Bargaining Outcomes in Patent Licensing: Asymptotic Results in a General Cournot Market[*]

Shin Kishimoto[†], Naoki Watanabe[‡], and Shigeo Muto[§]

December 24, 2009; revised October 21, 2010

Abstract

We study asymptotic bargaining outcomes in licensing a patented technology of an external patent holder to firms in a general Cournot market. Our results are as follows: When the number of firms is large, the bargaining set for each permissible coalition structure suggests that the patent holder should extract the entire profits of all licensees. The outcome that the bargaining finally reaches exactly coincides with the non-cooperative outcome, and it cannot be improved upon even by any objections with almost zero cost. Thus, it is strongly stable. The fair allocation represented by the Aumann-Drèze value is, however, not realized as such a stable bargaining outcome.

Keywords: licensing, asymptotic result, coalition structure, bargaining set, Aumann-Drèze value
JEL Classification: C71, D43, D45

[†]JSPS Research Fellow, Department of Mathematical and Computing Sciences, Graduate School of Information Science and Engineering, Tokyo Institute of Technology, Mail box W9-97, 2-12-1 Oh-Okayama, Meguro, Tokyo 152-8552 Japan, Tel: +81-3-5734-3622, Fax: +81-3-5734-3622, E-mail: kishimoto.s.aa@m.titech.ac.jp

[‡]Department of Social Systems and Management, Graduate School of Systems and Information Engineering, University of Tsukuba, 1-1-1 Tennodai, Tsukuba, Ibaraki 305-8573, Japan. E-mail: naoki50@sk.tsukuba.ac.jp

[§]Department of Social Engineering, Graduate School of Decision Science and Technology, Tokyo Institute of Technology 2-12-1 Oh-Okayama, Meguro, Tokyo 152-8552 Japan, E-mail: muto@soc.titech.ac.jp

1 Introduction: licensing and bargaining

We study asymptotic bargaining outcomes in licensing a patented technology of an external patent holder to firms in a general Cournot market. Our aim is to compare the bargaining outcomes with non-cooperative outcomes in the same situation traditionally studied in the literature, to consider how stable the bargaining outcomes are, and to examine whether or not the fair allocation can be realized as the stable bargaining outcomes, as the number of firms tends to infinity.

Patent licensing problems in oligopolistic markets have been investigated mainly through non-cooperative mechanisms; (fixed license) fee or (per-unit) royalty in Kamien and Tauman (1984, 1986), and auction in Katz and Shapiro (1985, 1986).[1] Many subsequent papers studied the optimal licensing mechanisms that maximize the patent holder's revenue. For example, among the above three non-cooperative mechanisms, Kamien et al. (1992, hereafter KOT) showed that in a Cournot market for a homogeneous good it is never optimal for an external patent holder to license his patented cost-reducing technology by means of royalty only. Muto (1993) found that in a Bertrand duopoly with differentiated goods there are cases where it is optimal for an external patent holder to license by means of royalty only.[2]

Licensing agreements are, on the other hand, contract terms signed by the patent holders and licensees resulting from bargaining. From this viewpoint, Tauman and Watanabe (2007) gave a cooperative interpretation of the payoff for an external patent holder: As the number of firms tends to infinity, the Shapley value of the patent holder, which measures his fair contribution to the total industry profit, approximates the payoff he obtains in the non-cooperative patent licensing games traditionally studied in the above literature. Jelnov and Tauman (2009) reconfirmed this result in another setup. Their analyses were, however, limited to payoff distributions of the monopoly profit. (i.e., all firms are licensed, and they form a cartel to coordinate their production level and market behavior.) In practice, monopoly is prohibited by the anti-trust law, and thus many papers in the literature do not allow firms to form a cartel both in production and in the market. Accordingly, the asymptotic equivalence they obtained is biased; thus it should be reconsidered.

---

[1]Katz and Shapiro analyzed games composed of an R&D stage followed by a licensing stage. The patent licensing problems typically do not address the R&D stage.

[2]For licensing a new product, Kamien et al. (1988) studied the optimal license fees, and Erutku and Richelle (2006) provided the optimal non-linear contracts that specifies a fixed upfront fee and a royalty. For licensing a cost-reducing innovation, Erutku and Richelle (2007) extended Kamien-Tauman model (1986) and provided the optimal non-linear contracts, and Sen and Tauman (2007) found the optimal combination of licensing schemes in which the upfront fee is determined by auction and royalty is determined by the patent holder. Stamatopoulos and Tauman (2008) considered a quality-improving innovation in a price-setting duopoly with the logit demand function.

Prohibiting firms from forming such a cartel, Watanabe and Muto (2008) investigated licensing agreements reached as bargaining outcomes. To consider the number of licensees that benefits an external patent holder most through bargaining, they used bargaining solutions for games with coalition structures where no side payments among coalitions are allowed as in Aumann and Drèze (1974). Watanabe-Muto model intends to deal with bargaining as a licensing policy other than non-cooperative mechanisms in a situation traditionally studied in the literature.

This paper is, in part, an outgrowth of Watanabe and Muto (2008). Their main result is that if the number of licensees that maximizes licensees' total surplus is greater than the number of existing non-licensees, each symmetric bargaining set for a coalition structure is a singleton. In this case, the optimal number of licensees from the viewpoint of the patent holder's revenue maximization is also uniquely determined. When this condition is not satisfied, however, the patent holder cannot determine the optimal number of licensees, because each symmetric bargaining set for a coalition structure is not necessarily a singleton. This paper red solves this problem in the case where the number of firms tends to infinity, and shows an asymptotic equivalence of a bargaining outcome to the non-cooperative one.

Our asymptotic results in a general Cournot market are as follows: (I) When the number of firms is large, the bargaining set for each permissible coalition structure suggests that the patent holder should extract the entire profits of all licensees. Moreover, the outcome that the bargaining finally reaches exactly coincides with the non-cooperative outcome derived by KOT. (II) The final bargaining outcome mentioned in (I) cannot be improved upon by any objections even if those objections entail almost zero cost, so it is strongly stable. (III) The fair allocation represented by the Aumann-Drèze value (an extension of the Shapley value to games with coalition structures) cannot be realized as such a stable bargaining outcome in our patent licensing game.

This paper shows as a minor result that, for every coalition structure, the core is empty in a general Cournot market. The core requires that there be no objection to a bargaining outcome.[3] This stability condition may be satisfied, when the objections entail some positive amounts of cost. Thus, in this paper, we say that a bargaining outcome is strongly stable when it is not improved upon by any objections with almost zero cost. On the other hand, the bargaining set for a coalition structure is always non-empty, which requires a weaker stability condition that there be a counter objection for every objection to a bargaining outcome. Throughout this paper, a stable bargaining outcome refers to the bargaining set for a coalition structure.

---

[3]Watanabe and Muto (2008) noted that, for every coalition structure, the core is empty in the linear Cournot market.

A key issue is how to define the characteristic function, i.e., the worth of a coalition of players (a patent holder and firms). In negotiations for licensees' payments to the patent holder, the worth of a coalition that forms to make an objection measures the power of the objection. It also measures the power of a counter objection. Tauman and Watanabe (2007) and Jelnov and Tauman (2009) assumed that when some players form a coalition deviating from the grand coalition, all the other players form the complementary coalition.[4] coalition can decide which firms in the coalition to activate (operate) and which firms to shut down, but (2) the non-active firms can share the total profit of the coalition through side payments; the side payments are made in reward for the non-active firms' cooperation to enhance the coalitional efficiency by softening the market competition. Thus, the grand coalition can work as a "monopolist" both in production and in the market. Under this setup, however, the asymptotic equivalence does not hold for the firms' profits.

Further, if firms in a coalition with the patent holder were to be shut down on an occasion for patent licensing, then those firms would incur disadvantage when another technology was newly invented, because they would not have the old one. In many practices, moreover, the new technology is not usable without the old one, because the patented technologies are, in reality, invented cumulatively one after another. This is another reason why we did not allow firms to form a cartel both in production and in the market, as in Watanabe and Muto (2008).

Driessen et al. (1992) applied another definition to an information trading game, not allowing firms either to shut-down their operations or to form cartels in the market. According to their definition, however, information (e.g., a patented technology) is not necessarily licensed to all potential buyers in a coalition for trading information, whereas the non-buyers in the coalition share the total profit of the coalition for efficient information sharing. So, against cumulative innovations, their definition has the same problem as in the above two papers.

The remainder of the paper is organized in the following way. Section 2 builds up our patent licensing game. Section 3 defines solutions applied to our game and shows our results. In three separate subsections, we investigate the characteristics of the stable bargaining outcomes described by the bargaining set for a coalition structure, comparing them with non-cooperative outcomes, considering how stable they are, and examining whether or not fair allocations are contained in the stable bargaining outcomes. Section 4 briefly refers to a property of our characteristic function and other related non-cooperative works.

---

[4]Jelnov and Tauman (2009) noted that the same asymptotic result can be regained also in the case where the complementary coalition is partitoned into some coalitions. These two papers further presumed that (1) each

2 A patent licensing game

We begin this section by describing the outline of our model, and then give a specification of bargaining in the model.

2.1 The outline

Consider a Cournot market with the set $N_n = \{1, \ldots, n\}$ of identical firms, where $2 \leq n < \infty$. Each firm $i \in N_n$ produces q_i (≥ 0) units of a homogeneous good with the constant unit cost c (> 0) of production. Let $q = \sum_{i \in N_n} q_i$ denote the total production level in the market. Each firm faces a downward sloping inverse demand function $P(q)$, where $P(0) > c$. Assume the following conditions on the demand function, according to KOT,

A1: The total revenue function $qP(q)$ is strictly concave in q.

A2: The demand function $Q(p)$ is decreasing, differentiable for $p > 0$, and the price elasticity $\eta(p) = -pQ'/Q$ (where $Q' = \mathrm{d}Q/\mathrm{d}p$) is a non-decreasing function of p.

An agent, who is not a producer, has a patent of a new technology that reduces the unit cost of production from c to $c - \varepsilon$, where $0 < \varepsilon < c$. This agent is called an external patent holder, and is denoted by player 0. Thus, the set of players of this game is $\{0\} \cup N_n$. The profit of firm i is $(P(q) - c + \varepsilon)q_i$ if it has access to the patented technology (licensee), and $(P(q) - c)q_i$ if it has no access to that technology (non-licensee). The external patent holder gains the revenue in return for licensing its patented technology to firms. Otherwise it gains nothing. According to the traditional literature, we assume that the patent is perfectly protected, namely no firm can use the patented technology without the patent holder's permission.[5]

The game has three stages. At stage (i), the patent holder selects a subset $S_n \subseteq N_n$ and invites the firms in S_n to negotiate on license issues. No firm in $N_n \setminus S_n$ can participate in that negotiation, so they are not licensed. At stage (ii), every firm in S_n negotiates with the patent holder over how much it should pay to the patent holder. It is assumed that all the firms in S_n that were invited to bargain will buy a license, thus focusing solely on the fees paid to the licensor.[6] All players in $\{0\} \cup S_n$ (the patent holder and licensees) can communicate within $\{0\} \cup S_n$. Firms in $N_n \setminus S_n$ (non-licensees) are not allowed to communicate with any players,

---

[5]So, there is neither piracy nor resale of the patented technology to non-licensees. Muto (1987) considered patent licensing under a resale-free situation. Muto (1990) and Nakayama and Quintas (1991) investigated resale-proof trades of information which is related to our patent licensing game.

[6]Even if firms in S_n could choose whether or not to buy the license, we would retain the same propositions due to the solution concepts we apply to this model.

because they do not take part in the negotiation; thus, they cannot observe how the negotiations run. The payment to the patent holder is made before the next stage. At stage (iii), firms compete *à la* Cournot (i.e., in quantities) in the market, knowing which firms are licensed or not. Firms are prohibited from forming a cartel to coordinate their production level and market behavior. This is the assumption under which we consider the same situation as in the literature, to compare the bargaining outcomes with the non-cooperative outcomes.

Remark 1. At stage (ii), a conference might be held by all members of $\{0\} \cup S_n$, or the patent holder might negotiate with each firm in S_n on a one-by-one basis. More important is that players in $\{0\} \cup S_n$ can communicate among themselves. This is a difference from the traditional non-cooperative patent licensing games.

2.2   Bargaining under a coalition structure

In Section 3, we analyze this model backwardly in the spirit of subgame perfection. Before that, we give a specification to stage (ii). Let us begin with stage (iii). Let $t_n = |T_n|$ for each $T_n \subseteq N_n$. When t_n firms are licensed, let $W(t_n)$ and $L(t_n)$ denote the Cournot equilibrium profits of each licensee and each non-licensee at stage (iii), respectively.[7] Because $\eta(p)$ is assumed by A2 to be non-decreasing in p, these equilibrium profits and the equilibrium price are uniquely determined for any t_n such that $0 \leq t_n \leq n$. Let $K \equiv c/(\varepsilon\eta(c))$. We assume $K > 1$, i.e., non-drastic innovations.[8] In general, K is not an integer, but for simplicity we treat it as an integer, according to the literature.[9] KOT showed that for any t_n such that $0 \leq t_n \leq n$ the Cournot equilibrium price $p = p(t_n)$ decreases in t_n and $p(K) = c$, and that $W(t_n)$ and $L(t_n)$ are as follows:

$$
W(t_n) = \begin{cases} -\frac{(p-c+\varepsilon)^2}{P'} & \text{if } 1 \leq t_n \leq K, \\ \frac{(p-c+\varepsilon)Q(p)}{t_n} & \text{if } K \leq t_n \leq n, \end{cases}
\tag{1}
$$

and

$$
L(t_n) = \begin{cases} -\frac{(p-c)^2}{P'}, & \text{if } 0 \leq t_n \leq K, \\ 0 & \text{if } K \leq t_n \leq n-1, \end{cases}
\tag{2}
$$

---

[7]To be more accurately, the Cournot equilibrium profits of each licensee and each non-licensee are functions of n as well, i.e., $W_n(\cdot)$ and $L_n(\cdot)$, respectively. Throughout this paper, for notational ease, we denote them by $W(\cdot)$ and $L(\cdot)$, when there are n firms in the market.

[8]Otherwise the monopoly price under the new technology is less than the competitive price under the old technology. In this case, the patent holder can extract the monopoly profit by licensing his patented technology to only one firm, so the patent licensing problem becomes trivial.

[9]Sen (2005) is the exception for this assumption.

where $P' = \mathrm{d}P/\mathrm{d}q < 0$. For any t_n such that $0 \leq t_n \leq n$, $W(t_n)$ decreases in t_n, while $L(t_n)$ decreases in t_n if $0 \leq t_n < K$. Thus, the Cournot equilibrium profits are summarized in the following order:

$$W(1) > \cdots > W(t_n) > \cdots > W(n) > L(0) > \cdots$$
$$> L(t_n) > \cdots > L(K) = \cdots = L(n-1) = 0. \tag{3}$$

Given these equilibrium profits are determined at stage (iii), we next formalize the bargaining at stage (ii) as a (cooperative) bargaining game with a coalition structure. Any non-empty subset of $\{0\} \cup N_n$ is called a coalition. At stage (ii), the firms that do not belong to S_n cannot participate in the negotiations on licensing issues, but play a relevant role in determining the outside options of negotiators in $\{0\} \cup S_n$. Therefore, for any coalition, we need to provide the worth of the coalition, which is the profit level that the players that belong to the coalition can guarantee for themselves in the worst anticipation because each player in $\{0\} \cup S_n$ should claim the credible outside options in the negotiation process. The worth of a coalition $T'_n \subseteq \{0\} \cup N_n$ is represented by $v(T'_n)$, which is generally called the characteristic function. As described above, every firm in coalition $\{0\} \cup S_n$ is licensed at stage (ii), and firms are not allowed to form a cartel both in production and in the market at stage (iii). So, the worth of each coalition is defined as the sum of the Cournot equilibrium profits of the players in the coalition.[10] Thus, the characteristic function $v : 2^{\{0\} \cup N_n} \to \mathbb{R}$ is given by

$$v(\{0\}) = v(\emptyset) = 0, \quad v(\{0\} \cup T_n) = t_n W(t_n) \quad \text{and} \quad v(T_n) = t_n L(n - t_n).$$

The patent holder can gain nothing without licensing his patented technology because he is not a producer; thus $v(\{0\}) = 0$. The total Cournot equilibrium profit of licensees in T_n is $t_n W(t_n)$; thus $v(\{0\} \cup T_n) = t_n W(t_n)$. $v(T_n)$ is the total Cournot equilibrium profit that firms in T_n can guarantee for themselves in the worst anticipation when firms in T_n jointly break off the negotiation. It is the worst case in our model that all the other $n - t_n$ firms are licensed, because of (3). We assume the worst case for coalition T_n in the spirit of von Neumann and Morgenstern (1944); thus, $v(T_n) = t_n L(n - t_n)$.[11]

For a non-empty set $S_n \subseteq N_n$ of licensees determined at stage (i), the permissible coalition structure is denoted by $P^{S_n} = \{\{0\} \cup S_n, \{\{i\}\}_{i \in N_n \setminus S_n}\}$, because players in $\{0\} \cup S_n$ can communicate with one another but non-licensees are not allowed to communicate with any players. (All firms behave independently in the market at

---

[10]Recall the detail discussions in Section 1.

[11]The worth $v(T_n)$ of a coalition T_n is defined from a pessimistic viewpoint. This definition plays no important role to obtain our propositions.

stage (iii).) Let $s_n = |S_n|$. The set of imputations under the coalition structure P^{S_n} is defined as

$$I^{S_n} = \{x^n = (x_0^n, x_1^n, \cdots, x_n^n) \in \mathbb{R}^{n+1} | x_0^n + \sum_{i \in S_n} x_i^n = s_n W(s_n),$$
$$x_0^n \geq 0, \ x_i^n \geq L(n-1) \ \forall i \in S_n, \ \text{and} \ x_i^n = L(s_n) \ \forall i \in N_n \setminus S_n\}.$$

Players in $\{0\} \cup S_n$ divide the total Cournot equilibrium profit of licensees, each $i \in \{0\} \cup S_n$ being guaranteed the worst payoff $v(\{i\})$. Each of non-licensees in $N_n \setminus S_n$ obtains the equilibrium profit $L(s_n)$, because s_n firms are licensed. Let $(\{0\} \cup N_n, v, P^{S_n})$ denote a (cooperative) bargaining game with the coalition structure P^{S_n}. Every vector of payoffs for players should be in I^{S_n}. (This requirement is slightly weakened in Subsection 3.2.) The solutions for this game are defined and derived within subsections in Section 3. We consider only a subset S_n of licensees with $S_n \neq \emptyset$, because the patent holder can guarantee the payoff zero by itself.

3 Asymptotic bargaining outcomes

In this section, for a coalition structure given at stage (i), we consider the bargaining set, the least core, and the Aumann-Drèze value as solutions that predict bargaining outcomes at stage (ii). Let $T_n \subseteq N_n$. For each coalition, the number of its elements is an integer. So, a sequence of $|T_n|$ is said to converge to an integer t (written as $\lim_{n \to \infty} |T_n| = t$) if there exists n' such that for all $n > n'$, we have $|T_n| = t$. Note that $|T_n| \leq n$, but $|T_n|$ may tend to infinity as n tends to infinity. In this paper, we confine our consideration to sequences of coalitions whose number of elements converges or diverges.[12]

We first shows the existence of the limits of $v(\{0\} \cup T_n)(= t_n W(t_n))$ and $v(T_n)(= t_n L(n - t_n))$ for each $T_n \subseteq N_n$ in the following lemma, which is used to prove our propositions. This is a variant of the Cournot limit theorem: As the number of firms in the Cournot market increases infinitely, the Cournot equilibrium price $p = p(t_n)$ falls to non-licensees' unit cost c of production or less. The formal proof is shown in the Appendix.

Lemma 1. *Let $t \equiv \lim_{n \to \infty} t_n$. In the Cournot market, the following four statements hold: (a) If $t \leq K$, then $\lim_{n \to \infty} t_n W(t_n) = t \cdot \varepsilon Q(c)/K$. (b) If $t > K$, then $\lim_{n \to \infty} t_n W(t_n) = (c - \varepsilon)Q(p)/(t\eta(p) - 1)$, where $p = p(t)$. (c) If t_n diverges, then $\lim_{n \to \infty} t_n W(t_n) = 0$. (d) Regardless of whether t_n converges or diverges, $\lim_{n \to \infty} t_n L(n - t_n) = 0$.*

---

[12]In addition to this restriction, we implicitly assume the following things: When a sequence of $|T_n|$ converges, if $i \in T_n$ for some $n > n'$, then $i \in T_{n+1}$. When it diverges, the player that belongs to some coalition in the sequence always belongs to the coalition in the limit.

3.1 The bargaining set for a coalition structure

When the solution is empty at stage (ii), we cannot answer our question on how many licenses the patent holder should sell to firms through negotiations. In a more general patent licensing game than ours, Watanabe and Muto (2008) showed that the core for a coalition structure is always empty, unless the grand coalition $\{0\} \cup N_n$ forms. On the other hand, the bargaining set for a coalition structure is always non-empty, which was shown by Davis and Maschler (1967) and Peleg (1967).[13]

Let us begin with defining the relevant notions. Let $i, j \in \{0\} \cup S_n$ and $x^n \in I^{S_n}$. We say that i has an objection (y^n, T_n) against j at x^n if $i \in T_n$, $j \notin T_n$, $T_n \subseteq \{0\} \cup N_n$, $y_k^n > x_k^n$ for any $k \in T_n$, and $\sum_{k \in T_n} y_k^n \leq v(T_n)$, and that j has a counter objection (z^n, R_n) to i's objection (y^n, T_n) if $j \in R_n$, $i \notin R_n$, $R_n \subseteq \{0\} \cup N_n$, $z_k^n \geq x_k^n$ for any $k \in R_n$, $z_k^n \geq y_k^n$ for any $k \in R_n \cap T_n$, and $\sum_{k \in R_n} z_k^n \leq v(R_n)$. We say that i has a *valid* objection (y^n, T_n) at x^n if there exists no counter objection to i's objection (y^n, T_n). The bargaining set for a coalition structure P^{S_n} is defined as

$$M^{S_n} = \{x^n \in I^{S_n} | \text{no player in } \{0\} \cup S_n \text{ has a valid objection at } x^n\}.$$

We simply call M^{N_n} the bargaining set.

Remark 2. When each player in $\{0\} \cup S_n$ makes his objection (or counter objection) at stage (ii), he makes it against another player via coalition T_n (or R_n) that does not actually form, because coalition $\{0\} \cup S_n$ eventually forms. Note that forming a coalition at stage (ii) does never imply cooperation either in production or in the market among players in the coalition.

Our first proposition suggests that when the number of firms is infinitely large, the patent holder should extract the entire profits of all licensees in the bargaining set for a permissible coalition structure except the grand coalition. We refer to the case of the grand coalition in the next subsection. The following result is due to the fact that it is harder for each firm in S_n to make objections and counter objections against the patent holder as the number of firms becomes infinitely large.

Proposition 1. *Suppose that $S_n \subsetneq N_n$. Take any $x^n \in M^{S_n}$. Then, in the Cournot market, $\lim_{n \to \infty} x_0^n = \lim_{n \to \infty} s_n W(s_n)$ and $\lim_{n \to \infty} x_i^n = 0$ for all $i \neq 0$.*

Proof. Take any $x^n \in M^{S_n}$ with $S_n \neq N_n$. First, we show that $\lim_{n \to \infty} \sum_{i \in N_n} x_i^n = 0$. Consider the following two cases. Case (i): Suppose that there exist $i' \in S_n$ such that $x_{i'}^n > L(s_n)$. Order all the n firms according to their profits in the non-decreasing order, and take the first s_n firms. Let T_n be the set of the first s_n firms.

---

[13]Some concepts of bargaining set for a coalition structure were provided in Aumann and Maschler (1964) as an earlier publication, but their non-emptiness was not shown there.

Note that $x_j^n = L(s_n)$ for $j \in N_n \setminus S_n$ because $x^n \in I^{S_n}$. Then, the patent holder has an objection $(y^n, \{0\} \cup T_n)$ against i' because $x_0^n + \sum_{i \in T_n} x_i^n < x_0^n + \sum_{i \in S_n} x_i^n = s_n W(s_n)$. But, i' can have a counter objection (z^n, N_n) because $x^n \in M^{S_n}$. Thus, $0 \leq \sum_{i \in N_n} x_i^n \leq \sum_{i \in N_n} z_i^n \leq nL(0) = v(N_n)$. ($0 \leq x_i^n$ for all $i \in N_n$, by (3) and $x^n \in I^{S_n}$.) By Lemma 1 (d) and the squeeze theorem, $\lim_{n \to \infty} \sum_{i \in N_n} x_i^n = 0$.

Case (ii): Suppose that $x_i^n \leq L(s_n)$ for all $i \in S_n$. Then, i has an objection (y^n, N_n) against the patent holder, because $\sum_{i \in N_n} x_i^n = \sum_{i \in S_n} x_i^n + (n - s_n)L(s_n) \leq nL(s_n) < nL(0)$. Note that $0 \leq \sum_{i \in N_n} x_i^n \leq nL(0)$. Thus, by Lemma 1 (d) and the squeeze theorem, $\lim_{n \to \infty} \sum_{i \in N_n} x_i^n = 0$.

Next, we complete the proof. Because $\lim_{n \to \infty} \sum_{i \in N_n} x_i^n = 0$ and $x_i^n \geq 0$ for all $i \in N_n$, $\lim_{n \to \infty} x_i^n = 0$. And, $\lim_{n \to \infty} \sum_{i \in S_n} x_i^n = 0$ because $0 \leq \sum_{i \in S_n} x_i^n \leq \sum_{i \in N_n} x_i^n$. By the definition of I^{S_n}, $x_0^n = s_n W(s_n) - \sum_{i \in S_n} x_i^n$. Therefore,

$$\lim_{n \to \infty} x_0^n = \lim_{n \to \infty} \left(s_n W(s_n) - \sum_{i \in S_n} x_i^n \right) = \lim_{n \to \infty} s_n W(s_n).$$

$\square$

Watanabe and Muto (2008) showed that the symmetric bargaining set for a coalition structure is a singleton under a certain condition. Proposition 1 shows that when the number of firms increases infinitely in the Cournot market, the patent holder's profit realized by the bargaining set for a coalition structure is uniquely determined, regardless of whether there are symmetric or asymmetric payoffs for the licensees, unless the grand coalition forms. We refer to the case of the grand coalition, i.e., M^{N_n}, as a corollary in Subsection 3.2. (We can extend Proposition 1 to $S_n = N_n$.)

Let us now consider the optimal number of licensees to be selected at stage (i). The next lemma suggests the answer; $s_n = K$ when the number of firms is infinitely large. So, there is no need for referring to the bargaining set M^{N_n}. The intuition is that all non-licensees are driven out of the market when K or more firms are licensed, and the Cournot equilibrium price goes down as the number of licensees increases, so the competition among licensees in the market results in the reduction of the total Cournot equilibrium profit $s_n W(s_n)$. The formal proof is shown in the Appendix.

Lemma 2. *Let s_n' be such that $s_n' W(s_n') \geq s_n W(s_n)$ for $s_n = 1, \ldots, n$. Then, $\lim_{n \to \infty} s_n' = K$.*

By this lemma, we can show the next proposition which suggests that when the number n of firms becomes infinitely large, the patent holder can gain the maximum

profit $\varepsilon Q(c)$ as a stable bargaining outcome by licensing his patented technology to K firms.

Proposition 2. *Take any $\hat{x}^n \in M^{S_n}$ with $\lim_{n \to \infty} |S_n| = K$. Then, in the Cournot market, $\lim_{n \to \infty} \hat{x}_0^n = \varepsilon Q(c) \geq \lim_{n \to \infty} s_n W(s_n)$ for any s_n.*

Proof. Suppose $S_n \subsetneq N_n$. By Proposition 1, for any $x^n \in M^{S_n}$, $\lim_{n \to \infty} x_0^n = \lim_{n \to \infty} s_n W(s_n)$. Lemma 2 suggests that the number of licensees that maximizes $s_n W(s_n)$ converges to K as the number of firms increases infinitely. Thus, by Lemma 1 (a), for any $S_n \subsetneq N_n$ with $\lim_{n \to \infty} |S_n| = K$, the patent holder obtains

$$\lim_{n \to \infty} \hat{x}_0^n = \left(\lim_{n \to \infty} |S_n| \right) \cdot \frac{\varepsilon Q(c)}{K} = \varepsilon Q(c).$$

$\square$

Among three non-cooperative mechanisms such as fixed license fee, per-unit royalty and auction, KOT showed that if the magnitude ε of innovation is not too small, then it is optimal for the patent holder to auction off K licenses, otherwise it is optimal to sell K licenses to firms by means of a fixed license fee. Eventually, when the Cournot industry size increases indefinitely, the market price drops to c, non-licensees exit the market, and the patent holder extracts the entire industry profit $\varepsilon Q(c)$.[14] Proposition 2 implies that the bargaining outcome obtained by applying the bargaining set for a coalition structure exactly coincides with the non-cooperative outcome. In other words, the non-cooperative outcome can be reached through negotiations as the stable bargaining outcome when the Cournot market is very large (i.e., the number of firms is infinitely large).

3.2   The least core for a coalition structure

In this subsection, we consider the least core for a coalition structure, in order to provide a stronger meaning for the result on the bargaining set suggested by Proposition 2, by investigating the relationship between these two solutions.

To define the least core, we begin with defining the ϵ-core for a coalition structure P^{S_n}, which is given for any $\epsilon \in \mathbb{R}$ as

$C_\epsilon^{S_n} = \{x^n \in I_p^{S_n} | \sum_{i \in T_n} x_i^n \geq v(T_n) - \epsilon,$

$\qquad\qquad \forall T_n \subseteq \{0\} \cup N_n \text{ with } T_n \cap (\{0\} \cup S_n) \neq \emptyset \text{ and } T_n \neq \{0\} \cup S_n\},$

where $I_p^{S_n} = \{x^n \in \mathbb{R}^{n+1} | x_0^n + \sum_{i \in S_n} x_i^n = s_n W(s_n) \text{ and } x_i^n = L(s_n) \ \forall i \in N_n \setminus S_n\}$. $I_p^{S_n}$ is called the set of pre-imputations for a coalition structure P^{S_n}. The real

---

[14]Kamien and Tauman (1984) showed that $\varepsilon Q(c)$ is also the patent holder's asymptotic payoff if he chooses to charge every licensee a pre-announced per-unit royalty.

number ϵ is interpreted as the cost that is needed to form an objecting coalition T_n. Evidently, $C_\epsilon^{S_n} \neq \emptyset$ if ϵ is large enough, so we can apply this solution to stage (ii) to find bargaining outcomes. When $\epsilon = 0$, $C_\epsilon^{S_n}$ is simply called the core C^{S_n} for a coalition structure P^{S_n}. Clearly, $C_{\epsilon'}^{S_n} \subseteq C_\epsilon^{S_n}$ whenever $\epsilon' < \epsilon$, with strict inclusion if $C_\epsilon^{S_n} \neq \emptyset$. The least core for a coalition structure P^{S_n} is defined as

$$LC^{S_n} = \bigcap_\epsilon C_\epsilon^{S_n} \quad \text{where} \quad C_\epsilon^{S_n} \neq \emptyset.$$

Let ϵ_0 be the smallest ϵ such that $C_\epsilon^{S_n} \neq \emptyset$, that is,

$$\epsilon_0 = \min_{x \in I_p^{S_n}} \quad \max_{T_n \subseteq \{0\} \cup N_n : T_n \cap (\{0\} \cup S_n) \neq \emptyset, \ T_n \neq \{0\} \cup S_n} \left(v(T_n) - \sum_{i \in T_n} x_i \right).$$

It is known that $LC^{S_n} = C_{\epsilon_0}^{S_n}$.

Let s_n^* denote the number of licensees that maximizes their total surplus, i.e., $s_n^*(W(s_n^*) - L(0)) \geq s_n(W(s_n) - L(0))$ for any $s_n = 1, \ldots, n$. This number plays an important role in this subsection, whose properties are shown in the next lemma.

Lemma 3. *In the Cournot market, the following properties on s_n^* hold: (a) $s_n^* \leq K$. (b) $\lim_{n \to \infty} s_n^* = K$.*

Proof. (a). We first show that, for any t_n such that $t_n \geq K$, $v(\{0\} \cup T_n)$ decreases in t_n. $v(\{0\} \cup T_n) = t_n W(t_n) = (p - c + \varepsilon)Q(p)$, where $p = p(t_n)$ is the Cournot equilibrium price when t_n firms are licensed, so

$$\frac{\partial v(\{0\} \cup T_n)}{\partial t_n} = \frac{\partial p}{\partial t_n} \left(Q(p) + (p - c + \varepsilon)Q' \right) = Q(p) \cdot \frac{\partial p}{\partial t_n} \left(1 - \frac{\eta(p)}{p}(p - c + \varepsilon) \right).$$

By a general property of the Cournot equilibrium price that $t_n(p - c) = p/\eta(p) - t_n \varepsilon$ if $t_n \geq K \equiv c/(\varepsilon \eta(c))$ (See, e.g., KOT),

$$1 - \frac{1}{\eta(p)} < 1 - \frac{1}{t_n \eta(p)} = \frac{c - \varepsilon}{p},$$

which implies that $1 > \eta(p)(p - c + \varepsilon)/p$. As noted in Subsection 2.2, KOT showed that, for any $t_n = 1, \ldots, n$, $p(t_n)$ decreases in t_n. Thus, $\partial v(\{0\} \cup T_n)/\partial t_n < 0$.

Let us now give the proof of (a). Suppose that there exists s_n^* with $s_n^* > K$. By the definition of s_n^*, $s_n^*(W(s_n^*) - L(0)) \geq K(W(K) - L(0))$, i.e.,

$$s_n^* W(s_n^*) - K W(K) \geq (s_n^* - K)L(0). \tag{4}$$

As shown above, for each t_n with $t_n \geq K$, $v(\{0\} \cup T_n)$ decreases in t_n, so the left-hand side of (4) is negative. The right-hand side is, however, positive by the supposition $s_n^* > K$ and (3). This contradiction implies $s_n^* \leq K$.

(b). Note first that, by Lemma 3 (a), $s_n^* \leq K$. By Lemma 1 (d), for any s_n such that $s_n \leq K \leq n$, $\lim_{n \to \infty} s_n L(0) = 0$, so $\lim_{n \to \infty} s_n(W(s_n) - L(0)) = \lim_{n \to \infty} s_n W(s_n)$. By Lemma 2, the number of licensees that maximizes $s_n W(s_n)$ becomes K as the number of firms increases infinitely. Thus, $\lim_{n \to \infty} s_n^* = K$. □

We here briefly refer to M^{N_n}. Assuming the same payoffs for all licensees, Watanabe and Muto (2008) showed in their Proposition 4 (a) that if $n > s_n^*$, then $n(W(n) - L(0)) \leq x_0^n \leq s_n^*(W(s_n^*) - L(0))$, where $x^n \in M^{N_n}$. Lemma 3 (a) suggests that $n > s_n^*$ holds for sufficiently large n. By Lemma 1 (d), $\lim_{n \to \infty} n L(0) = 0$. Thus, if $x^n \in M^{N_n}$, then $\lim_{n \to \infty} nW(n) \leq \lim_{n \to \infty} x_0^n$. On the other hand, $x_0^n \leq nW(n) = v(\{0\} \cup N_n)$. Therefore, if $x_0^n \in M^{N_n}$, then $\lim_{n \to \infty} x_0^n = \lim_{n \to \infty} nW(n)$ and $\lim_{n \to \infty} x_i^n = 0$ for all $i \neq 0$, by the squeeze theorem.

Corollary 1. *Take any $x^n \in M^{N_n}$. Then, in the Cournot market, $\lim_{n \to \infty} x_0^n = \lim_{n \to \infty} nW(n)$ and $\lim_{n \to \infty} x_i^n = 0$ for all $i \neq 0$.*

Before proceeding to the least core, confirm that the core C^{S_n} is empty for any permissible coalition structure in our model. This is the reason why we chose the bargaining set for a coalition structure as our solution.

Proposition 3. *In the Cournot market, if $n > K$, then $C^{S_n} = \emptyset$ for any $S_n \subseteq N_n$.*

Proof. Without specifying the market structure, Watanabe and Muto (2008) showed in their Propositions 1 and 2 that $C^{S_n} = \emptyset$ if $S_n \neq N_n$, and that $C^{N_n} \neq \emptyset$ if and only if $s_n^* = n$. In the Cournot market, by Lemma 3 (a), $s_n^* \neq n$ if $n > K$. Thus, $C^{S_n} = \emptyset$ for any permissible coalition structure. □

We now proceed to the relationship between the least core and the bargaining set for a coalition structure. Let us begin with showing the next lemma.

Lemma 4. *Let $S_n^* \subseteq N_n$ be the set of firms where $|S_n^*| = s_n^*$ and let ϵ_0^* be ϵ_0 such that $LC^{S_n^*} = C_{\epsilon_0^*}^{S_n^*}$. Then, in the Cournot market, $\lim_{n \to \infty} \epsilon_0^* = +0$.[15]*

Proof. By Proposition 3, $C^{S_n^*} = \emptyset$ if $n > K$. Thus, $\epsilon_0^* \geq 0$ if $n > K$. We next show that $C_{\epsilon'}^{S_n^*} \neq \emptyset$ where $\epsilon' = (n - s_n^*)(L(0) - L(s_n^*)) > 0$. Define $x'^n \in I_p^{S_n^*}$ by

$$
x_i'^n = \begin{cases} s_n^*(W(s_n^*) - L(0)) & \text{if } i = 0 \\ L(0) & \text{if } i \in S_n^* \\ L(s_n^*) & \text{if } i \in N_n \setminus S_n^*. \end{cases}
$$

---

[15]The notation $\lim_{n \to \infty} \epsilon_0^* = +0$ is a shorthand for the formal expression that $\epsilon_0^* > 0$, $\epsilon_0^* \to 0$ as $n \to \infty$.

Take any $T_n \subseteq \{0\} \cup N_n$ with $T_n \cap (\{0\} \cup S_n^*) \neq \emptyset$ and $T_n \neq \{0\} \cup S_n^*$. Let $t_n = |T_n \backslash \{0\}|$ and $t_n' = |(T_n \backslash \{0\}) \cap S_n^*|$. Then, $s_n^* + t_n - t_n' \leq n$. Thus, if $0 \in T_n$,

$$
\begin{aligned}
&\sum_{i \in T_n} x_i'^n - v(T_n) + \epsilon' \\
&= s_n^*(W(s_n^*) - L(0)) + (n - s_n^* + t_n')L(0) - t_n W(t_n) - (n + t_n' - s_n^* - t_n)L(s_n^*) \\
&\geq s_n^*(W(s_n^*) - L(0)) + (n - s_n^* + t_n')L(0) - t_n W(t_n) - (n + t_n' - s_n^* - t_n)L(0) \\
&= s_n^*(W(s_n^*) - L(0)) - t_n(W(t_n) - L(0)) \geq 0,
\end{aligned}
$$

and, if $0 \notin T_n$,

$$
\begin{aligned}
&\sum_{i \in T_n} x_i'^n - v(T_n) + \epsilon' \\
&= t_n' L(0) + (t_n - t_n')L(s_n^*) - t_n L(n - t_n) + (n - s_n^*)(L(0) - L(s_n^*)) \\
&\geq t_n' L(0) + (t_n - t_n')L(s_n^*) - t_n L(0) + (n - s_n^*)(L(0) - L(s_n^*)) \\
&= (n + t_n' - s_n^* - t_n)(L(0) - L(s_n^*)) \geq 0,
\end{aligned}
$$

which jointly imply $x'^n \in C_{\epsilon'}^{S_n^*}$. Consequently, we have $0 \leq \epsilon_0^* \leq (n - s_n^*)(L(0) - L(s_n^*))$ if $n > K$.

Finally, by Lemma 3 (b) and Lemma 1 (d),

$$
\lim_{n \to \infty} (n - s_n^*)(L(0) - L(s_n^*)) = \lim_{n \to \infty} (n - K)L(0) = 0.
$$

Therefore, by the squeeze theorem, $\lim_{n \to \infty} \epsilon_0^* = +0$. $\qquad\square$

Applying Lemma 4, we state the relationship between the least core and the bargaining set as the next proposition.

Proposition 4. *Take any $\hat{x}^n \in M^{S_n^*}$ and any $x^{*n} \in LC^{S_n^*}$. Then, in the Cournot market, $\lim_{n \to \infty} \hat{x}_0^n = \lim_{n \to \infty} x_0^{*n} = \varepsilon Q(c)$ and $\lim_{n \to \infty} \hat{x}_i^n = \lim_{n \to \infty} x_i^{*n} = 0$ for all $i \neq 0$.*

Proof. By Propositions 1 and 2 and Lemma 3 (b), for any $\hat{x}^n \in M^{S_n^*}$, $\lim_{n \to \infty} \hat{x}_0^n = \varepsilon Q(c)$ and $\lim_{n \to \infty} \hat{x}_i^n = 0$ for all $i \neq 0$. We first show that $\lim_{n \to \infty} x_0^{*n} = \varepsilon Q(c)$ if $x^{*n} \in LC^{S_n^*}$. Take an arbitrary $x^{*n} \in LC^{S_n^*}$. Then, $\sum_{i \in S_n^*} x_i^{*n} \geq v(S_n^*) - \epsilon_0^*$ by $LC^{S_n^*} = C_{\epsilon_0^*}^{S_n^*}$. By the definition of $I_p^{S_n^*}$, $x_0^{*n} + \sum_{i \in S_n^*} x_i^{*n} = s_n^* W(s_n^*)$, so

$$
x_0^{*n} \leq s_n^* W(s_n^*) - v(S_n^*) + \epsilon_0^* = s_n^* W(s_n^*) - s_n^* L(n - s_n^*) + \epsilon_0^*.
$$

On the other hand, for any sufficiently large n such that $2K < n$, $2s_n^* \leq 2K < n$, so we can take $T_n \subseteq N_n \backslash S_n^*$ such that $|T_n| = |S_n^*|$. Then, by $x^{*n} \in C_{\epsilon_0^*}^{S_n^*}$,

$$
x_0^{*n} \geq v(\{0\} \cup T_n) - \sum_{i \in T_n} x_i^{*n} - \epsilon_0^* = s_n^* W(s_n^*) - s_n^* L(s_n^*) - \epsilon_0^*.
$$

14

Accordingly, by Lemmas 1 (a), (d), 3 (b) and 4 and by the squeeze theorem,

$$\lim_{n\to\infty} x_0^{*n} = \lim_{n\to\infty} s_n^* W(s_n^*) = \left(\lim_{n\to\infty} s_n^*\right) \cdot \frac{\varepsilon Q(c)}{K} = \varepsilon Q(c),$$

as shown in the proof of Proposition 2.

Next, we show that $\lim_{n\to\infty} x_i^{*n} = 0$ for each licensee $i \in S_n^*$. As shown above, $\lim_{n\to\infty} x_0^{*n} = \lim_{n\to\infty} s_n^* W(s_n^*)$. Thus, $\lim_{n\to\infty} \sum_{i\in S_n^*} x_i^{*n} = \lim_{n\to\infty}(s_n^* W(s_n^*) - x_0^{*n}) = 0$. On the other hand, $x_i^{*n} \geq L(n-1) - \epsilon_0^*$ by $x^{*n} \in C_{\epsilon_0^*}^{S_n^*}$. Accordingly, by Lemmas 1 (d) and 4, $\lim_{n\to\infty} x_i^{*n} = 0$ for each licensee $i \in S_n^*$.

Lastly, for any non-licensee $i \in N_n \setminus S_n^*$, $\lim_{n\to\infty} x_i^{*n} = \lim_{n\to\infty} L(s_n^*) = 0$ by Lemma 3 (b) and (3). $\qquad\square$

For a given coalition structure, the least core LC^{S_n} is the subset of $I_p^{S_n}$ that cannot be improved upon by any objections even if those objections entail a cost of at least ϵ_0. In this sense, the bargaining outcomes are *strongly stable* if they are in the least core with very small ϵ (nearly or less than zero). Proposition 4 together with Proposition 2 jointly suggest that when the number of firms increases infinitely in the Cournot market, the bargaining outcome obtained by the bargaining set for a coalition structure $P^{S_n^*}$, where the patent holder can gain the maximum profit, cannot be improved upon even by any objections with almost zero cost. Therefore, we can say that the bargaining outcome that the patent holder gains the maximum profit $\varepsilon Q(c)$ is strongly stable, when the Cournot market is very large.

3.3 The Aumann-Drèze value

It is well known that the Shapley value is not necessarily in the core, but its relationship with the bargaining set has not been studied comprehensively. The Shapley value is frequently interpreted as a *fair* allocation, while the bargaining set is regarded as *stable* bargaining outcomes. Thus, the inclusion of the Shapley value in the bargaining set implies that the fair allocation can be realized as a stable bargaining outcome. Watanabe and Tauman (2003) showed, however, that the Shapley value of their patent licensing game is not in the bargaining set when the linear Cournot market is very large.

In this subsection, we reexamine the relationship of those solutions for a coalition structure in our model considering the practical situation. Aumann and Drèze (1974) defined the Shapley value for a coalition structure (as well as other solutions) and provided a set of axioms that characterizes the value. So, we hereafter call it the Aumann-Drèze value.

Let $\varphi^{S_n}(\in \mathbb{R}^{n+1})$ denote the Aumann-Drèze value of our bargaining game with a coalition structure P^{S_n}. Let $s_n = |S_n|$ and $t = |T|$ for $T \subseteq S_n$. The Aumann-Drèze

15

value $\varphi_0^{S_n}$ for the patent holder is represented by

$$\varphi_0^{S_n} = \sum_{T \subseteq S_n} \frac{t!(s_n - t)!}{(s_n + 1)!} (v(\{0\} \cup T) - v(T)).$$

There are $s_n!/(t!(s_n - t)!)$ orderings with the same marginal contribution $v(\{0\} \cup T) - v(T) = t(W(t) - L(n - t))$ of the patent holder because licensees in S_n are identical. Thus, the Aumann-Drèze value $\varphi_0^{S_n}$ of the patent holder is given by

$$\varphi_0^{S_n} = \frac{1}{s_n + 1} \sum_{t=0}^{s_n} t(W(t) - L(n - t)).$$

By the axioms of relative efficiency and symmetry (Aumann and Drèze (1974)), $\varphi_i^{S_n} = (v(\{0\} \cup S_n) - \varphi_0^{S_n})/s_n$ for all $i \in S_n$, and $\varphi_j^{S_n} = v(\{j\})$ for all $j \in N_n \setminus S_n$.

The Aumann-Drèze value is player i's average marginal contribution to coalitions in the coalition to which i belongs under a coalition structure P^{S_n}, so it is interpreted as representing a fair allocation.

Proposition 5. *In the Cournot market, $\lim_{n\to\infty} \varphi_0^{S_n} < \lim_{n\to\infty} s_n W(s_n)$ for all $S_n \subsetneq N_n$ with $\lim_{n\to\infty} s_n \leq K$.*

Proof. Let $s = \lim_{n\to\infty} s_n$. For any T_n such that $T_n \subseteq N_n$, $\lim_{n\to\infty} t_n L(n - t_n) = 0$, by Lemma 1 (d). Thus, for any S_n such that $T \subseteq S_n$,

$$\lim_{n\to\infty} \varphi_0^S = \lim_{n\to\infty} \frac{1}{s_n + 1} \sum_{t=0}^{s_n} t(W(t) - L(n - t)) = \frac{1}{s + 1} \sum_{t=0}^{s} \lim_{n\to\infty} tW(t).$$

When $s \leq K$, $\lim_{n\to\infty} s_n W(s_n) = s \cdot Q(c)\varepsilon/K$, by Lemma 1 (a). So, for any t such that $t \leq s \leq K$, $\lim_{n\to\infty} tW(t) \leq \lim_{n\to\infty} s_n W(s_n)$. Accordingly,

$$\frac{1}{s + 1} \sum_{t=0}^{s} \lim_{n\to\infty} tW(t) \leq \frac{s}{s + 1} \lim_{n\to\infty} s_n W(s_n) < \lim_{n\to\infty} s_n W(s_n).$$

$\square$

We briefly refer to the case of $\lim_{n\to\infty} s_n > K$. For any t_n such that $t_n \geq K$, $v(\{0\} \cup T_n) = t_n W(t_n)$ decreases in t_n, as shown at the beginning of the proof of Lemma 3, so $s_n W(s_n) < tW(t)$ when $K \leq t < s_n$. By an analogy to the Cournot limit theorem applied to non-licensees (Lemma 1 (d)), if we could obtain $\lim_{n\to\infty} tW(t) = \lim_{n\to\infty} s_n W(s_n)$, then it would be clearly that

$$\lim_{n\to\infty} \frac{1}{s_n + 1} \sum_{t=1}^{s_n} tW(t) \leq \lim_{n\to\infty} \frac{s_n}{s_n + 1} s_n W(s_n),$$

which plays an essential role in proving Proposition 5. Even if n tends to infinity, however, $t'W(t')$ does not vary whenever $K \leq t'$, because $n - t'$ non-licensees exit

the market and so the number of firms producing in the market does not change. Consequently, we cannot necessarily obtain a clear relationship between these two solutions when $\lim_{n\to\infty} s_n > K$.

As far as any $S_n \subsetneq N_n$ with $\lim_{n\to\infty} s_n \leq K$, however, we found that the Aumann-Drèze value is not in the bargaining set for a coalition structure P^{S_n} in a very large Cournot market. Proposition 1 and Lemma 3 suggest that, in such a very large Cournot market, the patent holder chooses $s_n^* = K$ firms at stage (i). Therefore, we can say that the fair allocation cannot be realized as a stable bargaining outcome.

Finally, let us compute the Aumann-Drèze value when the patent holder negotiates with s_n^* firms in a large Cournot market, to see how far the fair allocation is from the stable bargaining outcome.

Proposition 6. *In the Cournot market,*

$$\lim_{n\to\infty} \varphi_0^{S_n^*} = \frac{\varepsilon Q(c)}{2}, \quad \lim_{n\to\infty} \varphi_i^{S_n^*} = \frac{\varepsilon Q(c)}{2K} \ if \ i \in S_n^*, \ and \ \lim_{n\to\infty} \varphi_j^{S_n^*} = 0 \ if \ j \in N_n \setminus S_n^*.$$

Proof. By Lemma 3 (b), $\lim_{n\to\infty} s_n^* = K$. By Lemma 1 (a), for any t such that $t \leq K$, $\lim_{n\to\infty} tW(t) = tQ(c)\varepsilon/K$. By Lemma 1 (d), $\lim_{n\to\infty} tL(n-t) = 0$. Thus,

$$\lim_{n\to\infty} \varphi_0^{S_n^*} = \lim_{n\to\infty} \frac{1}{s_n^*+1} \sum_{t=0}^{s_n^*} t(W(t) - L(n-t)) = \frac{1}{K+1} \sum_{t=1}^{K} \frac{\varepsilon Q(c)t}{K} = \frac{\varepsilon Q(c)}{2},$$

For all $i \in S_n^*$, because $v(\{0\} \cup S_n^*) = s_n^* W(s_n^*)$,

$$\lim_{n\to\infty} \varphi_i^{S_n^*} = \lim_{n\to\infty} \frac{s_n^* W(s_n^*) - \varphi_0^{S_n^*}}{s_n^*} = \frac{\varepsilon Q(c)}{2K}.$$

For all $j \in N_n \setminus S_n^*$,

$$\lim_{n\to\infty} \varphi_j^{S_n^*} = \lim_{n\to\infty} v(\{j\}) = \lim_{n\to\infty} L(n-1) = 0.$$

$\square$

For a broad class of games, Wooders and Zame (1987) showed that the Shapley value is in the ϵ-core and ϵ is very small if the game has infinitely many players, i.e., fair allocations are strongly stable in such large games.[16] Proposition 6 indicates, however, that the fair allocation is far from the stable bargaining outcome by as

---

[16]Kats and Tauman (1985) studied the asymptotic inclusion relationship of the Shapley value in the core in replicated production economies with divisible and indivisible inputs, where only a limited number of permitted firms have access to a better production technology, assuming that every firm is a price taker.

much as $\varepsilon Q(c)/2$ from the patent holder's viewpoint when $s_n^* = K$, so the difference between the fair allocation and the stable outcome is not small.

In our patent licensing game, the patent holder acts as a *big boss* in the sense that no firm can use his patented technology without his permission and all non-licensees incur disadvantage compared with licensees. As Lemma 1 (d) shows, on the other hand, the bargaining power of objecting or counter-objecting coalitions of firms is (almost) nothing, when the number of firms is large. Therefore, even an external patent holder can extract the entire profits of licensees, although he can gain nothing without licensing his patented technology. Our patent licensing game is not formulated as a large game, but the existence of such a big boss is the essential point that induces our asymptotic result on the Aumann-Drèze (Shapley) value to be different from Wooders and Zame's result: They considered a class of games including private exchange economies (with divisible and indivisible goods), coalition-production economics, etc., where there is no such agent who plays a remarkably important role like a big boss.[17]

4 Remarks on the related literature

4.1 The super-additivity

The characteristic function we defined in Subsection 2.2 does not necessarily exhibit super-additivity that is often presumed in the cooperative analysis. Super-additivity is the feature of characteristic functions required in analyzing how to divide the total payoff in the grand coalition, because the grand coalition may not actually form without it. It would not be a pre-requisite in games where there is no need for players to form the grand coalition. In fact, Aumann and Drèze (1974) did not require the super-additivity for analysis of games with coalition structures.

This paper prohibits firms from forming any cartels in the market, because we wished to consider the same situation as in the non-cooperative analysis in the literature. A coalition is thus regarded as merely a group within which communication among its members is allowed. This is one of the reasons why our characteristic function does not necessarily satisfy the super-additivity.[18]

---

[17]Muto et al. (1989) characterized many solutions and the relationship among them in a class of games where there exists a big boss in the context of information trading. They required a monotonicity for the characteristic function and did not have to take into account any coalition structures. These are the major differences with our patent licensing games.

[18]Watanabe and Tauman (2003) proposed a sophisticated definition of the characteristic function under a subtle mixture of conflict and cooperation. Tauman and Watanabe (2007) gave a simpler interpretation to it. Their characteristic function satisfies the super-additivity.

4.2 The incumbent patent holder

In this paper, we considered the patent licensing problem with an external patent holder. If the patent holder is also a producer, he is called an incumbent patent holder. Wang (1998) showed that licensing by means of a per-unit royalty is better than that by means of a fixed license fee for the incumbent patent holder in a Cournot duopoly market. Kamien and Tauman (2002) extended his model to a Cournot oligopoly market. With a general demand function and convex cost, Ino and Kawamori (2009) examined whether or not a cost-reducing innovation is profitable for the incumbent patent holder in a large oligopolistic market, and showed that a partial-monopoly market, in which the incumbent patent holder chooses his output as a price maker while the other firms produce as price takers, arise when he does not license his (non-drastic) patented technology. It is left for a future research to study bargaining outcomes in the case of an incumbent patent holder.

Appendix

Proof of Lemma 1

Lemma 1. *Let $t \equiv \lim_{n \to \infty} t_n$. In the Cournot market, the following four statements hold: (a) If $t \leq K$, then $\lim_{n \to \infty} t_n W(t_n) = t \cdot \varepsilon Q(c)/K$. (b) If $t > K$, then $\lim_{n \to \infty} t_n W(t_n) = (c - \varepsilon)Q(p)/(t\eta(p) - 1)$, where $p = p(t)$. (c) If t_n diverges, then $\lim_{n \to \infty} t_n W(t_n) = 0$. (d) Regardless of whether t_n converges or diverges, $\lim_{n \to \infty} t_n L(n - t_n) = 0$.*

Proof. (a) We first show that, for each t_n such that $t \leq K$, $\lim_{n \to \infty} p(t_n) = c$. As a general property, for sufficiently large n, the Cournot equilibrium price $p = p(t_n)$ satisfies

$$n(p - c) = \frac{p}{\eta(p)} - t_n \varepsilon \text{ if } t_n \leq K, \tag{5}$$

where t_n is the number of licensees. (See, e.g., KOT.) As noted in Subsection 2.2, for any t_n with $0 \leq t_n \leq n$, the Cournot equilibrium price $p = p(t_n)$ decreases in t_n and $p(K) = c$, so $c \leq p(t_n)$ if $0 \leq t_n \leq K$. By A2, $\eta(p)$ is non-decreasing in p. Thus, $\eta(c) \leq \eta(p(t_n))$ whenever $0 \leq t_n \leq K$. Accordingly, by (5),

$$n(p - c) = \frac{p}{\eta(p)} - t_n \varepsilon \leq \frac{p}{\eta(c)} - t_n \varepsilon,$$

i.e.,

$$c \leq p(t_n) \leq \left(c - \frac{t_n \varepsilon}{n}\right) \bigg/ \left(1 - \frac{1}{n\eta(c)}\right) \text{ if } t_n \leq K.$$

Confirm that

$$\lim_{n \to \infty} \left(c - \frac{t_n \varepsilon}{n}\right) \bigg/ \left(1 - \frac{1}{n\eta(c)}\right) = c.$$

Therefore, for each t_n with $t \leq K$,

$$\lim_{n \to \infty} p(t_n) = c, \tag{6}$$

by the squeeze theorem.

Let us give the proof of Lemma 1 (a). When $t_n \leq K$, by $\eta(p) = -pQ'/Q$ and $Q' = 1/P'$ (i.e., $dQ/dp = 1/(dP/dq)$), (5) is rewritten as $np + P'Q(p) = nc - t_n\varepsilon$. Thus, by (1),

$$
\begin{aligned}
t_n W(t_n) &= -\frac{t_n(p - c + \varepsilon)^2}{P'} = \frac{t_n Q(p)(p - c + \varepsilon)^2}{n(p - c) + t_n\varepsilon} \\
&= \frac{t_n Q(p)(p - c)^2}{n(p - c) + t_n\varepsilon} + \frac{2t_n Q(p)(p - c)\varepsilon}{n(p - c) + t_n\varepsilon} + \frac{t_n Q(p)\varepsilon^2}{n(p - c) + t_n\varepsilon},
\end{aligned}
$$

where $p = p(t_n)$ is the Cournot equilibrium price. By (6), $\lim_{n \to \infty} p(t_n) = c$. Note that, by (5),

$$\lim_{n \to \infty} n(p - c) = \lim_{n \to \infty} \left(\frac{p}{\eta(p)} - t_n\varepsilon \right) = \frac{c}{\eta(c)} - t\varepsilon = \varepsilon(K - t).$$

Thus, by $0 < \varepsilon < \infty$,

$$\lim_{n \to \infty} t_n W(t_n) = \lim_{n \to \infty} \frac{t_n Q(p)\varepsilon^2}{n(p - c) + t_n\varepsilon} = t \cdot \frac{Q(c)\varepsilon}{K}.$$

(b) Let n be such that, for all $n' \geq n$, $t_{n'} = t$. Because the Cournot equilibrium price $p(t_n)$ decreases in t_n and $p(K) = c$, $p(t_n) < c$ if $t_n > K$. Then, only t_n firms produce in the market and $n - t_n$ firms exit the market. As noted in the proof of Lemma 3 (a), the Cournot equilibrium price $p = p(t_n)$ satisfies

$$p = (c - \varepsilon) \Big/ \left(1 - \frac{1}{t_n \eta(p)} \right), \tag{7}$$

when $t_n > K$. (7) does not depend on n because $t_n = t(> K)$ and only t firms produce in the market. Thus,

$$\lim_{n \to \infty} t_n W(t_n) = \lim_{n \to \infty} t_n \cdot \frac{(p - c + \varepsilon)Q(p)}{t_n} = (p - c + \varepsilon)Q(p) = \frac{(c - \varepsilon)Q(p)}{t\eta(p) - 1},$$

where $p = p(t_n) = p(t)$.

(c) We show that $\lim_{n \to \infty} p(t_n) = c - \varepsilon$ if t_n diverges. Let $t_n > K$ and $p = p(t_n)$. As shown in Lemma 1 (b), $p(t_n) < c$ if $t_n > K$, so

$$1 - \frac{1}{t_n \eta(p)} \leq 1 - \frac{1}{t_n \eta(c)} = 1 - \frac{\varepsilon K}{t_n c} \leq 1, \tag{8}$$

20

because $\eta(c) \geq \eta(p)$, $t_n \geq 1$ and $K = c/\varepsilon\eta(c) > 1$. By a general property of the Cournot equilibrium price that (7) holds if $t_n > K$ and (8), $p(t_n) \geq c - \varepsilon$. Furthermore, $\eta(p) \geq \eta(c - \varepsilon)$ because $\eta(p)$ is non-decreasing in p, so

$$p = (c - \varepsilon) \bigg/ \left(1 - \frac{1}{t_n\eta(p)}\right) \leq (c - \varepsilon) \bigg/ \left(1 - \frac{1}{t_n\eta(c - \varepsilon)}\right).$$

When t_n diverges to infinity,

$$\lim_{n\to\infty} (c - \varepsilon) \bigg/ \left(1 - \frac{1}{t_n\eta(c - \varepsilon)}\right) = c - \varepsilon,$$

which implies $\lim_{n\to\infty} p(t_n) = c - \varepsilon$ by the squeeze theorem. Then,

$$\lim_{n\to\infty} t_n W(t_n) = \lim_{n\to\infty} t_n \cdot \frac{(p - c + \varepsilon)Q(p)}{t_n} = (c - \varepsilon - c + \varepsilon)Q(c - \varepsilon) = 0.$$

(d) Consider the total Cournot equilibrium profit of t_n non-licensees. Then, there are $n - t_n$ licensees. If $n - t_n > K$, $t_n L(n - t_n) = 0$, by (3). Hence, when $n - t_n$ diverges or converges to more than K, $\lim_{n\to\infty} t_n L(n - t_n) = 0$. When $\lim_{n\to\infty}(n - t_n) \leq K$, for sufficiently large n, (5) is rewritten as

$$n(p - c) = \frac{p}{\eta(p)} - (n - t_n)\varepsilon, \tag{9}$$

where $p = p(n - t_n)$ and $n - t_n$ is the number of licensees. By $\eta(p) = -pQ'/Q$ and $Q' = 1/P'$ (i.e., $\mathrm{d}Q/\mathrm{d}p = 1/(\mathrm{d}P/\mathrm{d}q)$), (9) is rewritten as

$$np + P'Q(p) = nc - (n - t_n)\varepsilon, \tag{10}$$

If $n - t_n \leq K$, by (2), (9) and (10),

$$\begin{aligned}
t_n L(n - t_n) &= -\frac{t_n(p - c)^2}{P'} = \frac{t_n Q(p)(p - c)^2}{n(p - c) + (n - t_n)\varepsilon} \\
&= \frac{t_n\eta(p)Q(p)}{p} \cdot \left(\frac{p - \eta(p)(n - t_n)\varepsilon}{n\eta(p)}\right)^2 \\
&= \frac{t_n\eta(p)Q(p)}{n^2 p} \cdot \left(\left(\frac{p}{\eta(p)}\right)^2 - 2(n - t_n)\left(\frac{p}{\eta(p)}\right)\varepsilon + (n - t_n)^2\varepsilon^2\right) \\
&\leq \frac{t_n}{n} \cdot \frac{\eta(p)Q(p)}{p} \cdot \left(\frac{1}{n}\left(\frac{p}{\eta(p)}\right)^2 - 2\left(1 - \frac{t_n}{n}\right)\left(\frac{p}{\eta(p)}\right)\varepsilon + \frac{K^2\varepsilon^2}{n}\right),
\end{aligned}$$

where $p = p(n - t_n)$ is the Cournot equilibrium price. By (6), $\lim_{n\to\infty} p(n - t_n) = c$, where $n - t_n$ is the number of licensees. Note that $\lim_{n\to\infty} t_n/n = 1$ because $(n - K)/n \leq t_n/n \leq 1$. Thus, by $0 < \varepsilon < c$,

$$\lim_{n\to\infty} \left[\frac{t_n}{n} \cdot \frac{\eta(p)Q(p)}{p} \cdot \left(\frac{1}{n}\left(\frac{p}{\eta(p)}\right)^2 - 2\left(1 - \frac{t_n}{n}\right)\left(\frac{p}{\eta(p)}\right)\varepsilon + \frac{K^2\varepsilon^2}{n}\right)\right] = 0.$$

Because $0 \leq t_n L(n - t_n)$, $\lim_{n\to\infty} t_n L(n - t_n) = 0$, by the squeeze theorem. $\qquad\square$

Proof of Lemma 2

Lemma 2. *Let s'_n be such that $s'_n W(s'_n) \geq s_n W(s_n)$ for $s_n = 1, \ldots, n$. Then, $\lim_{n\to\infty} s'_n = K$.*

Proof. For any t_n such that $\lim_{n\to\infty} t_n \leq K$, $\lim_{n\to\infty} t_n W(t_n) = (\lim_{n\to\infty} t_n) \cdot Q(c)\varepsilon/K$, by Lemma 1 (a). Thus,

$$\left(\lim_{n\to\infty} t_n \right) \cdot \frac{Q(c)\varepsilon}{K} \leq \varepsilon Q(c) = \lim_{n\to\infty} KW(K).$$

For each t_n with $t_n \geq K$, $t_n W(t_n)$ decreases in t_n, as shown at the beginning of the proof of Lemma 3, and if t_n diverges or converges to more than K, $\varepsilon Q(c) \neq \lim_{n\to\infty} t_n W(t_n)$ by Lemmas 1 (b) and (c). Therefore, the number of licensees that maximizes $t_n W(t_n)$ becomes K as n tends to infinity. $\square$

References

Aumann, R. J., Drèze, J. H., 1974. Cooperative games with coalition structures. International Journal of Game Theory 3, 217–237.

Aumann, R. J., Maschler, M., 1964. The bargaining set for cooperative games. In: Dresher, M., Shaply, L. S., Tucker, A. W. (Eds.), Advances in Game Theory. Princeton University Press, Princeton, NJ, pp. 443–476.

Davis, M., Maschler, M., 1967. Existence of stable payoff configurations for cooperative games. In: Schubik, M. (Ed.), Essays in Mathematical Economics in honor of Oskar Morgenstern. Princeton University Press, Princeton, NJ, pp. 39–52, (The abstract appeared in Bulletin of the American Mathematical Society 69, 106-108 (1963).).

Driessen, T., Muto, S., Nakayama, M., 1992. A cooperative game of information trading: The core, the nucleolus and the kernel. Mathematical Methods of Operations Research 36, 55–72.

Erutku, C., Richelle, Y., 2006. Licensing a new product with non-linear contracts. Canadian Journal of Economics 39, 923–947.

Erutku, C., Richelle, Y., 2007. Optimal licensing contracts and the value of a patent. Journal of Economics and Management Strategy 16, 407–436.

Ino, H., Kawamori, T., 2009. Oligopoly with a large number of competitors: Asymmetric limit result. Economic Theory 39, 331–352.

Jelnov, A., Tauman, Y., 2009. The private value of a patent: A cooperative approach. Mathematical Social Sciences 58, 84–97.

Kamien, M. I., Oren, S. S., Tauman, Y., 1992. Optimal licensing of cost-reducing innovation. Journal of Mathematical Economics 21, 483–508.

Kamien, M. I., Tauman, Y., 1984. The private value of a patent: A game theoretic analysis. Journal of Economics Supp. 4, 93–118.

Kamien, M. I., Tauman, Y., 1986. Fees versus royalties and the private value of a patent. Quarterly Journal of Economics 101, 471–491.

Kamien, M. I., Tauman, Y., 2002. Patent licensing: The inside story. The Manchester School 70, 7–15.

Kamien, M. I., Tauman, Y., Zhang, I., 1988. Optimal license fees for a new product. Mathematical Social Sciences 16, 77–106.

Kats, A., Tauman, Y., 1985. Coalition production economies with divisible and indivisible inputs: Asymptotic results. Journal of Mathematical Economics 14, 19–42.

Katz, M. L., Shapiro, C., 1985. On the licensing of innovation. Rand Journal of Economics 16, 504–520.

Katz, M. L., Shapiro, C., 1986. How to license intangible property. Quarterly Journal of Economics 101, 567–589.

Muto, S., 1987. Possibility of relicensing and patent protection. European Economic Review 31, 927–945.

Muto, S., 1990. Resale-proofness and coalition-proof Nash equilibria. Games and Economic Behavior 2, 337–361.

Muto, S., 1993. On licensing policies in Bertrand competition. Games and Economic Behavior 5, 257–267.

Muto, S., Nakayama, M., Potters, J., Tijs, S., 1989. On big boss games. The Economic Studies Quarterly 39, 303–321.

Nakayama, M., Quintas, L., 1991. Stable payoffs in resale-proof trades of information. Games and Economic Behavior 3, 339–349.

Peleg, B., 1967. Existence theorem for the bargaining set $\mathcal{M}_1^i$. In: Schubik, M. (Ed.), Essays in Mathematical Economics in honor of Oskar Morgenstern. Princeton University Press, Princeton, NJ, pp. 53–56, (The abstract appeared in Bulletin of the American Mathematical Society 69, 109-110 (1963).).

Sen, D., 2005. Fee versus royalty reconsidered. Games and Economic Behavior 53, 141–147.

Sen, D., Tauman, Y., 2007. General licensing schemes for a cost-reducing innovation. Games and Economic Behavior 59, 163–186.

Stamatopoulos, G., Tauman, Y., 2008. Licensing of a quality-improvement innovation. Mathematical Social Sciences 56, 410–438.

Tauman, Y., Watanabe, N., 2007. The Shapley value of a patent licensing game: The asymptotic equivalence to non-cooperative results. Economic Theory 30, 135–149.

von Neumann, J., Morgenstern, O., 1944. Theory of Games and Economic Behavior, 1st Edition. Princeton University Press, Princeton, NJ, [2nd Edn. 1947], [3rd Edn. 1953].

Wang, X. H., 1998. Fee versus royalty licensing in a Cournot duopoly model. Economics Letters 60, 55–62.

Watanabe, N., Muto, S., 2008. Stable profit sharing in patent licensing: General bargaining outcomes. International Journal of Game Theory 37, 505–523.

Watanabe, N., Tauman, Y., 2003. Asymptotic properties of the Shapley value of a patent licensing game, mimeo., Kyoto University.

Wooders, M. H., Zame, W. R., 1987. Large games: Fair and stable outcomes. Journal of Economic Theory 42, 59–93.