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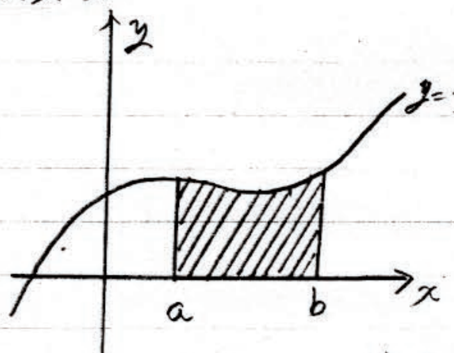
定積分



区間について積分

$$\int_a^b f(x)\,dx$$

$\Downarrow$

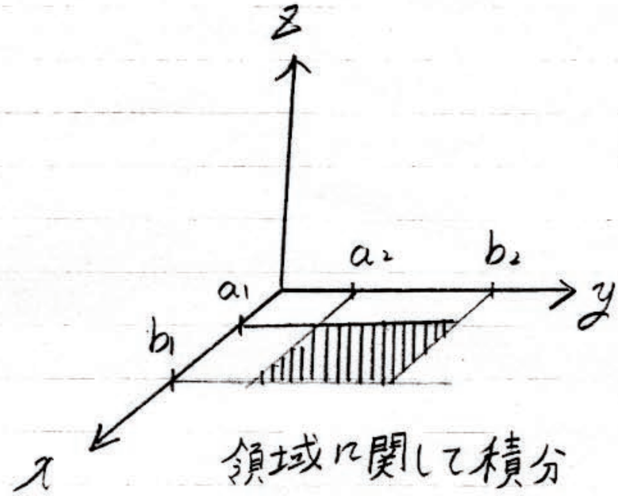
面積

$z=f(x,y)$

$$\int f(x,y)\,dx\,dy$$

$\Downarrow$

体積



領域に関して積分

今 y を固定すると

$y=y_0$ (平面)

$$=\int_{a_2}^{b_2}\Big\{\underbrace{\int_{a_1}^{b_1} f(x,y_0)\,dx}_{y\text{が動けば}y\text{の関数}}\Big\}\,dy \qquad (\text{二重積分})$$

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$$\int f(x,y)\,dx\,dy = \int_{a_2}^{b_2}\left\{\int_{a_1}^{b_1} f(x,y)\,dx\right\}dy$$

$$= \int_{a_1}^{b_1}\left\{\int_{a_2}^{b_2} f(x,y)\,dy\right\}dx$$

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pratonic　　　　　　　　　　pragmatic

ベクトル場 $\left\{\begin{array}{l} \text{力の場} \longleftrightarrow \text{1次の微分形式} \\ \text{流れの場} \longleftrightarrow \text{2次の微分形式} \end{array}\right.$

$\downarrow$

$f(x)\,e_1 + g(x)\,e_2 + h(x)\,e_3$

$\left(f, g, h : \underset{x}{\overset{\cup\!\!\mid}{\mathbb{R}^3}} \to \mathbb{R} \right)$

$\updownarrow$ 1次の微分形式

$f(x)\,\underline{dx} + g(x)\,\underline{dy} + h(x)\,\underline{dz}$

基底

2次の微分形式

$f(x)\,\underline{dy\wedge dz} + g(x)\,\underline{dz\wedge dx} + h(x)\,\underline{dx\wedge dy}$

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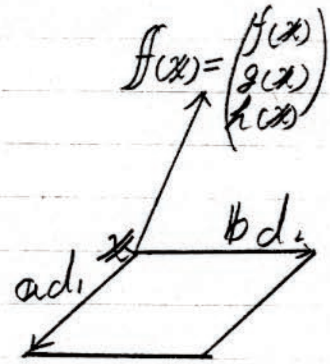
$$\begin{cases} a, b \in \mathbb{R}^3 \\ d_1, d_2 \in \mathcal{D} \end{cases}$$



ここに張られる平行六面体の体積

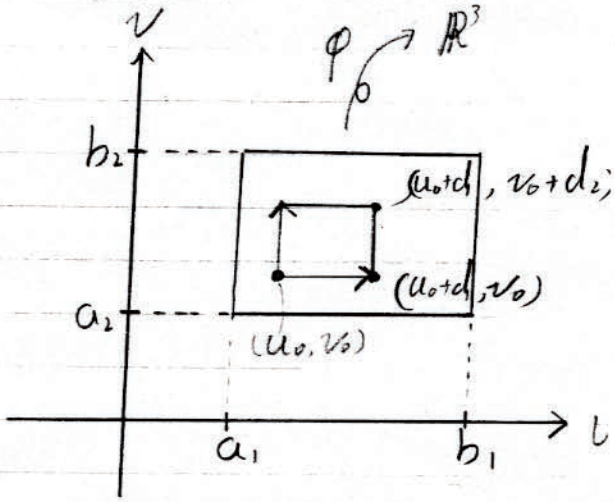
平行四辺形

$$\left\{ \begin{pmatrix} f(x) \\ g(x) \\ h(x) \end{pmatrix} \cdot a d_1 \times b d_2 \right\}$$

$$= \left\{ f(x) \underbrace{\begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}}_{(dy \wedge dz)(a,b)} + g(x) \underbrace{\begin{vmatrix} a_3 & b_3 \\ a_1 & b_1 \end{vmatrix}}_{(dz \wedge dx)(a,b)} + h(x) \underbrace{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}_{(dx \wedge dy)(a,b)} \right\} d_1 d_2$$

$$\varphi : [a_1, b_1] \times [a_2, b_2] \longrightarrow \mathbb{R}^3$$

曲面を単位時間に横切る水量



今、点の変化量 $d_1, d_2 \in \mathcal{D}$ だから
φで移された先でも直線

$$\varphi(u_0+d_1, v_0) - \varphi(u_0, v_0)$$
$$= \frac{\partial \varphi}{\partial u}(u_0, v_0)\, d_1$$

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同じように

$$\varphi(u_0, v_0+d_2) - \varphi(u_0, v_0) = \frac{\partial \varphi}{\partial v_0}(u_0, v_0)\, d_2$$

φで移った先でもベクトル $\frac{\partial \varphi}{\partial u_0}(u_0, v_0) d_1$ と $\frac{\partial \varphi}{\partial v_0}(u_0, v_0) d_2$ が直交しているとは限らない

$\Downarrow$

平行四辺形

$$\frac{\partial \varphi}{\partial u} \times \frac{\partial \varphi}{\partial v}$$

$\Downarrow$

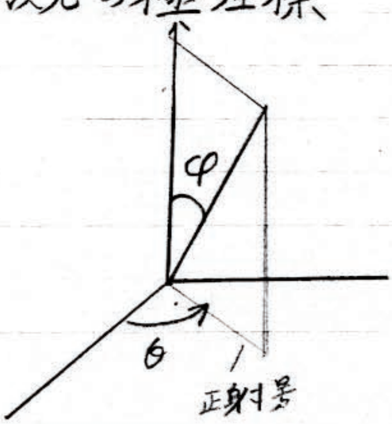
曲面のときの水量

$$\int_{a_2}^{b_2}\int_{a_1}^{b_1} f(x) \cdot \left(\frac{\partial \varphi}{\partial u} \times \frac{\partial \varphi}{\partial v}\right) du\, dv$$

半径aの球の体積　$\frac{4}{3}\pi a^3$

球面の面積　$4\pi a^2$

3次元の極座標



2次元　〃

$$\begin{cases} 0 \leq r < \infty \\ 0 \leq \theta < 2\pi \\ x = r\cos\theta \\ y = r\sin\theta \end{cases}$$

$$\begin{cases} 0 \leqq r < \infty \\ 0 \leqq \varphi < \pi \\ 0 \leqq \theta < 2\pi \\ z = r\cos\varphi \\ x = (r\sin\varphi)\cos\theta \\ y = (r\sin\varphi)\sin\theta \end{cases}$$

原点中心 $(0,0,0)$
半径 a の球面

球面上の点 $\boldsymbol{r}$

$$\boldsymbol{r} = \begin{pmatrix} x(\varphi,\theta) \\ y(\varphi,\theta) \\ z(\varphi,\theta) \end{pmatrix}$$

長さをとる
↕

$$\left| \frac{\partial \boldsymbol{r}}{\partial \varphi} \times \frac{\partial \boldsymbol{r}}{\partial \theta} \right| \Rightarrow \text{面積（スカラー）}$$

$$\int_0^{2\pi}\int_0^{\pi} \left| \frac{\partial \boldsymbol{r}}{\partial \varphi} \times \frac{\partial \boldsymbol{r}}{\partial \theta} \right| d\varphi\, d\theta$$

$$\frac{\partial \boldsymbol{r}}{\partial \varphi} = \begin{pmatrix} a\cos\varphi\cos\theta \\ a\cos\varphi\sin\theta \\ -a\sin\varphi \end{pmatrix}$$

$$\frac{\partial \boldsymbol{r}}{\partial \theta} = \begin{pmatrix} -a\sin\varphi\sin\theta \\ a\sin\varphi\cos\theta \\ 0 \end{pmatrix}$$

$$\left| \frac{\partial \boldsymbol{r}}{\partial \varphi} \times \frac{\partial \boldsymbol{r}}{\partial \theta} \right| = \sqrt{(-a^2\sin^2\varphi\cos\theta)^2 + (a^2\sin^2\varphi\sin\theta)^2 + (a^2\sin\varphi\cos\varphi\cos^2\theta + a^2\sin\varphi\cos\varphi\sin^2\theta}$$

$$= \sqrt{a^4\sin^4\varphi + a^4\sin^2\varphi\cos^2\varphi}$$

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$$= \sqrt{a^4 \sin^2\varphi \, (\sin^2\varphi + \cos^2\varphi)}$$

$$= a^2 \sin\varphi$$

$$\int_0^{\pi} \left| \frac{\partial r}{\partial \varphi} \times \frac{\partial r}{\partial \theta} \right| d\varphi = \int_0^{\pi} a^2 \sin\varphi \, d\varphi$$

$$= a^2 [-\cos\varphi]_0^{\pi}$$

$$= 2a^2$$

$$\int_0^{2\pi} \int_0^{\pi} \left| \frac{\partial r}{\partial \varphi} \times \frac{\partial r}{\partial \theta} \right| d\varphi \, d\theta = \int_0^{2\pi} 2a^2 \, d\theta$$

$$= 4\pi a^2$$