Noname manuscript No.
(will be inserted by the editor)

A note on separability and intra-household resource allocation in a collective household model

Tomoki Fujii · Ryuichiro Ishikawa

July 20, 2012

Abstract This paper shows that it is possible to track the changes in the distribution of power within a couple by focusing on the changes in the pattern of private consumption when the consumption decisions are efficient and private consumption is separable from public consumption in individual preferences. We first show that the separability of private consumption from public consumption at the individual level carries over to the household level. Hence, changes in public consumption only matters through a change in the residual budget available for private consumption. When the consumption decisions within the household is efficient, private consumption decisions can be modeled as the solution of a problem consisting in maximizing a weighted sum of the private-consumption sub-utility functions of the spouses under the residual budget, the weights being unique and representing the distribution of power over the allocation of private consumption. The model presented in this paper can be used to analyze the changes in the household resource allocation due to, for example, childbirth.

Keywords Collective model · Intra-household resource allocation · Bargaining · Separability.

JEL classification codes: C78 D01 D11

1 Introduction

Collective models of households with heterogeneous preferences among members have become a standard framework for analyzing household behavior. This is in

Ishikawa received Grants-in-Aid for Young Scientists (B), No. 23730184. We thank two anonymous referees and editors for helpful comments.

T. Fujii
School of Economics, Singapore Management University, 90 Stamford Road, Singapore 178903

R. Ishikawa
Faculty of Engineering, Information and Systems, University of Tsukuba, 1-1-1 Ten-nodai, Tsukuba, Ibaraki 305-8573, Japan
E-mail: ishikawa@sk.tsukuba.ac.jp

part because a number of empirical studies have rejected the traditional unitary household model (e.g., Thomas (1990) and Lundberg, Pollak, and Wales (1997)). The theoretical development of the Pareto efficient approach to collective household models following the seminal papers by Chiappori (1988, 1992) has also enabled us to test the collective rationality under a set of fairly weak assumptions (e.g., Browning and Chiappori (1998)) and the collective rationality has not been rejected in some countries, including Canada (Browning and Chiappori, 1998) and Mexico (Bobonis, 2009).[1] Blundell, Chiappori, and Meghir (2005) advanced the Pareto-efficient approach to households with children. They show that the sharing rule can be identified in the presence of either separability or one distribution factor.

In this note, we discuss the relationship between the Pareto-optimal allocation in the household and the allocation of private goods in the corresponding private sub-problem. We first show that the weak separability of private goods consumption from public goods consumption carries over to the household level. We then show the existence of a private sub-problem that supports the allocation of private goods in any Pareto-efficient outcome. We also provide a necessary and sufficient condition for the household Pareto weight and the Pareto weight in the private sub-problem to move in the same direction. When this condition is satisfied, we can focus on the space of private goods to track the changes in the overall decision power and resource allocation within a household.

The results presented in this note are directly applicable to the identification of changes in the resource allocation within a household. Using the analytical framework developed in this note, Fujii and Ishikawa (2012) estimate the impact of childbirth on intra-household allocation using the Japanese Panel Survey of Consumers (JPSC) dataset, which includes the measurements of consumption expenditure for the husband, wife, children, other household members and family as a whole. They find that the private goods allocation tends to move towards the wife's disadvantage when a new baby is born. Their study also suggests that the wife may be substituting more say in child-rearing for private consumption.[2]

This study is related to Blundell, Chiappori, and Meghir (2005) in the sense that both studies deal with the intra-household resource allocation in the presence of public goods. However, the purpose of this note is not to identify the sharing rule but to provide an analytical framework that allows us to identify the factors that affect the Pareto weight. This study is also different from Browning, Chiappori, and Lewbel (2010), who propose a way to estimate the indifference scales in a collective household, because we do not try to compare the welfare levels of individuals in different types of households.

In the next section, we formalize the intra-household allocation problem and derive the main results. Section 3 concludes.

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[1] Using experimental data, Bruyneel, Cherchye, and Rock (2012) find that their collective model outperforms the unitary model in terms of 'predictive success,' which simultaneously accounts for the goodness-of-fit and discriminatory power of a particular model specification.

[2] A related study is Peters (2011), who finds that access to family planning and maternal and child health services has a positive effect on female bargaining power in the household in Bangladesh. Because she only uses the ability to make certain purchases without permission as a measure of female bargaining power, her results may not be applicable to the outcome of bargaining (i.e., intrahousehold allocation of resources).

2 Main results

We consider a collective model for a household, which consists of a husband h, a wife w and possibly children and other household members. There are i private goods and c public goods in the economy. The total number of goods in the economy is $n \equiv i + c$. Private goods are enjoyed either by the husband or by the wife, whereas the public goods are enjoyed in common. Public goods may include those goods consumed by the children and other household members, because the husband and the wife care about their consumption or welfare.[3]

Let $\mathbb{R}^i_+$ and $\mathbb{R}^c_+$ be the consumption sets of private and public goods, respectively, for a household. Further, we let Y be the household income and $p \in \Delta(\mathbb{R}^n_{++})$ the price vector of the n goods, respectively, where $\Delta(\bullet)$ is a unit simplex for a set $\bullet$. With some slight abuse of notation, we denote by p^i and p^c the price vectors of private and public goods, respectively, with $p = (p^i, p^c) = (p_1, \ldots, p_i, p_{i+1}, \ldots, p_n)$. Similarly, we denote the index sets of private and public goods by $\mathbb{I}^i \equiv \{1, \ldots, i\}$ and $\mathbb{I}^c \equiv \{i+1, \ldots, n\}$, respectively.

Each member $m \in \{w, h\}$ has a cardinal utility function $U^m : \mathbb{R}^i_+ \times \mathbb{R}^c_+ \to \mathbb{R}_+$, which is strictly increasing, strictly concave, and twice continuously differentiable on $\mathbb{R}^n_+$. As with Blundell, Chiappori, and Meghir (2005), we assume that the allocation is Pareto-efficient for the husband and wife. Then, for any Pareto-efficient allocation $(\tilde{x}^w, \tilde{x}^h, \tilde{x}^c)$, there exists a unique *household Pareto weight* $\lambda \in [0, 1]$ such that $(\tilde{x}^w, \tilde{x}^h, \tilde{x}^c)$ is supported as follows:[4]

$$\begin{aligned}(\tilde{x}^w, \tilde{x}^h, \tilde{x}^c) = \underset{(x^w, x^h, x^c)}{\operatorname{argmax}} \ & \lambda U^w(x^w, x^c) + (1-\lambda) U^h(x^h, x^c) \\ \text{s.t.} \quad & p^i(x^w + x^h) + p^c x^c \leq Y. \end{aligned} \tag{1}$$

The household Pareto weight λ reflects the wife's overall decision power and summarizes how the resources are allocated within the household. In what follows, we shall exclude the degenerate cases where $\lambda \in \{0, 1\}$ because they are in effect cases of a single decision-maker.

We shall hereafter maintain the *weak separability*[5] of $\mathbb{I}^m$ from $\mathbb{I}^c$ in U^m. Formally, for any $j, k \in \mathbb{I}^m$ and any $l \in \mathbb{I}^c$,

$$\frac{\partial}{\partial x^c_l}\left(\frac{\partial U^m / \partial x^m_j}{\partial U^m / \partial x^m_k}\right) = 0,$$

where x^m_j and x^m_k are the j-th and k-th components of the vector of m's private goods consumption $x^m \in \mathbb{R}^i_+$, and x^c_l is the l-th component of the vector of public

[3] One can think of "private" and "public" simply as labels and adopt an alternative classification. As long as the weak separability assumption discussed below is maintained, the main points of this paper hold without any modification under the alternative classification. However, because the primary example of the application of this study is childbirth, we only consider the classification of private and public goods in this paper.

[4] We use parentheses and square brackets and to denote open and closed intervals, respectively. Therefore, $[0, 1]$ denotes a unit interval including both ends (i.e., 0 and 1), whereas $(0, 1)$ excludes them.

[5] The separability condition is often useful for obtaining the identification of the sharing rule Blundell, Chiappori, and Meghir (2005); Browning, Chiappori, and Lechene (2006), even though it is not always necessary. Blundell and Robin (2000) provide a generalization of weak separability. Thomas (1990) provides an empirical application of weak separability to intra-household resource allocation.

goods consumption $x^c \in \mathbb{R}^c_+$. The weak separability condition states that the marginal rate of substitution between any two private goods does not depend on the level of public goods consumption.

The weak separability assumption necessarily holds if the household budget is allocated in two stages. In the first stage, the total budget is split into private goods expenditure and public goods expenditure. Then, the bundle of private [public] goods to be consumed is determined solely by the private [public] goods expenditure and private [public] goods prices. For the Japanese households, the separability assumption was not rejected under a few alternative definitions of private and public goods (Fujii and Ishikawa, 2012).

Given the weak separability condition, there exist continuous, strictly increasing, and strictly quasi-concave functions u^m and $\bar{U}^m$ such that $U^m(x^m, x^c) = \bar{U}^m(u^m(x^m), x^c)$.[6] We call u^m the private sub-utility function because it represents the utility that the member m derives from his private goods consumption. Under the weak separability condition, the maximization problem Eq. (1) can be written as follows:

$$\begin{aligned} \max_{(x^w, x^h, x^c)} & \lambda \bar{U}^w(u^w(x^w), x^c) + (1-\lambda)\bar{U}^h(u^h(x^h), x^c) \\ \text{s.t.} \quad & p^i(x^w + x^h) + p^c x^c \leq Y. \end{aligned} \tag{2}$$

Let us denote the maximand of Eq. (2) by $W_\lambda(x^w, x^h, x^c)$. Then, it can be shown that $W_\lambda(x^w, x^h, x^c)$ is continuous, strictly increasing, and strictly quasi-concave. Further, for the j-th and k-th components in any $x^m \in \mathbb{R}^i_+$ and the l-th component in any $x^c \in \mathbb{R}^c_+$, the following holds:

$$\frac{\partial}{\partial x^c_l}\left(\frac{\partial W_\lambda / \partial x^m_j}{\partial W_\lambda / \partial x^m_k}\right) = \frac{\partial}{\partial x^c_l}\left(\frac{\partial U^m / \partial x^m_j}{\partial U^m / \partial x^m_k}\right) = \frac{\partial}{\partial x^c_l}\left(\frac{\partial u^m / \partial x^m_j}{\partial u^m / \partial x^m_k}\right) = 0.$$

The following proposition follows directly from this equation and the definition of weak separability:

Proposition 1 *For $m \in \{h, w\}$, $\mathbb{I}^m$ is weakly separable from $\mathbb{I}^c$ in W_λ.*

Proposition 1 states that weak separability at the individual level is preserved in the household utility function W_λ.[7] This suggests that we can follow the changes in the resource allocation within the household by looking at the space of private goods. Let us now formally define the private sub-problem for the household problem Eq. (1) as follows:

Definition 1 (Private sub-problem) Suppose that the allocation $(\tilde{x}^w, \tilde{x}^h, \tilde{x}^c)$ is Pareto-efficient. The problem $(u^w, u^h; y, \mu)$ is called *the private sub-problem* of the household problem $(U^w, U^h; Y, \lambda)$ if $(\tilde{x}^w, \tilde{x}^h)$ is supported in the following problem:

$$(\tilde{x}^w, \tilde{x}^h) = \operatorname*{argmax}_{(x^w, x^h)} \ \mu u^w(x^w) + (1-\mu)u^h(x^h) \quad \text{s.t.} \quad p^i(x^w + x^h) \leq y,$$

where $y (\leq Y)$ is the private-good expenditure and $\mu \in (0, 1)$ is the private Pareto weight.

[6] See Theorem 3.3b in Blackorby, Primont, and Russell (1978).

[7] Notice, however, that $\mathbb{I}^w \cup \mathbb{I}^h$ is not necessarily weakly separable from $\mathbb{I}^c$. We do not need such a condition for our purpose.

It can be shown that the private sub-problem exists under Pareto efficiency and the weak separability conditions.

Theorem 1 *Suppose that* $(\tilde{x}^w, \tilde{x}^h, \tilde{x}^c)$ *maximizes the maximand* W_λ *in Eq. (2). Then, a private sub-problem* $(u^w, u^h; y, \mu)$ *exists.*

Proof: Consider a Pareto efficient allocation $(\tilde{x}^w, \tilde{x}^h, \tilde{x}^c)$. This allocation must satisfy the first-order conditions of the maximization problem in Eq. (2). Hence, with some slight abuse of notation, we have the following for $m \in \{w, h\}$ and any $l_1, l_2, l_3, l_4 \in \mathbb{I}^i$:

$$\frac{\partial \bar{U}^m}{\partial u^m}\frac{\partial u^m}{\partial x^m_{l_1}} \Bigg/ \frac{\partial \bar{U}^m}{\partial u^m}\frac{\partial u^m}{\partial x^m_{l_2}} = \frac{\dfrac{\partial u^m}{\partial x^m_{l_1}}}{\dfrac{\partial u^m}{\partial x^m_{l_2}}} = \frac{p^i_{l_1}}{p^i_{l_2}}; \tag{3}$$

$$\frac{\lambda \dfrac{\partial \bar{U}^w}{\partial u^w}\dfrac{\partial u^w}{\partial x^w_{l_3}}}{(1-\lambda)\dfrac{\partial \bar{U}^h}{\partial u^h}\dfrac{\partial u^h}{\partial x^h_{l_4}}} = \frac{p^i_{l_3}}{p^i_{l_4}}. \tag{4}$$

This follows from the weak separability of W_λ.

Now, let us denote the marginal utility from the private sub-utility evaluated at $(\tilde{x}^m, \tilde{x}^c)$ by $\phi^m \equiv \partial \bar{U}^m / \partial u^m \big|_{(x^m, x^c) = (\tilde{x}^m, \tilde{x}^c)}$ for $m \in \{h, w\}$. Further, let $\mu^w \equiv \lambda \phi^w$ and $\mu^h \equiv (1-\lambda)\phi^h$, and set $\mu = \frac{\mu^w}{\mu^w + \mu^h}$ and $y = p^i(\tilde{x}^w + \tilde{x}^h)$. It is straightforward to verify that the first order conditions for the private sub-problem coincide with Eq. (3) and Eq. (4). Since u^m is strictly concave, the solution to the first-order conditions corresponds to the unique allocation in $(u^w, u^h; y, \mu)$. This proves the existence of the private sub-problem. □

Theorem 1 suggests that we can potentially track the changes of resource allocation within the household by focusing on the space of the private sub-problem. In general, however, the relationship between the household Pareto weight λ and the private Pareto weight μ may not be monotonic. It turns out that the following condition on the elasticity ρ of the ratio $\psi \equiv \phi^w / \phi^h$ of marginal utilities with respect to λ is important to ensure that λ and μ move in the same direction.

EL: The elasticity $\rho \equiv \psi/\lambda \cdot \partial\lambda/\partial\psi$ satisfies $\rho > -(1-\lambda)^{-1}$ for any $\lambda \in (0, 1)$, $Y \in \mathbb{R}_+$ and $p \in \Delta(\mathbb{R}^n_{++})$.

Note that ϕ^m is a function of Y, p and λ, because $\tilde{x}^w$, $\tilde{x}^h$ and $\tilde{x}^c$ are functions of Y, p and λ.

When λ increases, the wife's sub-utility increases while the husband's decreases. Then, ψ decreases because ϕ^m and u^m tend to move in the opposite direction. Assumption **EL** requires that the proportional change in ψ be sufficiently small in absolute terms relative to the proportional change in λ. While Assumption **EL** cannot be tested, it is violated only if $\rho < -1$. That is, it cannot be violated provided the proportional changes in λ exceed those in ψ in absolute terms. Under Assumption **EL**, the private Pareto weight μ moves in the same direction as the household Pareto weight λ as shown in the following theorem.

Theorem 2 $\partial\mu/\partial\lambda > 0$ *holds if and only if* **EL** *holds.*

Proof: We use the notations in the proof of Theorem 1. Then,

$$\begin{aligned}\frac{\partial\mu}{\partial\lambda} &= \frac{\mu^w_\lambda\mu^h - \mu^h_\lambda\mu^w}{(\mu^w+\mu^h)^2} = \frac{\phi^w\phi^h}{(\mu^w+\mu^h)^2}\left[1+\lambda(1-\lambda)\left(\frac{\phi^w_\lambda}{\phi^w}-\frac{\phi^h_\lambda}{\phi^h}\right)\right] \\ &= \frac{\phi^w\phi^h}{(\mu^w+\mu^h)^2}\left(1+(1-\lambda)\rho\right).\end{aligned}$$

Because ϕ^h, ϕ^w, and $1-\lambda$ are all positive, $\frac{\partial\mu}{\partial\lambda} > 0$ holds if and only if $\rho > -(1-\lambda)^{-1}$. □

As is clear from the construction of μ in the proof of Theorem 1, μ is a function of p, Y and λ. Theorem 2 shows that after controlling for the changes in p and Y we only need to examine the changes in the private Pareto weight μ in order to follow those in the household Pareto weight λ.

3 Conclusion

We have shown that the allocation of private goods in any Pareto-efficient allocation in a household can be represented as a Pareto-efficient allocation in the private sub-problem. This result follows from the weak separability assumption. While a similar result is presented in Blundell, Chiappori, and Meghir (2005), they treat the private and public goods as a Hicksian composite good. As discussed in Hicks (1946, p.33), a sufficient condition for the existence of a Hicksian composite good is that the relative prices of all the component goods are constant, which is highly restrictive.[8] Without making such a restrictive assumption, we derive the relationship between the household and private Pareto weights and provide a meaningful interpretation of the private Pareto weight under Assumption **EL**.

Our results are useful for analyzing the intrahousehold resource allocation when total individual private consumption data are available. Using the model presented here, Fujii and Ishikawa (2012) analyzed the impact of childbirth on intrahousehold resource allocation in Japan using the JPSC data that include the total private consumption expenditures. While the availability of total individual private consumption data is currently limited, a number of interesting studies on intrahousehold resource allocation could be generated using our analytical framework when more data become available.

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[8] Weiss and Sharir (1978) show a slightly less restrictive sufficient condition, but their condition is still restrictive. Chiappori (2011) develops a collective model that does not use Hicksian composite goods. His model, however, lacks public goods.

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