2. FUNDAMENTAL THEORY

2.1 Outline of Boundary Layer Theory

[34-38]

2.1.1 The Boundary Layer Concept

The influence of viscosity at high Reynolds numbers is confined to a very thin layer near a solid wall. If the condition of no slip were not to be satisfied for the case of a real fluid, there would be no appreciable difference between the field of flow of the real fluid and that of a perfect fluid. However, the fact that the fluid adheres to the surface at the solid wall means that frictional forces retard the motion of the fluid in a thin layer near the wall. In the thin layer, the velocity of the fluid increases from zero on the solid wall (no slip) to its full value which corresponds to external frictionless flow (main stream). The thin layer which has a large velocity gradient exists near the solid wall. This thin layer is called the boundary layer.

The thickness of this boundary layer increases along a plate in a downstream direction. Figure 2.1 indicates diagrammatically the velocity distribution in a boundary layer on a flat plate. In front of the leading edge of the plate the velocity distribution is uniform. With increasing distance from the leading edge in the downstream direction, the thickness δ of the retarded layer increases continuously. Evidently, the thickness of the boundary layer increases with increasing viscosity.

When a region with an adverse pressure gradient exists along the wall, the retarded fluid particles near the wall cannot, in general, move far downstream against the increased pressure owing to their small kinetic energy. Thus the decelerated fluid particles in the boundary layer do not remain in the thin layer which adheres to the body. In some cases, if the boundary layer increases its thickness considerably in the downstream direction, the downstream movement of the fluid near the wall which has small momentum is prevented and the flow in the boundary layer becomes reversed. This causes the decelerated fluid particles to be forced outwards, which means that the boundary layer is separated from the wall. This phenomenon is called boundary layer separation. Boundary layer separation is always associated with the formation of vortices and with large energy losses in the wake of the body. It occurs primarily near the sharp corner and blunt bodies, such as circular cylinders and spheres. Behind such a body there exists a region of strongly decelerated flow, in which the pressure distribution deviates considerably from that in a frictionless fluid. The large drag of such bodies can be explained by the existence of this large deviation in pressure distribution which is produced by a consequence of boundary layer separation.

Boundary layer separation is an undesirable phenomenon in engineering. For example, if flow separation occurs on airfoils, lift markedly decreases and a large drag of the body is produced. A deterioration in the performance of the diffuser and the impeller in the fluid machinery is produced by flow separation. Furthermore, the machinery is vibrated violently and broken by the surge from flow separation.

The Navier-Stokes equations are modified for the purpose of deriving the boundary-layer equation. In the case of steady flow two-dimensional incompressible boundary-layer equations are written as

$$\left.\begin{aligned} \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \\ u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\mathrm{d}p}{\mathrm{d}x}+\nu\frac{\partial^2 u}{\partial y^2}. \end{aligned}\right\} \quad (2.1)$$

The fact that separation in steady flow occurs only in decelerated flow (dp/dx>0) can be inferred from a consideration of the relation between the pressure gradient dp/dx and the velocity distribution $u(y)$ with the aid of the boundary-layer equation. From Eq. (2.1) with the boundary conditions u=v=0 we have at y=0

$$\mu\left(\frac{\partial^2 u}{\partial y^2}\right)_{y=0} = \frac{dp}{dx} \quad .$$

(2.2)

In the immediate neighborhood of the wall the curvature of the velocity profile depends only on the pressure gradient, and the curvature of the velocity profile at the wall changes its sign with the pressure gradient. For flow with decreasing pressure (accelerated flow, dp/dx<0)) we have from Eq. (2.2) that $\partial^2 u/\partial y^2 < 0$ over the whole width of the boundary layer (see Fig. 2.2). For flow with increasing pressure (decelerated flow, dp/dx>0) we find $\partial^2 u/\partial y^2 > 0$ (see Fig. 2.3). However, in any case where $\partial^2 u/\partial y^2 < 0$ at a large distance from the wall, there must exist a point for which $\partial^2 u/\partial y^2 = 0$. This is a point of inflexion of the velocity profile in the boundary layer.

2.1.2 Boundary Layer Thickness

In general, the boundary layer thickness is defined as that distance from the wall where the velocity differs by one percent from the external velocity. However, it is impossible to determine a boundary layer thickness in an unambiguous way, because the influence of viscosity in the boundary layer decreases asymptotically outwards. Instead of the boundary layer thickness, another quantity, the displacement thickness δ_1 or the momentum thickness δ_2, is sometimes used in a physically meaningful measure for the boundary layer thickness. The displacement thickness indicates the distance by which the streamlines of the external potential flow are displaced as a consequence of the decrease in velocity in the boundary layer owing to the effect of friction near the wall. The decrease in volume flow due to the influence of friction is

$$\int_0^\infty (U_0 - u)dy \quad ,$$

so that the displacement thickness δ_1 is given by

$$U_0\delta_1 = \int_0^\infty (U_0 - u)dy \quad ,$$

or

$$\delta_1 = \int_0^\infty (1 - \frac{u}{U_0})dy \quad .$$

(2.3)

The loss of momentum in the boundary layer due to the viscosity, as compared with potential flow, is given by

$$\int_0^\infty \rho u(U_0 - u)dy,$$

so that the momentum thickness δ_2 can be defined by

$$\rho U_0^2\delta_2 = \rho\int_0^\infty u(U_0 - u)dy \quad ,$$

or

$$\delta_2 = \int_0^\infty \frac{u}{U_0}(1 - \frac{u}{U_0})dy \quad .$$

(2.4)

The displacement thickness and the momentum thickness have a relation to the conservation law of mass and momentum, respectively. The ratio H_{12} is called the shape factor and is defined by

$$H_{12} = \frac{\delta_1}{\delta_2} \quad .$$

(2.5)

The shape factor indicates $H_{12} \fallingdotseq 2.6$ and $H_{12} \fallingdotseq 1.4$ for laminar flow and turbulent flow, respectively, in the boundary layer along a flat plate at zero incidence. As the value of H_{12} becomes smaller, the curvature of the velocity profile in the boundary layer indicates the type of flow with decreasing pressure (accelerated flow, see Fig. 2.2). As the value of H_{12} becomes larger, the curvature of the velocity profile in the boundary layer indicates the type of the flow with increasing pressure (decelerated flow, see Fig. 2.3).

2.2 Flow in Diffusers [35]

For the fluid transfer, pressure losses usually become smaller with decreasing velocity, because the pressure losses increase in proportion to the squared velocity. Therefore, it is important that the kinetic energy is converted to pressure by gradually decreasing the velocity. The flow path which can convert the kinetic energy to pressure energy is called a diffuser. Flow separation occurs easily for flows in the diffuser due to the adverse pressure gradient. Flow separation produces a decrease in the effective cross section of the flow path and therefore the pressure recovery in the diffuser markedly decreases.

Figure 2.4 indicates the characteristics of the flow in the diffuser with rectangular cross-sections. The area ratio *AR* (inlet to exit, $AR = W_e / W_i$) is defined as the ratio between the width of the inlet W_i and the width of the exit W_e of the diffuser. When the divergence angle of diffuser (2α) is small, flow along the surface is produced (see Fig. 2.4(a)). However, if the divergence angle becomes a little wider, flow separation occurs locally because the flow in the boundary layer cannot overcome the pressure gradient (see Fig. 2.4(b)). As the divergence angle increases, the flow is perfectly separated from either wall and produces back-flow (see Fig. 2.4(c)). However, the flow adheres to the wall at the other side (see Fig. 2.4(c)). In this case, the flow is very unstable and therefore the losses then reach a maximum. If the divergence angle further increases, the flow is perfectly separated from the wall on both sides and changes into a jet flow (see Fig. 2.4(d)).

The pressure recovery coefficient C_p is defined as

$$C_p = \frac{p_e - p_i}{(1/2)\rho \bar{u}_i^2} \quad . \tag{2.6}$$

Here p denotes the static pressure and $\bar{u}$ denotes the mean velocity over the cross section, whereas subscript i and e refer to conditions at the inlet and exit of the diffuser, respectively. The performance of the diffuser is evaluated by the diffuser effectiveness η. This is defined as

$$\eta = \frac{Cp}{Cp_{th}} \quad ,$$

(2.7)

where

$$Cp_{th} = 1 - \frac{1}{(AR)^2} \quad ,$$

is the ideal pressure recovery coefficient. However, for the design of diffuser we, in general, prefer a shorter length diffuser to the maximum diffuser effectiveness. The pressure recovery in the diffuser becomes

$$\Delta p = p_e - p_i = \frac{1}{2}\rho(\alpha_i \bar{u}_i^2 - \alpha_e \bar{u}_e^2) - \tilde{p}_l \quad ,$$

(2.8)

where

$$\alpha = \frac{1}{A}\int\left(\frac{u_L}{\bar{u}}\right)^3 \mathrm{d}A \ .$$

In Eq. (2.8) α denotes the modified coefficient of kinetic energy due to the velocity fluctuation in the vertical plane, u_L denotes the local streamwise velocity, A denotes the sectional area of the diffuser, and $\tilde{p}_l$ denotes the pressure losses (*cf.* Chapter 2 in Reference [35]). The value of $\tilde{p}_l$ is usually large in the diffuser but the influence of the pressure losses can be neglected near the center axis of the diffuser. Therefore, if the streamwise velocity near the center axis is defined as u_c, the pressure recovery is given as

$$\Delta p \approx \frac{1}{2}\rho(u_{ci}^2 - u_{ce}^2) \quad .$$

(2.9)

If Eq. (2.9) is introduced into Eq. (2.6) and $\bar{u}$ is replaced by u_c, we obtain the local pressure recovery coefficient Cp_L written as

$$Cp_L = \frac{1}{2}\rho(u_{ci}^2 - u_{ce}^2) / \frac{1}{2}\rho u_{ci}^2 \quad .$$

(2.10)

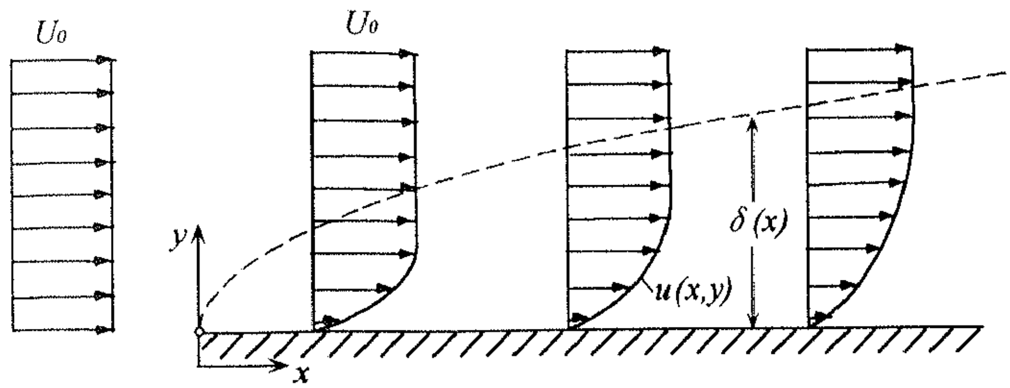

Figure 2.1 Image of boundary layer along a flat plate at zero incidence.

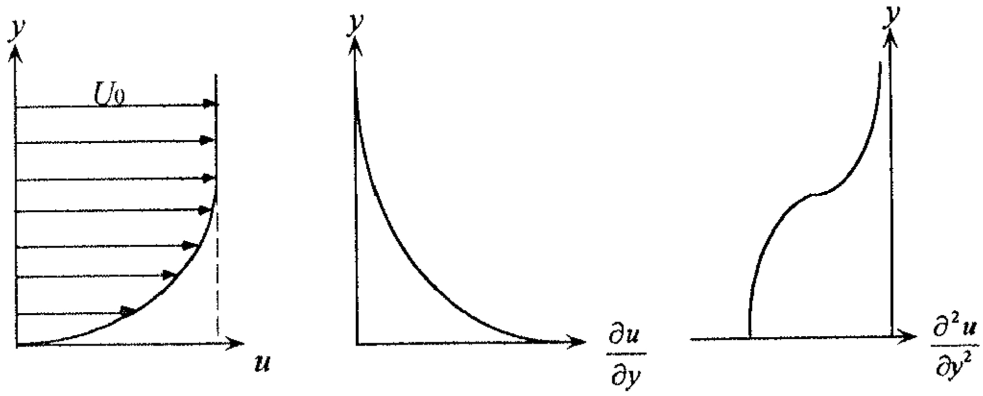

Figure 2.2 Velocity profile in a boundary layer with pressure decrease.

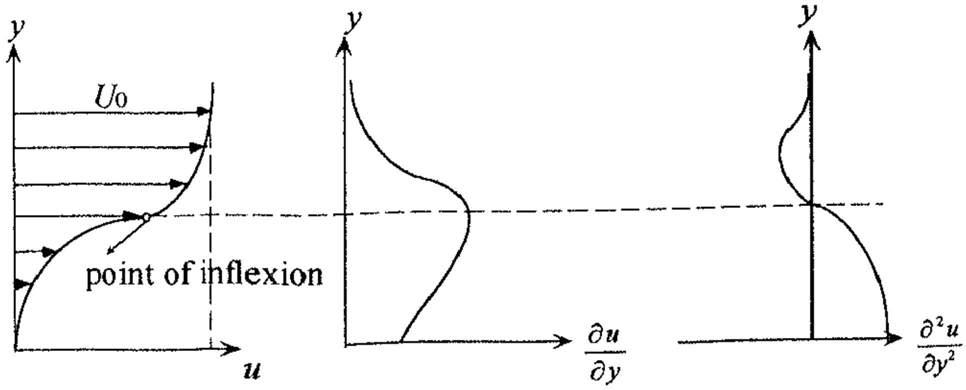

Figure 2.3 Velocity profile in a boundary layer with pressure increase.

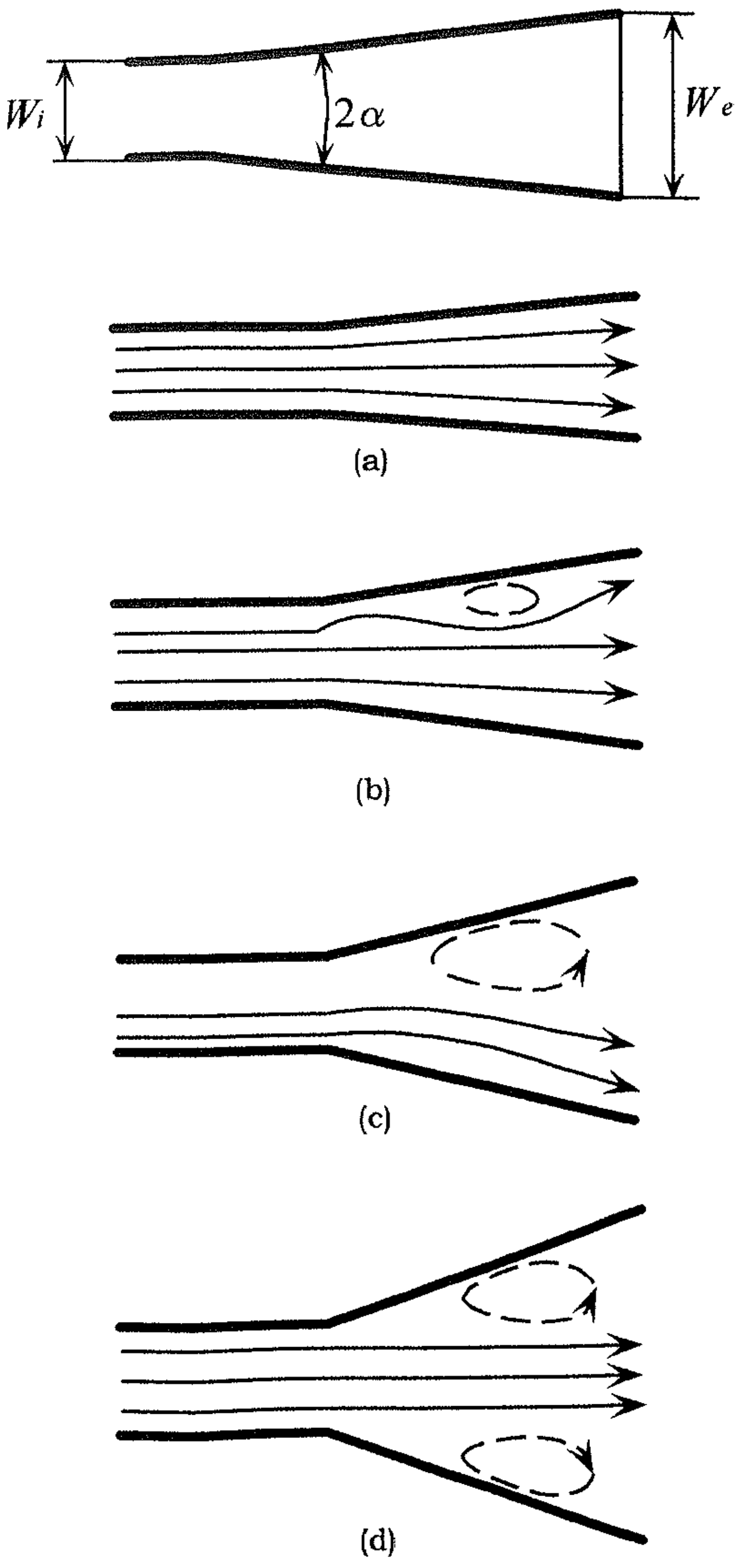

Figure 2.4 Flow in a diffuser with rectangular cross-section.