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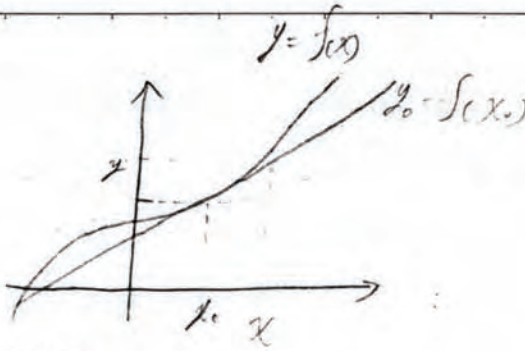
$$\underbrace{y - y_0}_{\Delta y} = f'(x_0)\underbrace{(x - x_0)}_{\substack{\Delta x \\ \cap \\ D=\{d^2=0 \mid d\in\mathbb{R}\}}}$$

これを認めれば等号成立



比例関数

線形関数

@ $f: \mathbb{R} \to \mathbb{R}^2$ でもよい

$$f(x) = \begin{pmatrix} f^1(x) \\ f^2(x) \end{pmatrix}$$

$$y^1 - y^1_0 = (f^1)'(x_0)(x - x_0)$$

$$y^2 - y^2_0 = (f^2)'(x_0)(x - x_0)$$

$$\begin{pmatrix} y^1 - y^1_0 \\ y^2 - y^2_0 \end{pmatrix} = \begin{pmatrix} (f^1)'(x_0) \\ (f^2)'(x_0) \end{pmatrix}(x - x_0)$$

@ $f: \mathbb{R}^2 \to \mathbb{R}$

$$y - y_0 = \underbrace{\left(\frac{\partial f}{\partial x^1}, \frac{\partial f}{\partial x^2}\right)}_{1\times 2}\underbrace{\begin{pmatrix} x^1 - x^1_0 \\ x^2 - x^2_0 \end{pmatrix}}_{2\times 1}$$

$$\left(\frac{\partial f}{\partial x^1}, \frac{\partial f}{\partial x^2}\right) = \left(\frac{\partial y}{\partial x^1}, \frac{\partial y}{\partial x^2}\right)$$

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$$y_0^1 = f^1(x_0^1, x_0^2)$$

◎ $f: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$

$$f(x^1, x^2) = \begin{pmatrix} f^1(x^1, x^2) \\ f^2(x^1, x^2) \end{pmatrix}$$

偏微分

$$\begin{pmatrix} y^1 - y_0^1 \\ y^2 - y_0^2 \end{pmatrix} = \underbrace{\begin{pmatrix} \frac{\partial f^1}{\partial x^1} & \frac{\partial f^1}{\partial x^2} \\ \frac{\partial f^2}{\partial x^1} & \frac{\partial f^2}{\partial x^2} \end{pmatrix}}_{2\times 2} \begin{pmatrix} x^1 - x_0^1 \\ x^2 - x_0^2 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\partial y^1}{\partial x^1} & \frac{\partial y^1}{\partial x^2} \\ \frac{\partial y^2}{\partial x^1} & \frac{\partial y^2}{\partial x^2} \end{pmatrix} \begin{pmatrix} x^1 - x_0^1 \\ x^2 - x_0^2 \end{pmatrix}$$

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◎ $f : \mathbb{R}^n \to \mathbb{R}^m$

$$f(x^1, \cdots, x^n) = \begin{pmatrix} f^1(x^1, \cdots, x^n) \\ \vdots \\ f^m(x^1, \cdots, x^n) \end{pmatrix}$$

$$\begin{pmatrix} y^1 - y_0^1 \\ \vdots \\ y^m - y_0^m \end{pmatrix} = \begin{pmatrix} \frac{\partial f^1}{\partial x^1} & \cdots & \frac{\partial f^1}{\partial x^n} \\ \vdots & & \vdots \\ \frac{\partial f^m}{\partial x^1} & \cdots & \frac{\partial f^m}{\partial x^n} \end{pmatrix} \begin{pmatrix} x^1 - x_0^1 \\ \vdots \\ x^n - x_0^n \end{pmatrix}$$

$m \times n$ 行列　　$n \times 1$ 行列

$\mathbb{R}^2 \to \mathbb{R}$ への線形写像

(例)

$$y = \alpha_1 x^1 + \alpha_2 x^2$$

$$y = \begin{pmatrix} \alpha_1 & \alpha_2 \end{pmatrix} \begin{pmatrix} x^1 \\ x^2 \end{pmatrix}$$

$\begin{cases} x^2 = 0 \text{ とすると } x^1 \text{ と } y \text{ の間の比例関係} \\ x^1 = 0 \quad 〃 \quad x^2 \text{ と } y \quad 〃 \end{cases}$

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少し一般化

$$\mathbf{a} = \begin{pmatrix} a^1 \\ a^2 \end{pmatrix} \qquad \mathbf{a} : \text{const}$$

$$\begin{pmatrix} x^1 \\ x^2 \end{pmatrix} = \begin{pmatrix} a^1 x \\ a^2 x \end{pmatrix}$$ とすると (独立な変数は x のみ)

$$\begin{aligned} y &= \alpha_1 a^1 x + \alpha_2 a^2 x \\ &= \underbrace{(\alpha_1 a^1 + \alpha_2 a^2)}_{\text{比例定数}} x \end{aligned}$$

y と x が比例：比例関数。

$\mathbf{a}$ を動かす (a_1, a_2 を動かす)

$$y = \boxed{(\alpha^1\ \alpha^2) \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}} x$$

$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \mathbf{a}$

線形写像

$$\begin{pmatrix} a^1 \\ a^2 \end{pmatrix} = \mathbf{a} \in \mathbb{R}^2 \longrightarrow (\alpha^1, \alpha^2) \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \in \mathbb{R}$$

$f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ 一般の関数を考える

$$\mathbf{a}, \mathbf{x}_0 \in \mathbb{R}^2$$

$$\partial_{\mathbf{a}} f(\mathbf{x}_0)$$

$\Leftrightarrow$「f の $\mathbf{x}_0$ における $\mathbf{a}$ 方向の微分」 これって何だったんだっけ？

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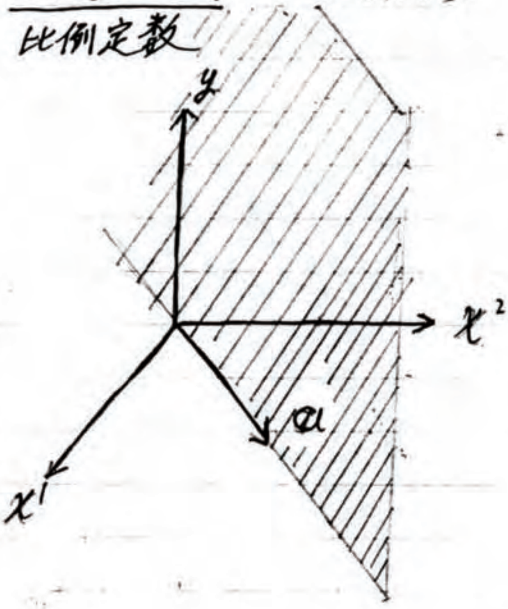
Kock-Lawvere の公理

$$d \in D \longmapsto f(x_0 + a d) \in R$$

$$f(x_0 + a d) - f(x_0) = \underbrace{\partial_a f(x_0)}_{\text{比例定数}} \, d \quad (x_0^1, x_0^2)$$



$\partial_a f(x_0) = \underline{d} f(x_0)$

d は $d \in D$ の d ではない。

x と y 軸で張られる平面を考える

この平面で問題つ $\begin{cases} \text{曲面 } y = f(x^1, x^2) \\ \quad \downarrow \\ \text{曲線} \end{cases}$

であり

接平面 ⟶ 接線

さきほどの

$(\alpha^1, \alpha^2) \begin{pmatrix} a^1 \\ a^2 \end{pmatrix} x$ を考える

$\Downarrow$

巾零無限小 ならば 接平面中を動く.

$\Downarrow$

$(\alpha^1, \alpha^2) \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ が線形性をもつのは

感覚的に分かった!

$$\left.\begin{array}{l} f : \mathbb{R}^2 \to \mathbb{R}^2 \\ g : \mathbb{R}^2 \to \mathbb{R}^2 \end{array}\right\}$$ 線形

$$f : \begin{pmatrix} y^1 \\ y^2 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} x^1 \\ x^2 \end{pmatrix} = \begin{pmatrix} a_{11}x^1 + a_{12}x^2 \\ a_{21}x^1 + a_{22}x^2 \end{pmatrix}$$

$$g : \begin{pmatrix} z^1 \\ z^2 \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \begin{pmatrix} y^1 \\ y^2 \end{pmatrix}$$

合成写像はどのような式で表されるのだろうか

$g \circ f$

$$\begin{pmatrix} z^1 \\ z^2 \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \begin{pmatrix} a_{11}x^1 + a_{12}x^2 \\ a_{21}x^1 + a_{22}x^2 \end{pmatrix}$$

$$= \begin{pmatrix} b_{11}(a_{11}x^1 + a_{12}x^2) + b_{12}(a_{21}x^1 + a_{22}x^2) \\ b_{21}(a_{11}x^1 + a_{12}x^2) + b_{22}(a_{21}x^1 + a_{22}x^2) \end{pmatrix}$$

$$= \begin{pmatrix} (b_{11}a_{11} + b_{12}a_{21})x^1 + (b_{11}a_{12} + b_{12}a_{22})x^2 \\ (b_{21}a_{11} + b_{22}a_{21})x^1 + (b_{21}a_{12} + b_{22}a_{22})x^2 \end{pmatrix}$$

$$= \underline{\begin{pmatrix} b_{11}a_{11} + b_{12}a_{21} & b_{11}a_{12} + b_{12}a_{22} \\ b_{21}a_{11} + b_{22}a_{21} & b_{21}a_{12} + b_{22}a_{22} \end{pmatrix}} \begin{pmatrix} x^1 \\ x^2 \end{pmatrix}$$

$$\stackrel{\mathrm{def}}{=} BA$$

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$$\left.\begin{array}{l} f : \mathbb{R}^2 \to \mathbb{R}^2 \\ g : \mathbb{R}^2 \to \mathbb{R}^2 \end{array}\right\} : \underline{\underline{一般}}$$ とする

fの微分は
$$\begin{pmatrix} \dfrac{\partial y^1}{\partial x^1} & \dfrac{\partial y^1}{\partial x^2} \\ \dfrac{\partial y^2}{\partial x^1} & \dfrac{\partial y^2}{\partial x^2} \end{pmatrix}$$

gは
$$\begin{pmatrix} \dfrac{\partial z^1}{\partial y^1} & \dfrac{\partial z^1}{\partial y^2} \\ \dfrac{\partial z^2}{\partial y^1} & \dfrac{\partial z^2}{\partial y^2} \end{pmatrix}$$

$$\frac{\partial z^1}{\partial x^2} = \frac{\partial z^1}{\partial y^1} \times \frac{\partial y^1}{\partial x^2} + \frac{\partial z^1}{\partial y^2} \times \frac{\partial y^2}{\partial x^2}$$

$g \circ f$ は?

先程の記号 d を使えば

答

$\underline{dg(y_0) \cdot df(x_0)}$

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一変数にしてみよう

$f: \mathbb{R} \to \mathbb{R}$

$g: \mathbb{R} \to \mathbb{R}$

$$(g \circ f)'(x_0) = \underset{\mathbb{R}}{\underset{\cap}{[g'(y_0)]}} \underset{\mathbb{R}}{\underset{\cap}{[f'(x_0)]}}$$ ⇐ つまり 1×1 行列の行列同士の積としてとらえる

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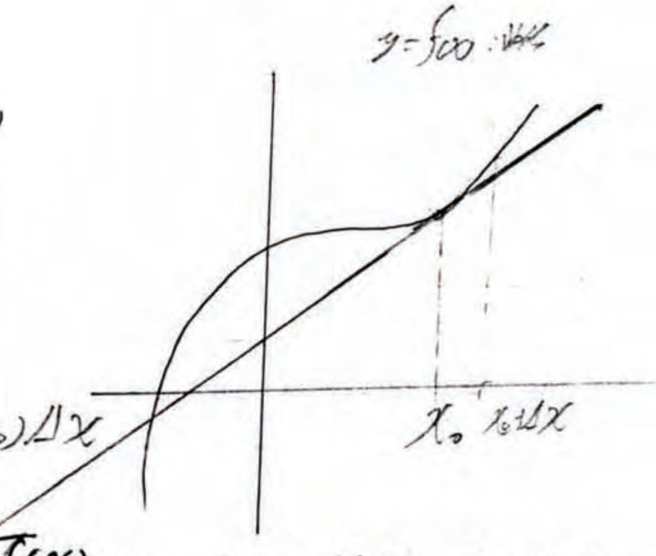
x_0 を固定すれば

$\Delta y = f(x_0 + \Delta x) - f(x_0) = \underbrace{g(\Delta x)}$

一般には複雑



$\bar{f}(x_0 + \Delta x) - f(x_0)$

$= \bar{f}(x_0 + \Delta x) - \bar{f}(x_0) = f'(x_0)\Delta x$

$y = \underline{\bar{f}(x)}$ （バーをつけて接線関数を表す）

$D = \{ d \in \mathbb{R} \mid d^2 = 0 \}$

$\underbrace{\Delta x \in D}$ の場合

$\underline{f(x_0 + \Delta x) = \bar{f}(x_0 + \Delta x)}$