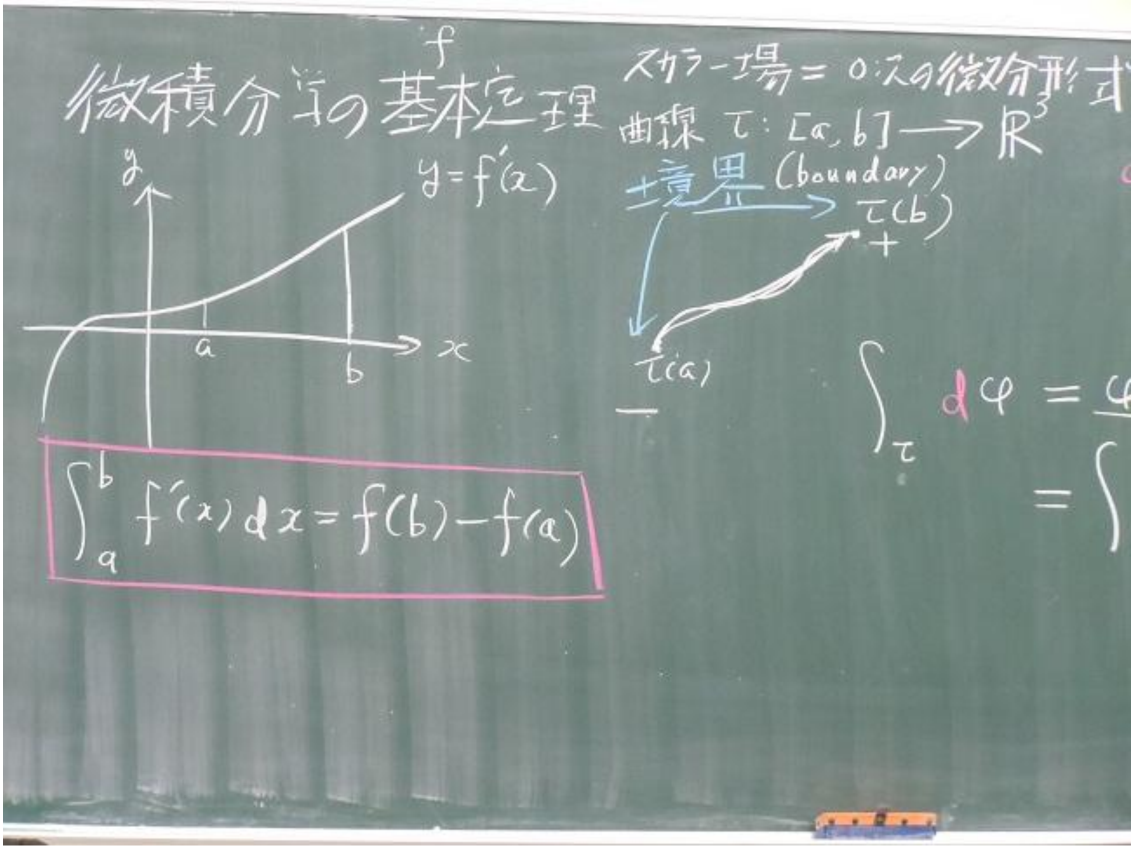
$$\left(\begin{array}{l} \{f'(x)(a)b_1 + g'(x)(a)b_2 + h'(x)(a)b_3\}d_1d_2 \\ - \{f'(x)(b)a_1 + g'(x)(b)a_2 + h'(x)(b)a_3\}d_1d_2 \end{array} \right)$$

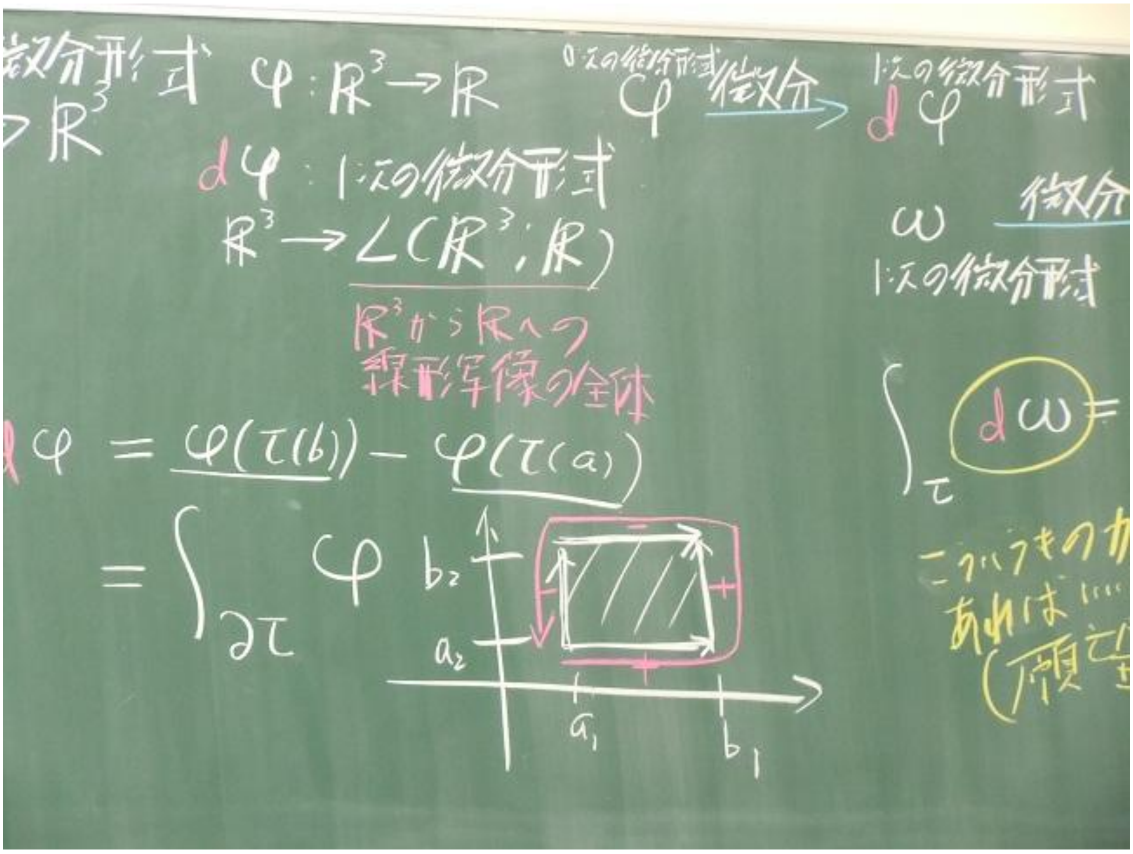
$$\left(\begin{array}{l} f'(x) = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial z}dz \\ g'(x) = \frac{\partial g}{\partial x}dx + \frac{\partial g}{\partial y}dy + \frac{\partial g}{\partial z}dz \\ h'(x) = \frac{\partial h}{\partial x}dx + \frac{\partial h}{\partial y}dy + \frac{\partial h}{\partial z}dz \end{array} \right)$$

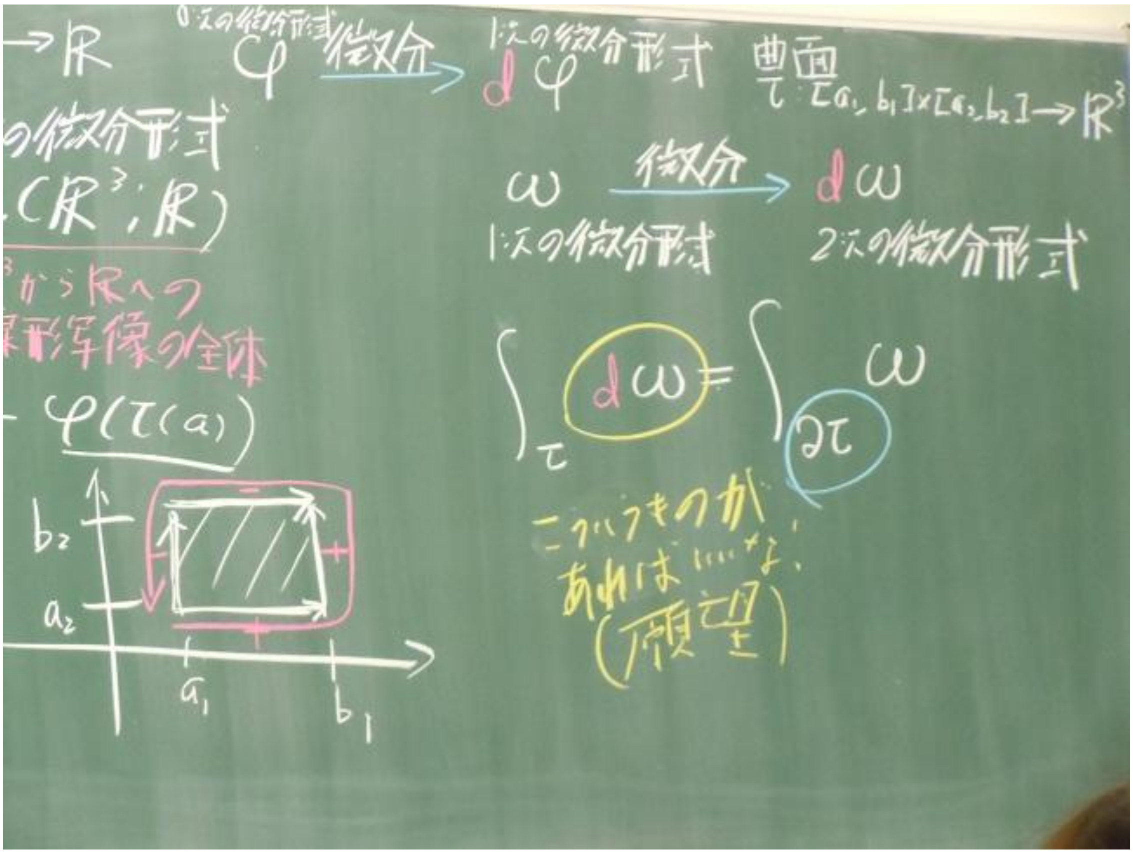
レポート：当てはめて計算

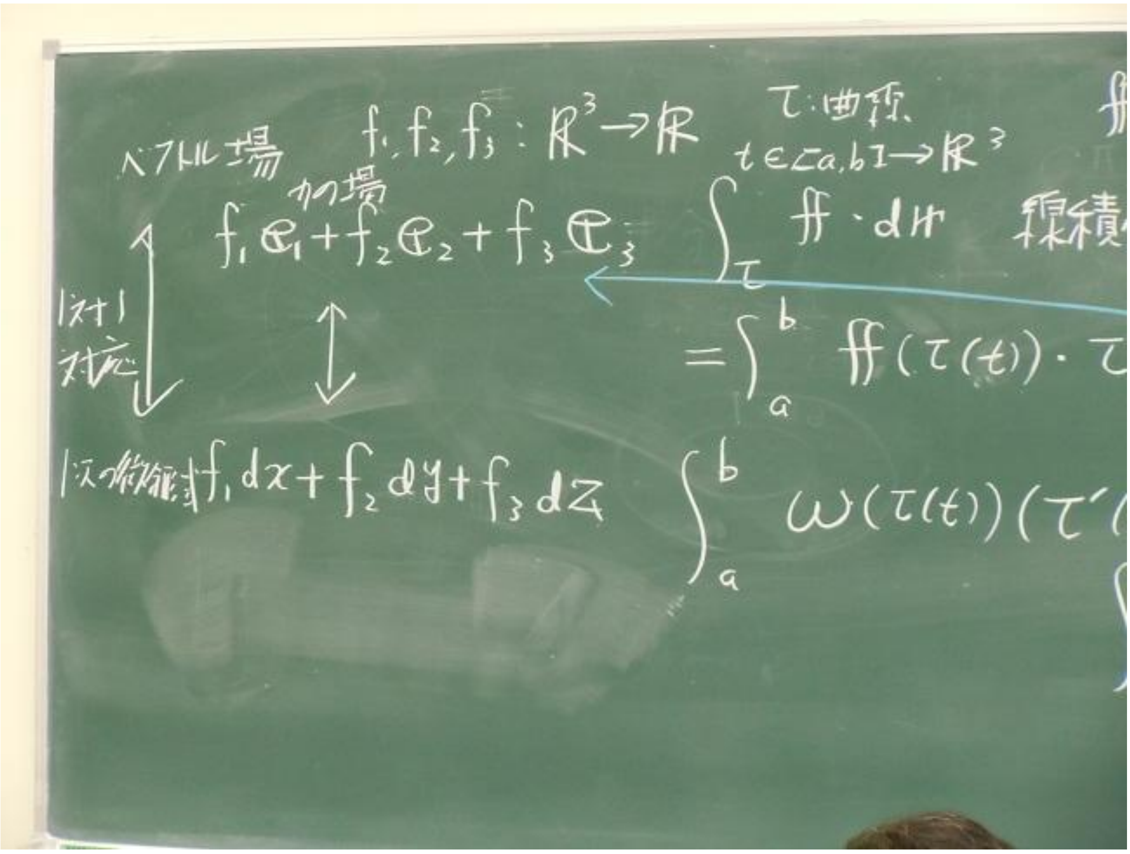

球の表面積
半径 a
adθ
dθ
θ
a
a cosθ
上半分
0≦θ≦π/2
2π acosθ
adθ
長方形
2πa cosθ
adθ
2∫_0^{π/2} 2π


2πa² cosθ dθ = 4πa² [sinθ]_0^{π/2} = 4πa²
dr
r
中心
4/3 πr³
0≦r≦a
∫_0^a S(r) dr = 4/3 πa³


微積分学の基本定理
y = f'(x)
∫_a^b f'(x)dx = f(b) − f(a)
スカラー場 = 0次の微分形式
曲線 τ: [a, b] → R^3
境界 (boundary)
τ(b)
τ(a)
∫_τ dφ =


φ: R^3 → R
0次の微分形式 φ 微分 dφ 1次の微分形式
dφ : 1次の微分形式
R^3 → L(R^3; R)
R^3からRへの線形写像の全体
dφ = φ(τ(b)) − φ(τ(a))
= ∫_∂τ φ
ω 微分
1次の微分形式
∫_τ dω =


$\to \mathbb{R}$
0次の微分形式 φ 微分 $d\varphi$ 1次の微分形式
曲面
$\tau : [a_1, b_1] \times [a_2, b_2] \to \mathbb{R}^3$
の微分形式
$(\mathbb{R}^3 ; \mathbb{R})$
^{3}から$\mathbb{R}$への
線形写像の全体
$\varphi(\tau(a))$
ω 微分 $d\omega$
1次の微分形式
2次の微分形式
$\int_\tau d\omega = \int_{\partial\tau} \omega$
こういうものがあればいいな
（願望）
b_2
a_2
a_1
b_1


ベクトル場
$f_1, f_2, f_3 : \mathbb{R}^3 \to \mathbb{R}$
τ: 曲線
$t \in [a,b] \to \mathbb{R}^3$
力の場
$f_1 \mathbb{e}_1 + f_2 \mathbb{e}_2 + f_3 \mathbb{e}_3$
$\int_\tau \mathbb{F} \cdot d\mathbb{r}$ 線積
1対1対応
$= \int_a^b \mathbb{F}(\tau(t)) \cdot \tau$
1次の微分形式 $f_1 dx + f_2 dy + f_3 dz$
$\int_a^b \omega(\tau(t))(\tau'($

$\mathbb{f} = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix}$ (水の)流れの場

曲面 $\tau : [a_1, b_1] \times [$

線積分 $\displaystyle\int_\tau \mathbb{f} \cdot dS = \int_{a_2}^{b_2} \int_{a_1}^{b_1} \mathbb{f}(\tau(s, t)) \cdot \Big(\frac{\partial \tau}{\partial s}$

$)) \cdot \tau'(t)\, dt$

$\omega = f_1\, dy \wedge dz + f_2\, dz \wedge dx$

$(\tau'(t))\ dt$

$\displaystyle\int_\tau \omega = \int_{a_2}^{b_2} \int_{a_1}^{b_1} \omega(\tau(s, t)) \Big(\frac{\partial \tau}{\partial s},$

)流れの場

曲面 $\tau : [\underset{s}{a_1, b_1}] \times [\underset{t}{a_2, b_2}] \longrightarrow \mathbb{R}^3$

$\displaystyle\int_{a_2}^{b_2} \int_{a_1}^{b_1} \mathbb{f}(\tau(s, t)) \cdot \Big(\frac{\partial \tau}{\partial s} \times \frac{\partial \tau}{\partial t}\Big)\, ds\, dt$

$\omega = f_1\, dy \wedge dz + f_2\, dz \wedge dx + f_3\, dx \wedge dy.$

$\displaystyle\int_{a_2}^{b_2} \int_{a_1}^{b_1} \omega(\tau(s, t)) \Big(\frac{\partial \tau}{\partial s}, \frac{\partial \tau}{\partial t}\Big)\, ds\, dt$

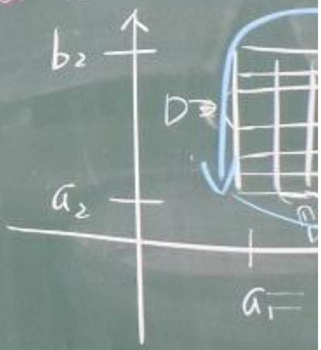
1次の微分形式

$\omega = f\,dx + g\,dy + h\,dz \xrightarrow{d\,(微分)} d\omega$

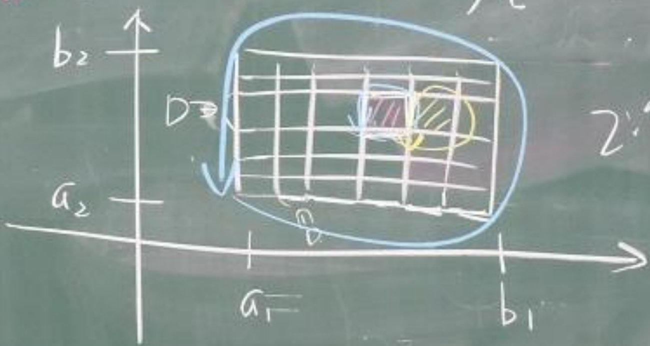
曲面τ: $[a_1, b_1] \times [a_2, b_2] \longrightarrow \mathbb{R}^3$

$$\boxed{\int_\tau d\omega = \int_{\partial\tau} \omega}$$ 願望

$\boxed{\text{無限小のlevelで成立したと仮定}}$



$d\omega$

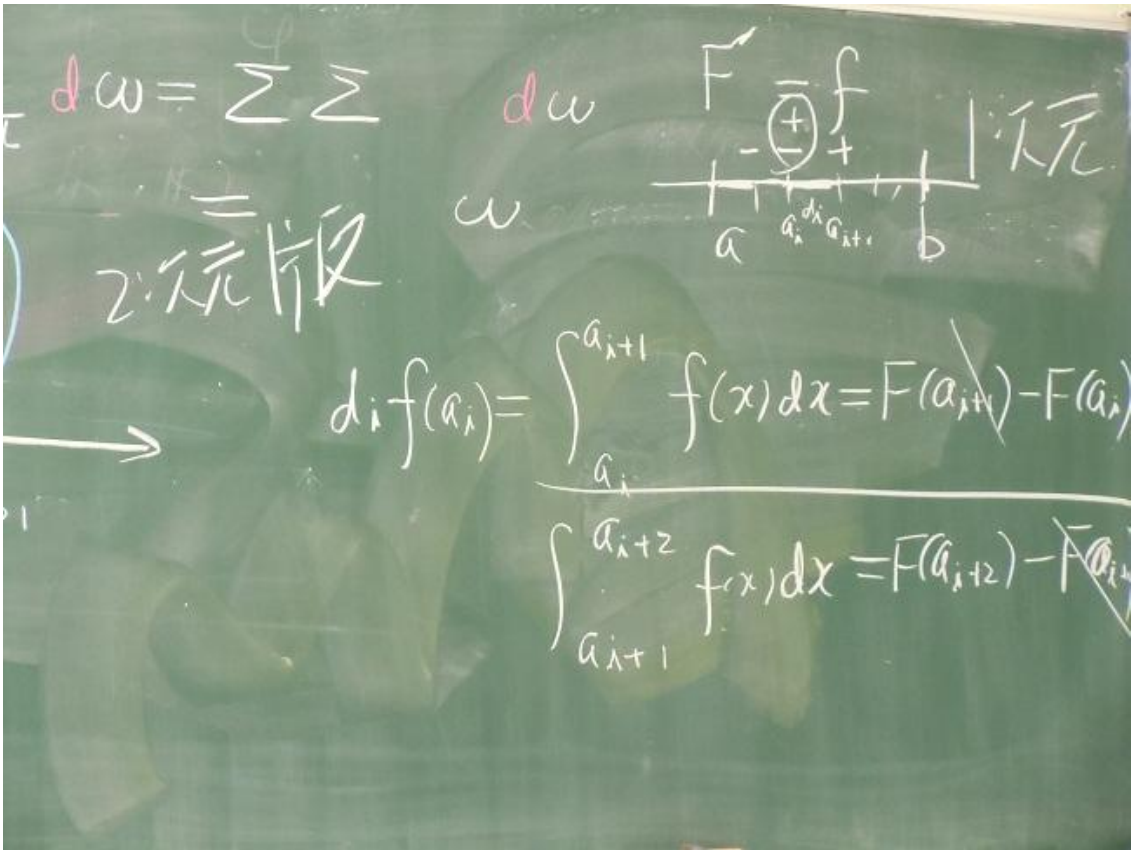


$\int_\tau d\omega = \Sigma\Sigma\ d\omega$

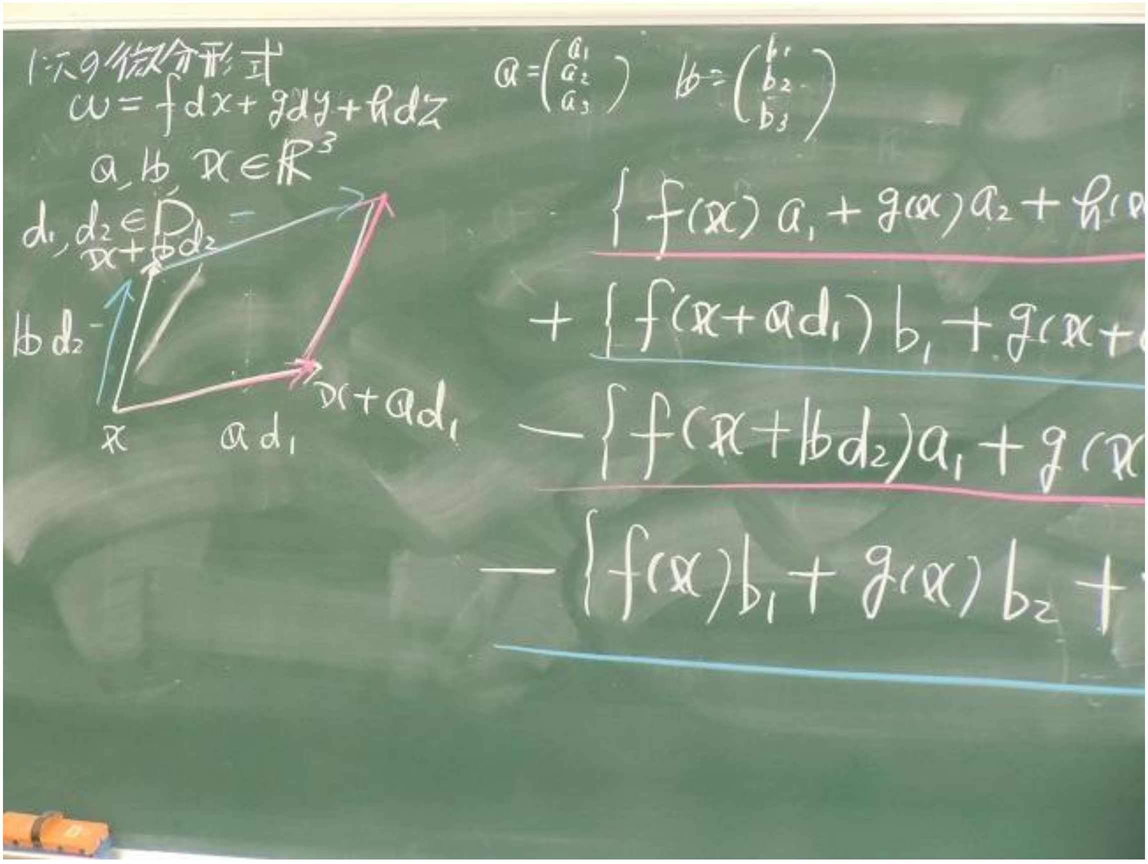
$= \Sigma\Sigma\ \omega$

2次元版

$d_i f(a_i) = \dfrac{\int_{a_i}^{a_{i+1}}}{\int_{a_{i+}}^{a_{i+}}}$



$d\omega = \sum\sum$
$d\omega$
$= \omega$
2次元版
$F \xrightarrow{} f$
1次元
$a \quad a_i \quad d_i \quad a_{i+1} \quad b$
$d_i f(a_i) = \int_{a_i}^{a_{i+1}} f(x)\,dx = F(a_{i+1}) - F(a_i)$
$\int_{a_{i+1}}^{a_{i+2}} f(x)\,dx = F(a_{i+2}) - F(a_{i+1})$



1次の微分形式
$\omega = f\,dx + g\,dy + h\,dz$
$a, b, x \in \mathbb{R}^3$
$d_1, d_2 \in \mathbb{R}$
$a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ $b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$
$x + b d_2$
$b d_2$
x
$a d_1$
$x + a d_1$
$\{ f(x) a_1 + g(x) a_2 + h(x)$
$+ \{ f(x + a d_1) b_1 + g(x +$
$- \{ f(x + b d_2) a_1 + g(x$
$- \{ f(x) b_1 + g(x) b_2 +$

$$\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

$$(x)\, a_1 + g(x)\, a_2 + h(x)\, a_3 \}\, d_1$$

$$(x + \mathbf{a} d_1)\, b_1 + g(x + \mathbf{a} d_1)\, b_2 + h(x + \mathbf{a} d_1)\, b_3 \}\, d_2$$

$$(x + \mathbf{b} d_2)\, a_1 + g(x + \mathbf{b} d_2)\, a_2 + h(x + \mathbf{b} d_2)\, a_3 \}\, d_1$$

$$x)\, b_1 + g(x)\, b_2 + h(x)\, b_3 \}\, d_2$$

$$= \{ f'(x)(\mathbf{a})\, b_1 + g'(x)(\mathbf{a})\, b_2 + h'(x)(\mathbf{a}$$

$$- \{ f'(x)(\mathbf{b})\, a_1 + g'(x)(\mathbf{b})\, a_2 + h'($$

$$f'(x) = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial z} dz$$

$$f(x)(\mathbf{b}) = \frac{\partial f}{\partial x} b_1 + \frac{\partial f}{\partial y} b_2 + \frac{\partial f}{\partial z} b_3$$

$$_1 + g'(x)(a)\,b_2 + h'(x)(a)\,b_3 \{ d_1 d_2$$

$$a_1 + g'(x)(b)\,a_2 + h'(x)(b)\,a_3 \{ d_1 d_2$$

$$x + \frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial z} dz$$

$$b_1 + \frac{\partial f}{\partial y} b_2 + \frac{\partial f}{\partial z} b_3$$