No.

Date

No. 9　11/11

◎ 微分形式

R^3（空間）

0次の微分形式

$\varphi : R^3 \to R$　スカラー関数

（各点の温度，気圧など）

微分する

$x, a \in R^3$

$$d\varphi(x)(a)d = \varphi(x+ad) - \varphi(x)$$

φをxで微分したもの

$D = \{d \in R \mid d^2 = 0\}$

$d\varphi(x)$ は $R^3 \to R$ の線形写像

$$x \in R^3 \to d\varphi(x)$$

⇓

$d\varphi$ は1次の微分形式

◎ 微分形式の積分

$$\int_a^{a+d} f(x)\,dx = d f(a)$$

曲線 $\gamma : e \in D \longmapsto x + ae \in R^3$

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◎ 微分形式

$\mathbb{R}^3$（空間）

0次の微分形式

$\varphi : \mathbb{R}^3 \to \mathbb{R}$　スカラー関数

（各点の温度，気圧など）

微分する

$x, a \in \mathbb{R}^3$

$$\underbrace{d\varphi(x)}_{\varphi\text{を}x\text{で微分したもの}}(a)\,d = \varphi(x + ad) - \varphi(x)$$

$D = \{d \in \mathbb{R} \mid d^2 = 0\}$

$d\varphi(x)$は$\mathbb{R}^3 \to \mathbb{R}$の線形写像

$x \in \mathbb{R}^3 \to d\varphi(x)$

$\Downarrow$

$d\varphi$は1次の微分形式

◎ 微分形式の積分

$$\int_a^{a+d} f(x)\,dx = d\,f(a)$$

曲線 $\gamma : e \in D \longmapsto x + ae \in \mathbb{R}^3$

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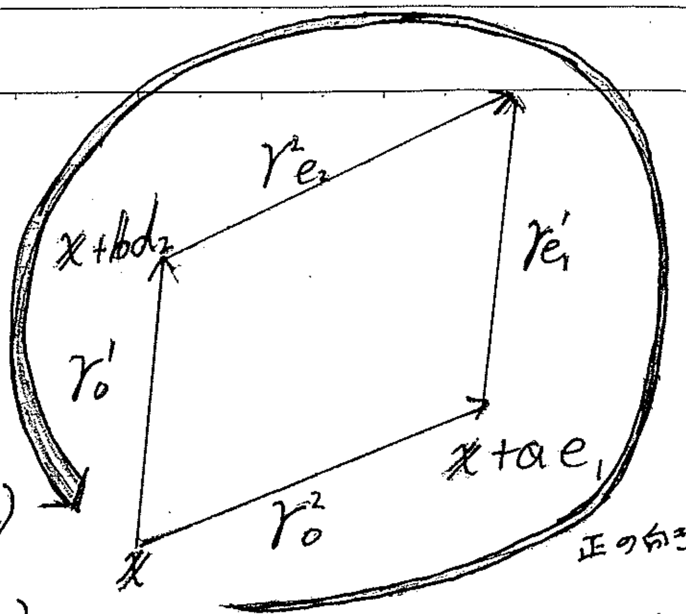
[原理]

$$\int_{\partial(\gamma, e_1, e_2)} \omega = \int_{(\gamma, e_1, e_2)} d\omega$$



イメージ図

$$\partial(\gamma; e_1, e_2) = (\gamma^2_0; e_1) + (\gamma^1_{e_1}; e_2) - (\gamma^2_{e_2}; e_1) - (\gamma^1_0; e_2)$$

$$\gamma^2_0 : d \in D \longmapsto \gamma(d, 0) = x + ad$$

$$\gamma^1_{e_1} : d \in D \longmapsto \gamma(e_1, d) = x + ae_1 + bd$$

$$\gamma^2_{e_2} : d \in D \longmapsto \gamma(d, e_2) = x + ad + be_2$$

$$\gamma^1_0 : d \in D \longmapsto \gamma(0, d) = x + bd$$

$$\int_{\partial(\gamma; e_1, e_2)} \omega = \int_{(\gamma^2_0; e_1)} \omega + \int_{(\gamma^1_{e_1}; e_2)} \omega - \int_{(\gamma^2_{e_2}; e_1)} \omega - \int_{(\gamma_0; e_2)} \omega$$

$$= \{f(x)a_1 + g(x)a_2 + h(x)a_3\}e_1 + \left\{\begin{array}{l} f(x+ae_1)b_1 + g(x+ae_1)b_2 \\ + h(x+ae_1)b_3 \end{array}\right\}e_2$$

$$- \{f(x+be_2)a_1 + g(x+be_2)a_2 + h(x+be_2)a_3\}e_1$$
$$- \{f(x)b_1 + g(x)b_2 + h(x)b_3\}e_2 = \mathrm{I}$$

ここで

$$f(x+ae_1) - f(x) = df(x)(a)e_1$$

$$df(x)(a) = \frac{\partial f}{\partial x}(x)a_1 + \frac{\partial f}{\partial y}(x)a_2 + \frac{\partial f}{\partial z}(x)a_3$$

に注意して 展開整理

$$\mathrm{I} = \left\{\frac{\partial g}{\partial x}(x) - \frac{\partial f}{\partial y}(x)\right\}(a_1b_2 - a_2b_1)$$
$$+ \left\{\frac{\partial h}{\partial y}(x) - \frac{\partial g}{\partial z}(x)\right\}(a_2b_3 - a_3b_2)$$
$$+ \left\{\frac{\partial f}{\partial z}(x) - \frac{\partial h}{\partial x}(x)\right\}(a_3b_1 - a_1b_3)$$

$$= \left\{\frac{\partial g}{\partial x}(x) - \frac{\partial f}{\partial y}(x)\right\} dx \wedge dy(a, b)$$
$$+ \left\{\frac{\partial h}{\partial y}(x) - \frac{\partial g}{\partial z}(x)\right\} dy \wedge dz(a, b)$$
$$+ \left\{\frac{\partial f}{\partial z}(x) - \frac{\partial h}{\partial x}(x)\right\} dz \wedge dx(a, b)$$