PAPER *Special Section on Description Models for Concurrent Systems and Their Applications*

Introduction of Economic-Oriented Fairness to Process Algebras

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SUMMARY Fairness is one of the important notion for programming language, such as process algebras like CCS, that includes concurrency (or parallelism) and nondeterminism. This ensures that while repeatedly choosing among a set of alternatives, no alternative will be postponed forever. However, the fairness does not mention at what frequency these alternatives are selected. In this paper, we propose a quantitative fairness, which is called economic-oriented fairness, to each alternatives. This fairness ensures that the expected number of selection for each alternatives are same. We give a condition for probability assignment of selection of each alternative to be satisfied for economic-oriented fairness. First we show a simple probability assignment rule. In this assignment, between any two alternatives, if an alternative is selected n times and the other m times then the probability to select the former alternative is $(m+1)/(n+1)$ times the probability for the latter. We prove that this assignment satisfies the condition of economic-oriented fairness. For a model of the economic-oriented fairness, we adopt a probabilistic process algebra. Finally, we elaborate with two process models of the economic-oriented fairness. The first one is a server and client model, where each client communicates only with the server, but not among them. In this model, the expected number of communication by each client are equal. The second model considers communication between two processes. In practice, a process has several subprocesses. Each subprocess communicates via a communication port. In the second model, there is economic-oriented fairness where the expected number of communications via each communication port are equal.

key words: *fairness, economic-oriented fairness, process algebra, CCS*

1. Introduction

Fairness [5], [10] is one of the important notion for programming language, such as process algebras like CCS [8], which includes concurrency (or parallelism) and nondeterminism. This ensures that while repeatedly choosing among a set of alternatives, no alternative will be postponed forever. For example, in a network system a node P sends a message to either a node Q or R repeatedly. When this network system does not satisfy the fairness, one of Q or R may not be able to receive any message.

In general, fairness is implicitly assumed. So both Q and R will receive a message eventually in a practical system. However, the fairness does not say anything about the frequency at which Q and R receive messages. Even though Q receives several thousand messages, R may receive only one. Is it really "fair" in practice? This paper introduces the concept of quantitative fairness. Paper [5] named such fairness as "economic-oriented." So we also call this fairness economic-oriented fairness. In this paper, we consider the economic-oriented fairness as a probabilistic fairness. In the above example, if economic-oriented fairness is satisfied, the probability that Q and R receive a message from P are same. More precisely, the expected number of received messages of each process at different step must be same. Therefore, P must assign probability while sending messages to different destinations.

Section 2 briefly introduces probabilistic process algebra and related definition of economic-oriented fairness. This probabilistic process algebra is a subset of probabilistic CCS [6], [8] with a special operator $\prod$. This is a compositional operator whose subprocesses satisfy the economic-oriented fairness.

Section 3 demonstrates two process models as the examples of the economic-oriented fairness. The models do not consider any silent actions. The first one is a server and client model, where each client communicates only with the server, but not among them. In this model, the expected number of communication by each client are equal. The second model considers communication between two processes. In practice, a process has several subprocesses. Subprocesses communicate via communication ports. In the second model, there is economic-oriented fairness where the expected number of communication via each communication port are equal.

For these models, we give a condition for probability assignment of selection of each alternative to be satisfied for economic-oriented fairness. We show a simple probability assignment rule. In this assignment, between any two alternatives, if an alternative is selected n times and the other m times then the probability to select the former alternative is $(m+1)/(n+1)$ times the probability for the latter. The addition of 1 is necessary for the case where $n=0$. We prove that this assignment satisfies the condition of economic-oriented fairness.

Finally, we conclude this paper in Sect. 4.

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2. Process Algebras and Its Economic-Oriented Fairness

First, we briefly introduce a probabilistic process algebra. It is a subset of probabilistic CCS [6], [8] with a special operator $\prod$. This is a compositional operator whose subprocesses satisfy the economic-oriented fairness.

Let $\mathcal{A}$ be a finite or infinite set of *actions*. $\mathcal{E}$ is a set of *process expressions*, which is a minimum set satisfying the following conditions, where $\mathcal{X}$ is a set of all process variables:

1. $\mathbf{0} \in \mathcal{E}$,
2. $\mathcal{X} \subseteqq \mathcal{E}$,
3. if $a \in \mathcal{A}$ and $E \in \mathcal{E}$ then $a.E \in \mathcal{E}$,
4. if $E, F \in \mathcal{E}$ then $E + F \in \mathcal{E}$,
5. if $X \in \mathcal{X}$ and $E \in \mathcal{E}$ then $\mathbf{fix}\, X.E \in \mathcal{E}$.

$\mathbf{0}$ is called an *inaction*, which does not perform any actions. $a.E$ is called an *action prefix*. It performs an action a and becomes a process E. $E + F$ is a *summation*, which performs a process E or F. The choice between E and F are nondeterministic. The semantics of a process expression is given by the *transition relation* $\rightarrow\ \subseteqq \mathcal{E} \times \mathcal{A} \times \mathcal{E}$, which is defined as follows. We omit $(E, a, F) \in\rightarrow$ by $E \xrightarrow{a} F$. Intuitively, $E \xrightarrow{a} F$ means that E performs the action a and becomes F. For any $E, F \in \mathcal{E}$, $a \in \mathcal{A}$ and $X \in \mathcal{X}$, the transition relation $\rightarrow$ is a minimum relation satisfying the followings:

1. $a.E \xrightarrow{a} E$,
2. if $E \xrightarrow{a} E'$ then $E + F \xrightarrow{a} E'$,
3. if $F \xrightarrow{a} F'$ then $E + F \xrightarrow{a} F'$,
4. if $E\{\mathbf{fix}\, X.E/X\} \xrightarrow{a} E'$ then $\mathbf{fix}\, X.E \xrightarrow{a} E'$,

where $E\{F/X\}$ is a process expression E. Here all free occurrences of X in E are replaced by F. The notions of free and boundness are as usual. A bound process expression is called a *process*. In the rest of this paper, we deal with processes only.

The economic-oriented fairness in this paper ensures that while one has to repeatedly choose among a set of alternatives, i.e. processes, expected number of selection of each alternative are equal at each transition step. The rest of this section introduces the new process operator $\prod$, which is a compositional operator whose subprocesses satisfy the economic-oriented fairness.

First, we introduce a probabilistic transition relation for a process to calculate the expected number of each alternatives.

Definition 2.1: Let $N \geqq 1$ and each P_i a process where $0 \leqq i \leqq N-1$. Then for any $n_i \geqq 0$, $\prod_{i=0}^{N-1} n_i : P_i$ is a *compositional process*. In the case of $N = 2$, $\prod_{i=0}^{1} n_i : P_i$ could be simply expressed as $P_0\ {}_{n_0}\|_{n_1} P_1$. □

The semantic of a compositional process is defined by the *probabilistic labeled transition relation* $\rightarrow$. The probabilistic transition relation $\rightarrow\ \subseteqq \mathcal{P} \times \mathcal{A} \times \mathcal{P} \times Pr$, where $\mathcal{P}$ is a set of all compositional processes and $Pr = \{pr \mid 0 \leq pr \leq 1\}$. We express $(P, a, Q, p) \in\rightarrow$ by $P \xrightarrow{a}_p Q$, which means P can perform the action a with probability p, and after action a, it becomes Q. The probability p of $\rightarrow_p$ is defined by the function $\mu : \mathcal{P} \times \mathcal{A} \times \mathcal{P} \longrightarrow [0, 1]$. This is a total function called the *probabilistic transition function*, satisfying the following restriction: $\forall P \in \mathcal{P}$,

$$\sum_{a \in \mathcal{A}, Q \in \mathcal{P}} \mu(P, a, Q) = 1$$

To satisfy the economic-oriented fairness, the probability of a transition depends on n_is of the compositional process $\prod_{i=0}^{N-1} n_i : P_i$. The number n_i can be considered as characteristic number. It may be changed at each transition. In the next section, two process models are illustrated.

We can easily extend the above transition relation to an action sequence. That is, $\mu(P, \varepsilon, P) = 1$ and $\forall t \in \mathcal{A}^*$, if $\mu(P, t, Q) = p$ and $\mu(Q, a, R) = q$, then $\mu(P, t \cdot a, R) = p \cdot q$. It is obvious that for any process P and $m \geqq 0$:

$$\sum_{Q \in \mathcal{P}, t \in \mathcal{A}^m} \mu(P, t, Q) = 1.$$

From the probabilistic transition function μ, we can calculate the expected number to define the economical-oriented fairness. For a compositional process P, the expected number can be calculated at each transition step m. It depends on the number of alternatives N. The definition of expected number also depends on what is fair with respect to the model. So there exists model dependent parameter t. We can describe the expected number by $EX(P, N, m, t)$.

We define the economic-oriented fairness for compositional processes as follows.

Definition 2.2: Let $P \in \mathcal{P}$ and N the number of subprocesses of an operator $\prod$ in P. P satisfies the economic-oriented fairness, if for any transition step m and model dependent parameter t, the expected number $EX(P, N, m, t) = m/N$. □

3. Process Models of Economic-Oriented Fairness

This section demonstrates two process models as examples of the economic-oriented fairness. Both models do not consider any silent actions. Thus, we assume that the set of actions $\mathcal{A}$ has no silent actions.

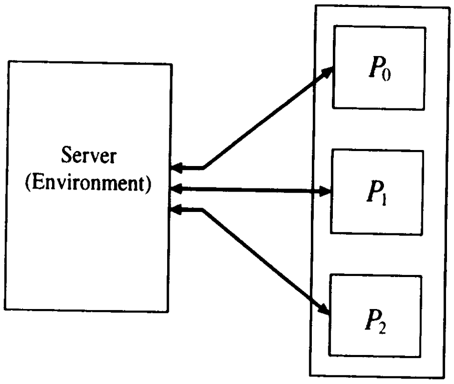

Fig. 1 A server and client model.

3.1 A Server and Client Model

The first one is a server and client model, where each client communicates only with the server, but not among them. The number of server is one, as we consider the server as the environment of the model. The clients are expressed by subprocesses of $\prod$. Figure 1 illustrates a server and client model, which has three clients P_0, P_1 and P_2. This model is expressed by $\prod_{i=0}^{2} n_i : P_i$.

If the economic-oriented fairness is satisfied in this model, then the server (i.e. the environment) must extend services (in this case, communication opportunities) equally to each client. Therefore, the expected number in Definition 2.2 is defined as the number of actions from each subprocesses of $\prod$.

The number n_i of a composition process $\prod_{i=0}^{N-1} n_i : P_i$ expresses how many actions the subprocess P_i performs. So the number n_i at the initial state of a composition process is zero. For example, $P\, {}_2\|_3 Q$ shows that five actions are performed from its initial composition process $P_0\, {}_0\|_0 Q_0$. Two are by P and three are by Q in some order. At this point, if P performs an action and becomes P', then its result composition process is $P'\, {}_3\|_3 Q$.

The probabilistic transition relation under the server and client model is defined as follows.

Definition 3.1: The probabilistic transition relation under the server and client model $\rightarrow$ is a minimum relation which satisfies the following condition: for any $a \in \mathcal{A}$, and $0 \leqq j \leqq N-1$,

$$\text{if } P_j \xrightarrow{a}_p Q \text{ then } \prod_{i=0}^{N-1} n_i : P_i \xrightarrow{a}_{p'} \prod_{i=0}^{N-1} n'_i : P'_i,$$

where $n'_i = n_i (i \neq j)$ or $n_j + 1 (i = j)$ and $P'_i = P_i (i \neq j)$ or $Q(i = j)$. p and p' are defined by $\mu(P_j, a, Q)$ and $\mu(\prod_{i=0}^{N-1} n_i : P_i, a, \prod_{i=0}^{N-1} n'_i : P'_i)$ respectively. □

The condition to be satisfied for economic-oriented fairness is as follows. Before introducing the theorem, we define some notations. Let N be the number of alternatives. Corresponding to each alternative, a number from 0 to $N-1$ is used. At the transition step m, we can assign a number 0 to $N^m - 1$ to each transition state as follows. Let the state at the transition step 0 correspond to 0. From x-th transition state at the transition step m, suppose the y-th alternative is selected. Then the transition state after this selection, i.e. at the $(m+1)$-th transition step, is assigned as $xN + y$. With this rule, each transition state corresponds to a unique number. Let $p_m(i)$ be the probability at the i-th transition state and the transition step m. $p_m(i,j)$ is the probability to select j-th alternative at the transition state i and the transition step m. It is obvious that $p_{m+1}(iN+j) = p_m(i) \cdot p_m(i,j)$.

In the case of compositional processes, a model dependent parameter t is each subprocess number j. Thus, $EX(P, N, m, j)$ means the expected number of actions which P_j performed. Let $n_m(i,j)$ be the number of actions which subprocess P_j performed from the initial process to the one at the transition state i and the transition step m. Until m-th transition step, m actions are performed. Thus for any i and m, $\sum_{j=0}^{N-1} n_m(i,j) = m$. At the transition state x and the transition step m, if the y-th subprocess is selected, then the transition state x becomes the $(xN+y)$-th transition state at the transition step $(m+1)$, where $0 \leqq x \leqq N^m - 1$ and $0 \leqq y \leqq N-1$. Therefore, $n_{m+1}(xN+j, j) = n_m(x,j) + 1$ and $n_{m+1}(xN+y, j) = n_m(x,y)$ if $y \neq j$. The expected number $EX(P, N, m, j)$ is defined as follows:

$$EX(P, N, m, j) = \sum_{i=0}^{N^m - 1} n_m(i,j) \times p_m(i)$$

The theorem to be satisfied for economic-oriented fairness is introduced.

Theorem 3.2: Let $P \in \mathcal{P}$ under the server and client model, and N the number of subprocesses of an operator $\prod$ in P. P satisfies the economic-oriented fairness, iff for any transition step $m \geqq 1$:

$$\forall j. \sum_{i=0}^{N^m - 1} p_m(i) \cdot p_m(i,j) = 1/N$$

Proof: $(\Longrightarrow)$ Since P satisfies the economic-oriented fairness, the expected number $EX(P, N, m, j) = \sum_{i=0}^{N^m - 1} n_m(i,j) \cdot p_m(i) = m/N$. For $EX(P, N, m+1, j)$, the followings are derived.

$$\begin{aligned} & EX(P, N, m+1, j) \\ &= \sum_{i=0}^{N^{m+1}-1} n_{m+1}(i,j) \cdot p_{m+1}(i) \\ &= \sum_{x=0}^{N^m - 1} \sum_{y=0}^{N-1} n_{m+1}(xN+y, j) \cdot p_{m+1}(xN+y) \\ &= \sum_{x=0}^{N^m - 1} \left\{ \left(\sum_{y=0}^{N-1} n_m(x,j) \cdot p_m(x) p_m(x,y) \right) \right. \end{aligned}$$

$$+ p_m(x)p_m(x,j)\Bigg\}$$

$$= \sum_{x=0}^{N^m-1}\left\{\left(n_m(x,j)p_m(x)\sum_{y=0}^{N-1}p_m(x,y)\right) + p_m(x)p_m(x,j)\right\}$$

$$= EX(P,N,m,j) + \sum_{x=0}^{N^m-1} p_m(x)p_m(x,j)$$

$$= (m+1)/N.$$

Therefore, $\sum_{x=0}^{N^m-1} p_m(x)p_m(x,j) = 1/N$.

($\Longleftarrow$) Same as the above proof. □

Next, we discuss assignment of the transition probability, i.e. $p_m(i,j)$. Indeed, for any m, i and j, if $p_m(i,j) \equiv 1/N$, where N is the number of alternatives, then Theorem 3.2 is satisfied. Thus, this is one of the definition for economic-oriented fairness. However this assignment is too simple to match any practical situation. Let a compositional process $P = \mathbf{fix}\, X.a.X\ {}_0\|_0 \mathbf{fix}\, X.b.X$. For an action sequence $t = a \cdots a$, where the number of a is m, suppose $P \xrightarrow{t} P'$. From the above assignment, even though no b is performed, the probability of selecting a or b is 1/2 respectively. We would like to increase probability of selecting b over a.

Here, we define another probability assignment rule. In this assignment, between any two alternatives, if an alternative is selected n times and the other m times then the probability to select the former alternative is $(m+1)/(n+1)$ times the probability for the latter. The addition of 1 is necessary for the case where $n = 0$. Let P be a compositional process which has N subprocesses. As we have already mentioned, a number could be assigned to a transition state of P. Let its transition probability $p_m(i,j)$ be the sum of all transition probabilities from j-th subprocess at i-th transition state and m-th transition step. The following definition of $p_m(i,j)$ satisfies the above property.

Definition 3.3: Let P be a compositional process under the server and client model which has N subprocesses. After m actions are performed, suppose P becomes $\prod_{i=0}^{N-1} n_i : P_i$. The transition probability $p_m(i,j)$, which has occurred from j-th subprocess at i-th transition state and m-th transition step, is defined as

$$p_m(i,j) = \left(\prod_{k \neq j}(n_k+1)\right) / \sum_{y=0}^{N-1}\prod_{k\neq y}(n_k+1)$$

□

It is easy to check that $p_m(i,x)/p_m(i,y) = (n_y+1)/(n_x+1)$ for any x and y. By the definition of $p_m(i,j)$, the probability $p_m(i)$ is defined as the rule mentioned after Definition 3.1. The following proposition shows that a compositional process satisfying the previous definition satisfies economic-oriented fairness.

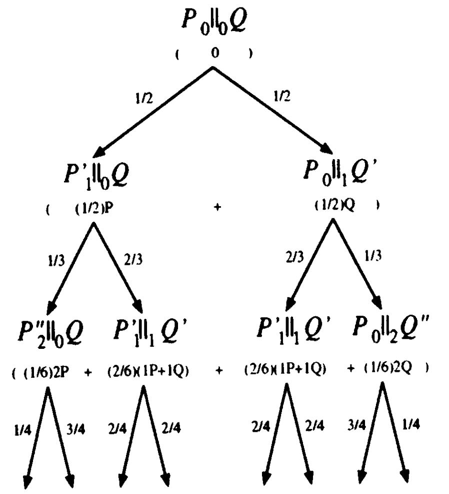

Fig. 2 An example of a part of an economical-fair probabilistic transition tree.

Proposition 3.4: Let P be a compositional process under the server and client model which has N subprocesses. For any transition step m, transition state i and subprocess j:

$$\forall j.\ \sum_{i=0}^{N^m-1} p_m(i)\cdot p_m(i,j) = 1/N$$

Proof: By induction on m. □

Figure 2 shows an example of a transition tree according to Definition 3.3. In this figure, the number p at each transition relation $R \xrightarrow{p} S$ shows the value of transition probability $p_m(i,j)$, where R is a process at the transition state i and the transition step m. By performing an action at j-th subprocess, R becomes S. $mP+nQ$ under each compositional process in this figure means that the sum of the expected number of actions by P's subprocesses is m and that by Q's is n. For example, at $P'\ {}_1\|_1 Q'$, expected number of actions both by P and Q are $2/6 \cdot 1$. The expected number of actions from P and Q at second transition step is 1.

3.2 Communication between Two Processes

The second model considers communication between two processes. In practice, a process has several subprocesses. Each subprocess communicates via respective communication port. In Fig. 3, processes P and Q have two subprocesses each, P_0, P_1 and Q_0, Q_1 respectively. P_0 communicates to Q_0 via the port a. P_1 communicates to Q_1 via either port b or c. The assignment of the transition probability in Definition 3.3 ensures that subprocesses of P and Q satisfy the economic-oriented fairness, i.e. the expected number of actions which are performed by each subprocess is equal. However, with what frequency the communication ports are selected? P_1 and

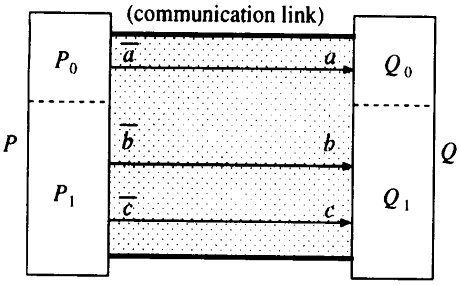

Fig. 3 An example of a system of two communicating processes.

Q_1 can communicate only via port b, even though the economic-oriented fairness is satisfied. In such models, each communication port is not ensured to be fair. In the rest of this section, we introduce the new assignment of the transition probability based on the fairness for communication ports. For this assignment, the transition system in Definition 3.1 is also modified.

In general, there is one physical communication link between P and Q. This link is logically divided and used by several ports. The economic-oriented fairness for communication ports provides the fairness about link throughput.

First, we introduce new notation of a process to count the number of actions at each communication port. It is a process with a *counting set*, expressed as P_C, where P is a process and C is a counting set. The counting set is a set of 2-tuples. The first term is a port name, and the second term a natural number. They represent how many communications the process performs via each communication port. A set of actions included in a process is called a *sort*. Let $Sort(P)$ be a sort in P. The counting set C of an initial process P is $C = \{(a, 0) \mid a \in Sort(P)\}$.

For example, $P_{\{(a,2),(b,3)\}}$ shows that the sort of the initial process of P is $\{a, b\}$, and two as and three bs are executed in some order from the initial process of P. If P executes an a and becomes Q, then the counting set of Q is $\{(a,3),(b,3)\}$. The transition probability is decided from this counting set.

Note that if the sort of a process is finite, then the expression of a process with counting set can be transfered to a compositional process in Definition 2.1. Therefore, the definition of fairness in Definition 2.2 can be applied to a process with counting set if the sort is finite. Let N be the number of actions in a sort. Then, we can correspond a number from 0 to $N-1$ to each action in the sort. As in the case of client-server model, we can assign a number 0 to $N^m - 1$ to each transition state at the transition step m. We can also define the probability $p_m(i,j)$ and $p_m(i)$. The former is the probability to select j-th action at the transition state i and the transition step m. The latter is the probability at the i-th transition state and the transition step m. Then, Theorem 3.2 is also satisfied.

The semantics of a process with a counting set is defined by the following probabilistic transition system.

Definition 3.5: The probabilistic transition relation $\rightarrow$ between two processes with each counting set is a minimum relation which satisfies the following condition: for any $a \in Sort(P)$,

$$\text{if } (a,n) \in C \text{ and } P \xrightarrow{a} Q \text{ then } P_C \xrightarrow{a}_p Q_{C'}$$

where $p = \mu(P, a, Q)$ and $C' = (C - \{(a,n)\}) \cup \{(a, n+1)\}$. □

From the above definition, we can define the assignment of a transition probability $p_m(i,j)$ as in Definition 3.3. From Proposition 3.4, this assignment satisfies the economic-oriented fairness.

4. Conclusion

In this paper, the notion of economical-oriented fairness for a compositional processes is introduced. We demonstrated the economic-oriented fairness with two common models, a client and server model and a system of two communicating processes. We also proved that the defined conditions actually ensured economic-oriented fairness. The economical-oriented fairness is defined under compositional processes. To extend the economical-oriented fairness to general processes, we need to solve several other problems.

First, there are some restrictions on the compositional processes. The compositional operator $\prod$ is applicable only to interleaved processes, and thus the subprocesses of $\prod$ are not allowed to communicate among them. We need to deal with communication between subprocesses, i.e. a silent action, to define the economic-fairness for general processes. However, the environment cannot recognize silent actions. Therefore, it is difficult to count the number of actions performed by each subprocess.

The process operators in this paper are only basic ones. Other process operators, especially a restriction operator should be introduced. However, the following problem will appear when dealing with the restriction operator. That is, the effective transition probability of a probabilistic process is modified by the restriction operator (\ in CCS). The restriction operator restricts performing some actions. For example, let $P = (a.\mathbf{0} \mid b.\mathbf{0})\backslash\{b\}$, where the operator '|' is a compositional operator in CCS. By the restriction operator, P cannot perform b (and $\bar{b}$). Even if the transition probabilities of two subprocesses are equal (i.e. 0.5), the latter subprocess cannot perform any action. Therefore, the effective transition probability of the former subprocess should be 1.

In CCS, only communications between two processes are allowed. It would be interesting to introduce a process in a multi synchronization system, such as CSP [7] or LOTOS [2].

Second, in this paper, the economic-fairness is defined for communication between two (sub)processes.

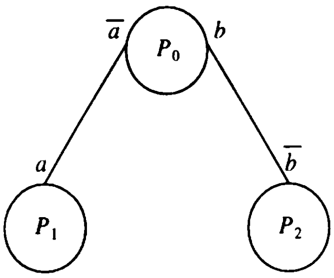

Fig. 4 A tree type topology of a process $P_0|P_1|P_2$.

In this case, it is sufficient to consider the communication between a process and its environment. However, the economic-fairness between more than three (sub)processes will be much complicated. For example, consider the process $P = P_0|P_1|P_2$. There are three kind of internal communications, i.e. communications between P_0 and P_1, P_1 and P_2, and P_2 and P_0. In network systems, the network topology of a process is very important to define its fairness. Figure 4 shows a tree type topology of P. In this topology, P_0 and P_1 are connected by a port a, and P_0 and P_2 are connected by a port b. Assume the three subprocess satisfies the economic-oriented fairness in Definition 2.2. Therefore the expected number of actions which each subprocess performs is equal. However, if the port a and b are concealed by the environment, i.e. no messages are sent to or received from the environment, this assumption is inconsistent. This is because, if the process P_1 and P_2 send or receive the same number of messages, then P_0 send or receive twice. When the economic-fairness based on communication ports is defined, there is another problem. In Fig. 4, there is no connection between P_1 and P_2. Thus, if P_2 sends a message to P_1 then the message is transmitted via P_0. We will need to discriminate messages from P_2 to P_1 and those from P_2 to P_0, to define the fairness based on communication ports between P_2 and P_0.

Third, the expected number is always fixed to m/N in Definition 2.2. In practice, it is useful to be able to define the expected number by each subprocess. This is related to the notion of a priority, i.e. larger expectation numbers has to be assigned to processes with higher priorities.

The economic-oriented fairness in Sect. 3 is defined as the expected number of performed actions from each subprocess or each communication port is fair. The number of performed actions is related to throughput, which is the amount of communications per a unit time. An another important characteristic appreciating networks performance is communication delay. In many process algebras including CCS, however, an action is atomic and instantiation. That is, there is no notion of time in CCS. To represent the fairness for the communication delay, we must introduce timed process([1], [4], for examples).

As mentioned in the introduction, Papers [5], [10] introduce the strong or weak fairness, which is general fairness in process algebras. This ensures that while repeatedly choosing among a set of alternatives, no alternative will be postponed forever. Though the economic-oriented fairness does not ensure the above fact, we can prove that the probability that some alternative will be postponed forever, is zero. This could be derived from the property of probabilistic process [9].

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